Emaranhamento e os fundamentos da Física Estatística

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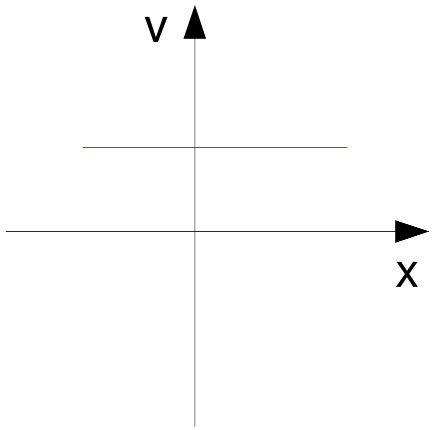
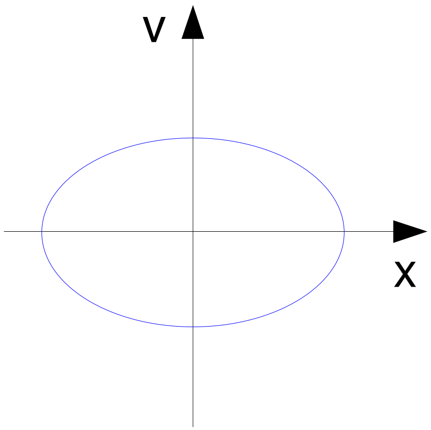
Paradigma da Física Clássica

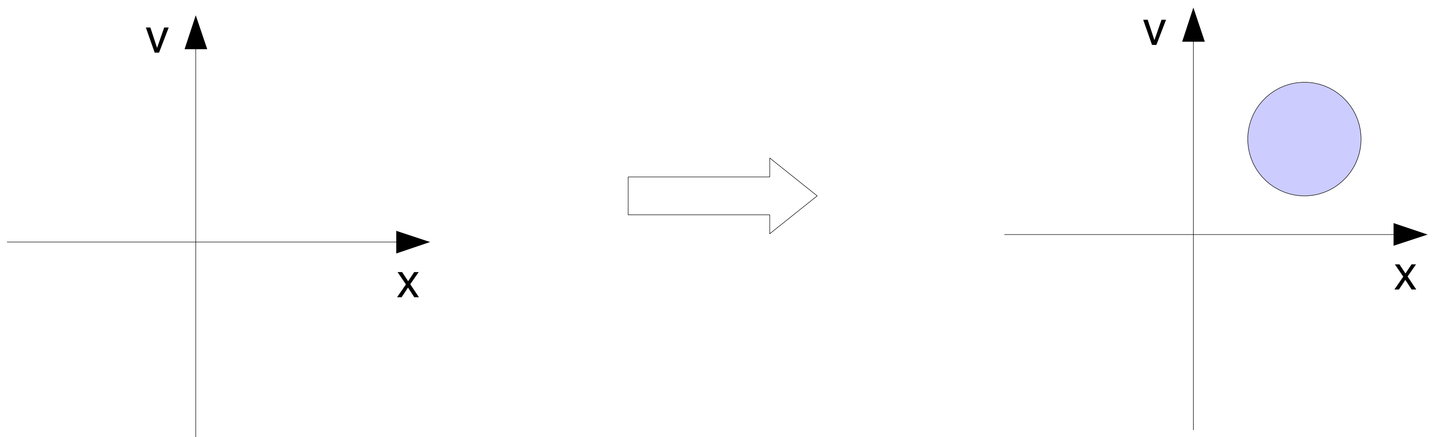
- Determinismo e reducionismo
- Uma partícula em 1D
  - Conheço as forças e {x(0), v(0)}
  - Leis de Newton
  - Obtenho x(t) e v(t)
  - Espaço de Fase





N Partículas

- Objetos Macroscopicos: Termodinâmica!
- Espaço de Fase tem 6N dimensões
- Conhecer todas as posições e velocidades
  - Complicado
- Tenho Informação Macroscópica: energia
  - Restringe os estados possíveis



“Receita” da Física Estatística

- Introduz probabilidades: ***Pr(x,v)***
  - Ignorância subjetiva sobre o estado
  - Obter propriedades fazendo medias
- Funciona, mas porque
  - O sistema está em um estado bem definido e evoluindo deterministicamente!
  - Irreversibilidade!?
  - Joel L. Lebowitz
- Qual ***Pr(x,v)*** usar?
  - Iguais a priori: Postulado? Máxima ignorância?

Cenário Quântico

- Informação macroscópica sobre o sistema: $\mathcal{H} \rightarrow \mathcal{H}_R$
- Duas possibilidades:
  - Assumo $\Omega = \frac{1}{d_R}$
  - Olho o sistema
  - $\Omega_S = Tr_B[\Omega]$

  - Escolho $|\psi\rangle$ aleatoriamente: ***Pr(ψ)***
  - $\rho = |\psi\rangle\langle\psi|$
  - $\rho_S = Tr[|\psi\rangle\langle\psi|]$
- ***Pr(ψ)*** uniforme: $\bar{\rho}_S = \Omega_S$
- Há diferença?
  - $D_1 = \|\rho_S - \bar{\rho}_S\|_1$

Concentração de Medida

- Função f: $V^n \rightarrow \mathbb{C}$
- Se não oscila muito, então é praticamente constate para n>>1

$$\text{Prob}\left\{|f-\bar{f}| \geq \epsilon\right\} \leq k_1 exp(-k_2\epsilon^2 n/\eta^2)$$

$$\eta \equiv sup_{U_1 \neq U_2} \frac{|f(U_1)-f(U_2)|}{\|U_1-U_2\|_2}$$

ARTICLES

Entanglement and the foundations of statistical mechanics

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$$\overline{D}_1 \leq \sqrt{\frac{d_S^2}{d_B}} \qquad Prob\left[D_1 \geq \epsilon + \sqrt{\frac{d_S}{d_B}}\right] \leq 2\, e^{-C\, d_B \epsilon^2}$$

- Tipicamente $\rho_S \approx \Omega_S$
  - Princípio das probabilidades iguais a priore é aparente
  - Emaranhamento é responsável pela ignorância local:
  - Resultados de concentração de medida

Phys. Rev. Lett. 96, 050403 (2006) [3 pages]

Canonical Typicality

Abstract References Citing Articles (37)

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Sheldon Goldstein[1,*], Joel L. Lebowitz[1,†], Roderich Tumulka[2,‡], and Nino Zanghì[3,§]
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It is well known that a system *S* weakly coupled to a heat bath *B* is described by the canonical ensemble when the composite *S*+*B* is described by the microcanonical ensemble corresponding to a suitable energy shell. This is true for both classical distributions on the phase space and quantum density matrices. Here we show that a much stronger statement holds for quantum systems. Even if the state of the composite corresponds to a single wave function rather than a mixture, the reduced density matrix of the system is canonical, for the overwhelming majority of wave functions in the subspace corresponding to the energy interval encompassed by the microcanonical ensemble. This clarifies, expands, and justifies remarks made by Schrödinger in 1952.

Phys. Rev. Lett. 99, 160404 (2007) [4 pages]

Typicality for Generalized Microcanonical Ensembles

Abstract | References | Citing Articles (15)

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See Also: Publisher's Note

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For a macroscopic, isolated quantum system in an unknown pure state, the expectation value of any given observable is shown to hardly deviate from the ensemble average with extremely high probability under generic equilibrium and nonequilibrium conditions. Special care is devoted to the uncontrollable microscopic details of the system state. For a subsystem weakly coupled to a large heat bath, the canonical ensemble is recovered under much more general and realistic assumptions than those implicit in the usual microcanonical description of the composite system at equilibrium.

LECTURE NOTES IN PHYSICS

J. Gemmer
M. Michel
G. Mahler

Quantum Thermodynamics

Emergence of Thermodynamic Behavior Within Composite Quantum Systems

Springer

Problemas?

- Como selecionar estados aleatoriamente com uma probabilidade (medida) uniforme (Haar)
- Geração não é eficiente: há muitos parâmetros.
- Ficar restrito a estados realizáveis/físicos
- Como escolher tal conjunto de estados?

Estados de Produto de Matrizes (MPS)

$$|\psi\rangle = \mathrm{tr}[A_{i_1}^{[1]} A_{i_2}^{[2]} ... A_{i_N}^{[N]}]|i_1 i_2 ... i_N\rangle$$

- $\boldsymbol{A_k}$ é uma matriz χ x χ : $d^N \Rightarrow N\, d\, \chi^2$
- χ é o parâmetro fundamental
- DMRG é um método variacional no conjunto de MPS
- Boa aproximação para Hamiltonianas 1-D locais e com gap
- Obedecem a lei da área para o emaranhamento
- Pode descrever qualquer estado para $\chi \sim \boldsymbol{d^N}$

Estados de Produto de Matrizes

- Liberdade de Gauge: $A \rightarrow X^{-1} A X$
- Impor $\sum A^i A^i = I$

– Útil numericamente

– A vem de unitárias dχ por dχ

$$U = \left[\begin{array}{c|c|c|c} A^1 & A^2 & A^3 & A^4 \\ \hline & & & \end{array} \right]$$

- Unitárias podem exibir concentração de medida

Tipicalidade para MPS

- Observável em L partículas: Ô
  - Constante Lipschitz para $O(U)=\langle\psi|\hat{O}|\psi\rangle$
  - Usando cálculo diferencial de matrizes
  - $Pr\left[\left|O-\overline{O}\right|\geq\epsilon\right]c_1\exp[-c_2\epsilon^2\frac{\chi(N)}{\eta^2}]$
  - $\eta\leq 4\mathrm{d}^{2\mathrm{L}+2}N\left\|\hat{O}\right\|^L$
  - $Pr\left[\left|O-\overline{O}\right|\geq\epsilon\right]c_1\exp[-c_2\epsilon^2\frac{\chi(N)}{N^2}]$
- Limite superior é exponencial em N

Resultados Numéricos

- Partículas de Spin ½
  - Ô=S_x
  - Variância

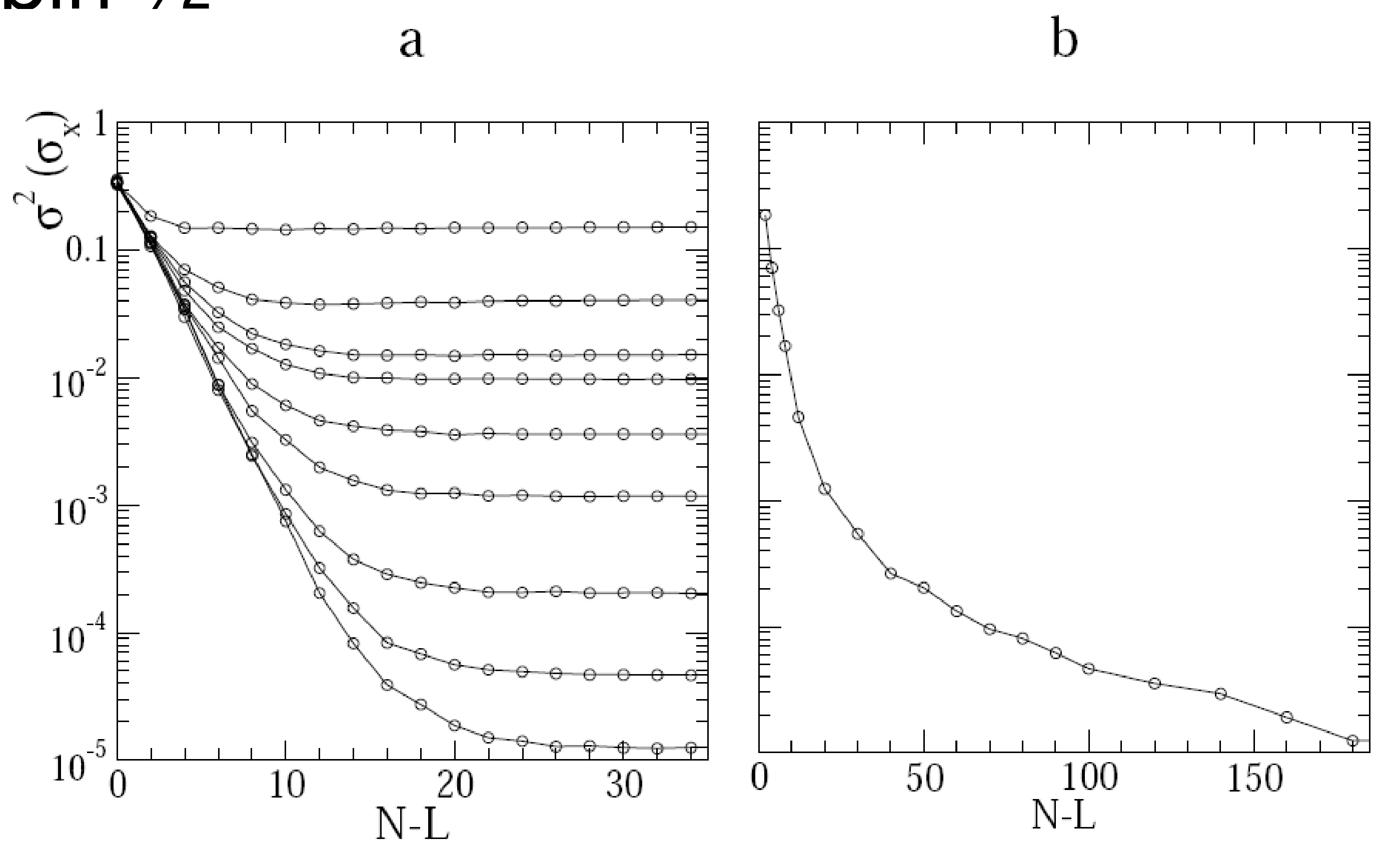

Figure 5: (a) The variance of the expectation value of σ_x ($L = 1$) increasing the size of the system and for fixed but different values of $\chi = 2, 4, 6, 8, 12, 20, 50, 100,$ and 180 (from top to bottom). (b) The variance of the expectation value of σ_x ($L = 1$) for increasing system size when the MPS dimension increases linearly with the number of particles in the bath: $\chi = N - L$.

Conclusão

- Tipicalidade não surge somente no espaço de Hilbert todo, mas também numa região pequena de estados fisicamente acessíveis
- Tipicalidade pode ter um papel num melhor entendimento dos fundamentos da Mecânica Estatísica
- MPS médio

Perspectivas

- Resultados são estáticos
  - Existem resultados dinâmicos para estados gerais
  - Reiman, Popescu e outros
- Como selecionar MPS com certas propriedades
  - White

Quantum mechanical evolution towards thermal equilibrium

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(Dated: 12 December 2008)

The circumstances under which a system reaches thermal equilibrium, and how to derive this from basic dynamical laws, has been a major question from the very beginning of thermodynamics and statistical mechanics. Despite considerable progress, it remains an open problem. Motivated by this issue, we address the more general question of equilibration. We prove, with virtually full generality, that reaching equilibrium is a universal property of quantum systems: Almost any subsystem in interaction with a large enough bath will reach an equilibrium state and remain close to it for almost all times. We also prove several general results about other aspects of thermalisation besides equilibration, for example, that the equilibrium state does not depend on the detailed micro-state of the bath.

Foundation of Statistical Mechanics under Experimentally Realistic Conditions

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(Received 19 September 2008; published 7 November 2008)

We demonstrate the equilibration of isolated macroscopic quantum systems, prepared in nonequilibrium mixed states with a significant population of many energy levels, and observed by instruments with a reasonably bound working range compared to the resolution limit. Both properties are satisfied under many, if not all, experimentally realistic conditions. At equilibrium, the predictions and limitations of statistical mechanics are recovered.

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