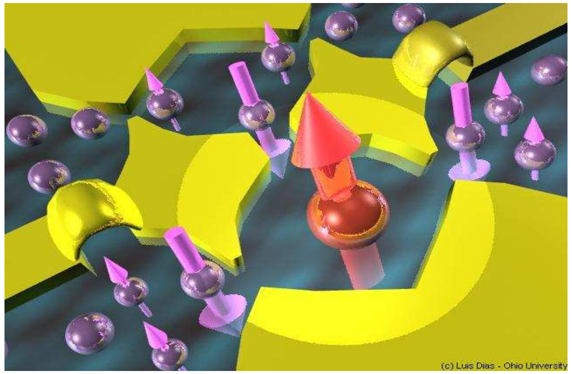
1998: O encontro de "Kondo" com "Nano"



Luis Gregório Dias da Silva

Depto. de Física dos Materiais e Mecânica - DFMT

Instituto de Física, Universidade de São Paulo - IFUSP

Mapa do Seminário

♦ **15 anos do Efeito Kondo em nanoestruturas.**

- Review: Efeito Kondo em metais com impurezas.
- 1998: “Revival of the Kondo effect”: pontos quânticos e átomos em superfícies.

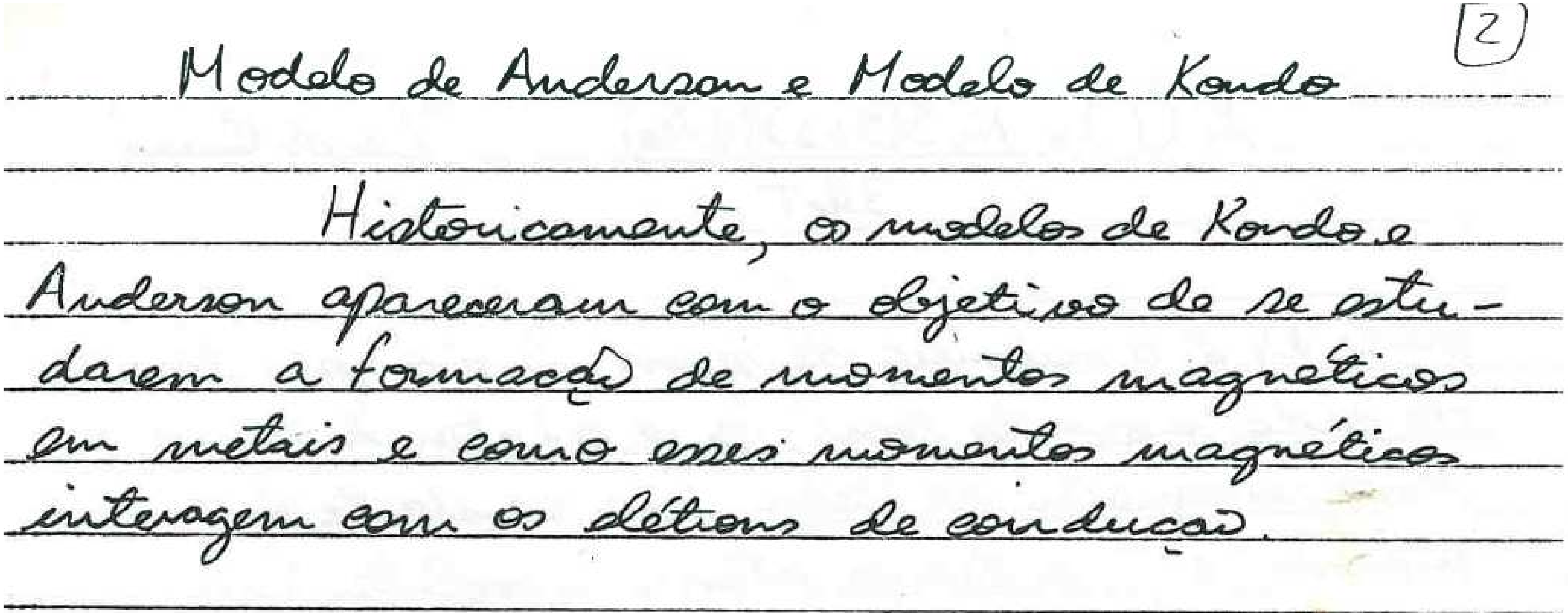
♦ **E hoje? Alguns desenvolvimentos recentes.**

- Efeito Kondo com Férmions de Dirac.
- Ação combinada com outros efeitos quânticos (graus de liberdade orbitais, efeito Zeeman, etc.): transições de fase quânticas e “filtros de spin”.

1998: “The Kondo year”

- Meu primeiro “Kontato”:

1o semestre de 1998



Modelo de Anderson e Modelo de Kondo

2

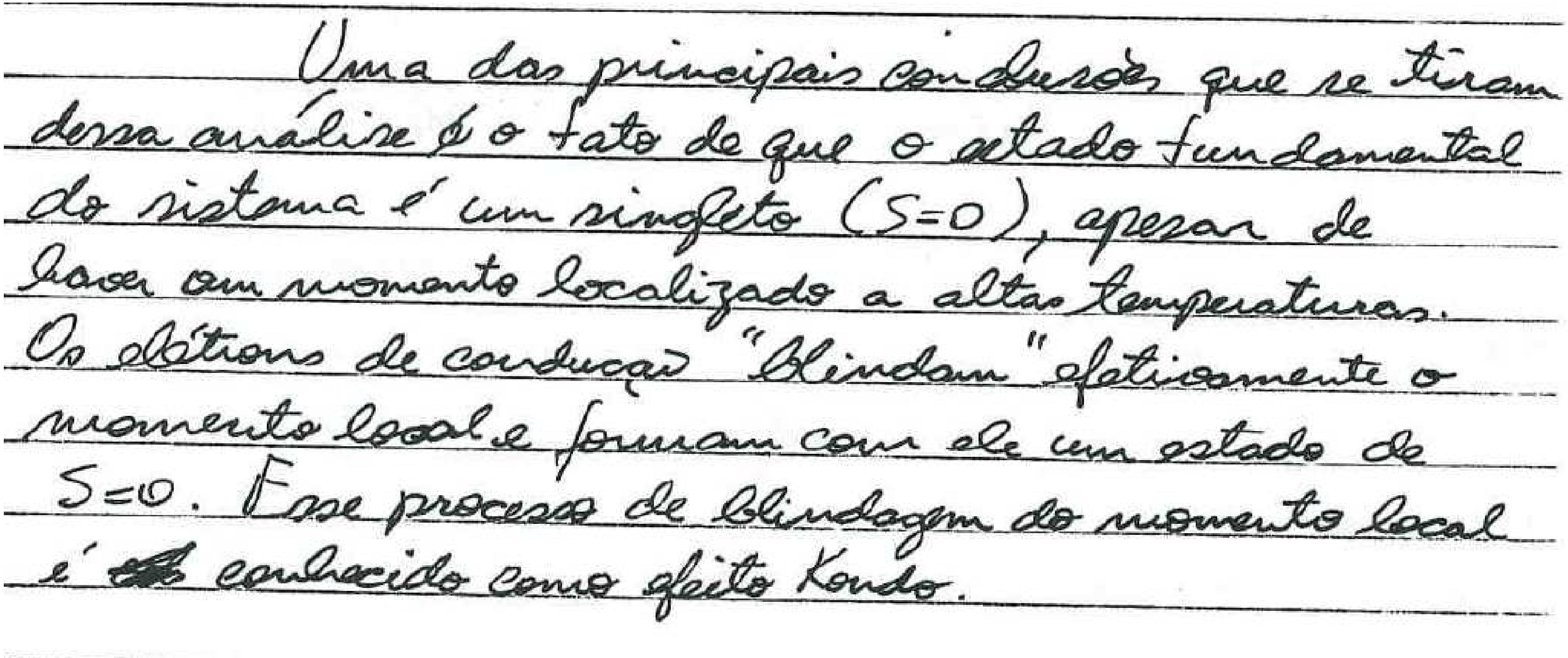
Historicamente, os modelos de Kondo e Anderson apareceram com o objetivo de se estudarem a formação de momentos magnéticos em metais e como esses momentos magnéticos interagem com os elétrons de condução.

E. Miranda, Notas de aula, curso de Muitos Corpos. (1o. semestre 1998)

1998: "The Kondo year"

- Meu primeiro "Kontato":

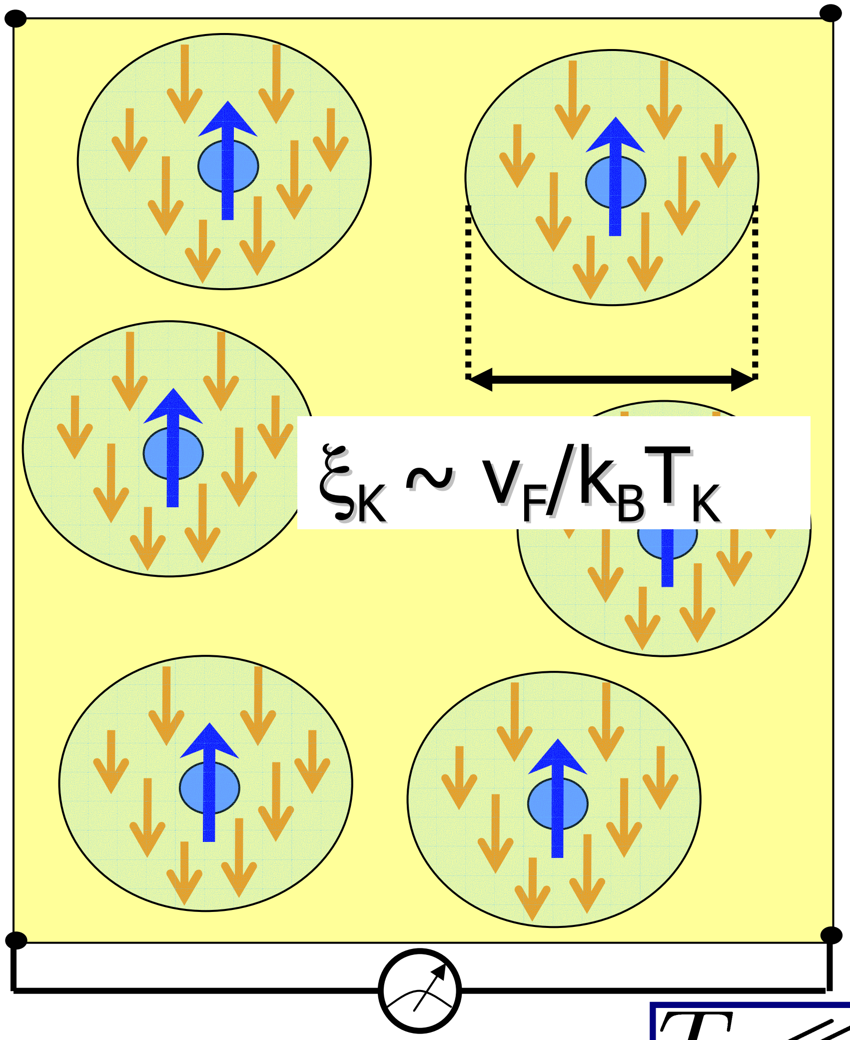
1o semestre de 1998



Uma das principais conclusões que se tiram dessa análise é o fato de que o estado fundamental do sistema é um singleto (S=0), apesar de haver um momento localizado a altas temperaturas. Os elétrons de condução "blindam" efetivamente o momento local e formam com ele um estado de S=0. Esse processo de blindagem do momento local é conhecido como efeito Kondo.

E. Miranda, Notas de aula, curso de Muitos Corpos. (1o. semestre 1998)

Kondo effect

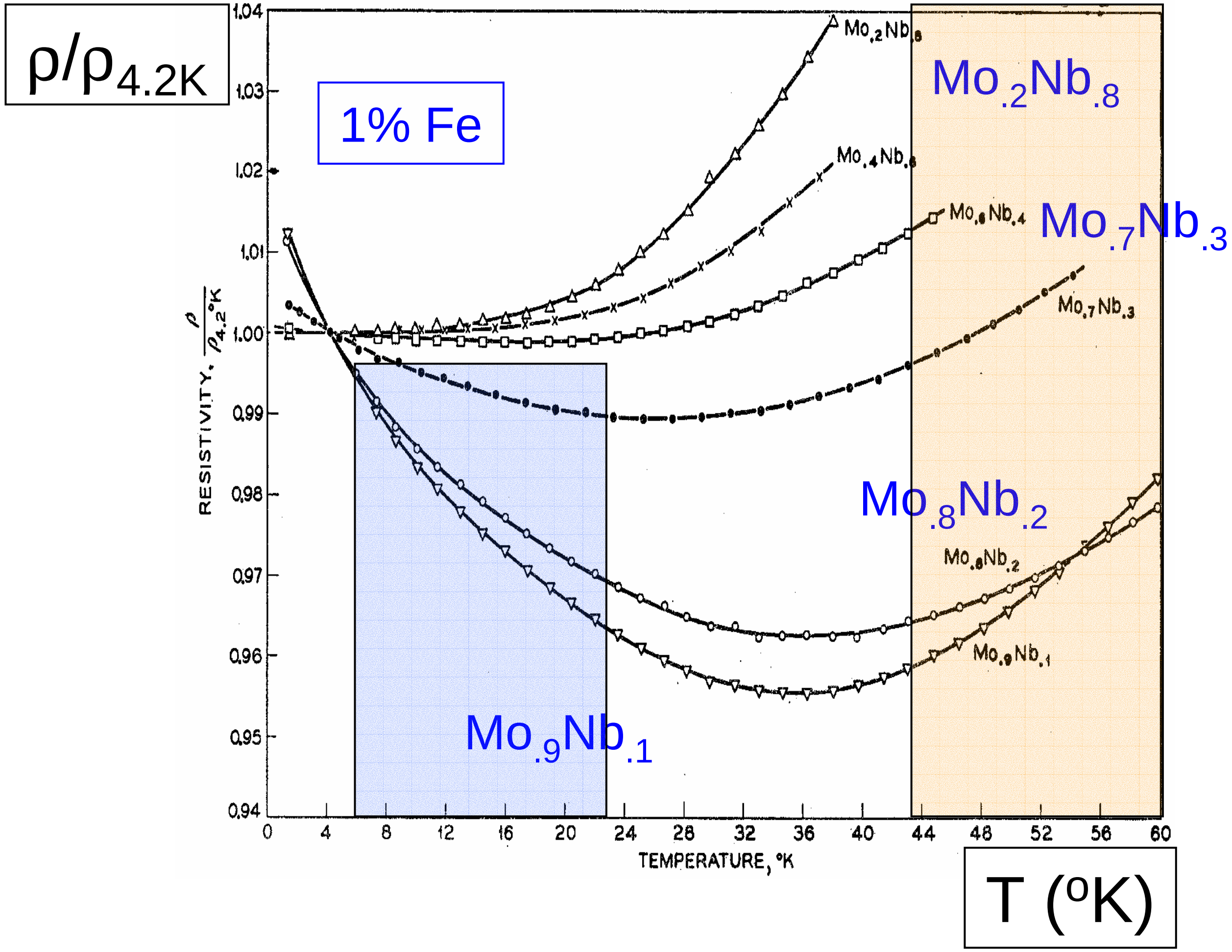


$\xi_K \sim v_F/k_B T_K$

ρ(T)

$\boxed{T \ll T_K}$

M.P. Sarachik *et al* Phys. Rev. **135** A1041 (1964).



Resistivity *increases* with *decreasing* T (Kondo effect)

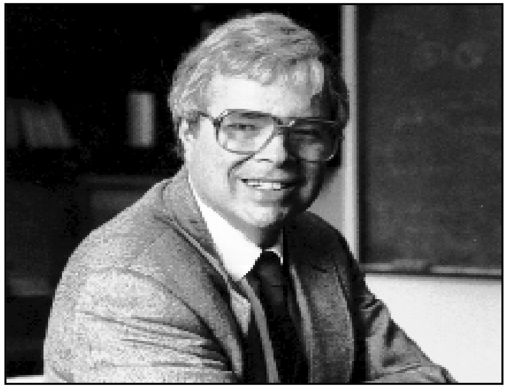
Characteristic energy scale: the *Kondo temperature* (around) T_K

Brief History of Kondo Phenomena

- First Observations: 1930's
- Kondo's Explanation for the resistance minumum: 1964
- Anderson's Poor Man's scaling: 1970
- Wilson's NRG: early 1970's (RMP 1975).
- Nozières Fermi liquid picture: 1974
- Bethe Ansatz solution: Andrei, Wiegmann: 1980

(Andrei, Furuya, Lowenstein, "Solution of the Kondo Problem" RMP 1983)

Numerical Renormalization Group (NRG):



Kenneth G. Wilson – Physics Nobel Prize in 1982
"for his theory for critical phenomena in connection
with phase transitions"

- Developed by Ken Wilson in the 70's.
- Designed to address the "Kondo problem" (magnetic impurities in metals): infrared divergencies in perturbative expansions.

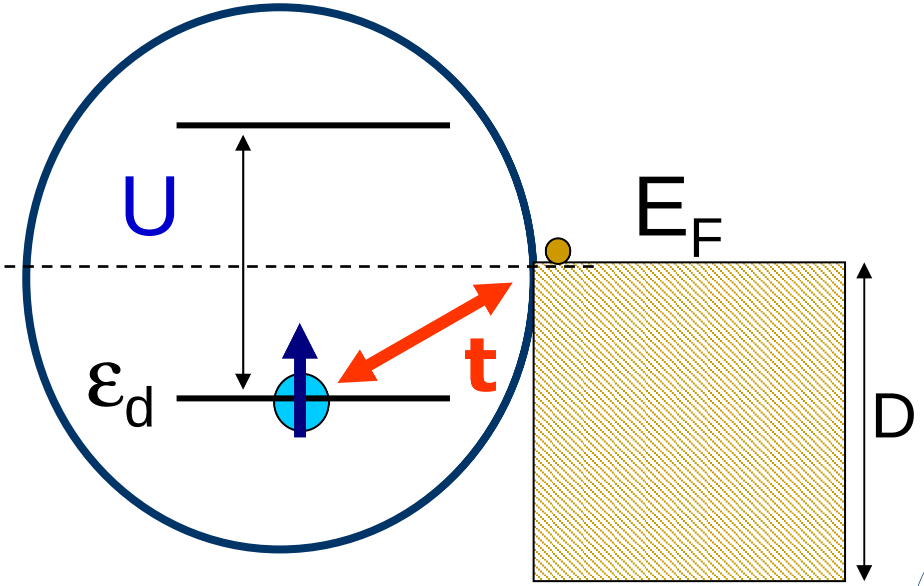
Key elements in the NRG procedure:

1. Logarithmic separation of energy scales → Mapping into a tight binding chain.
2. "Selective sampling" of the Hilbert space: keep some states, discard others.
3. Iterative numerical solution: RG "flow" unveils low-energy physics.

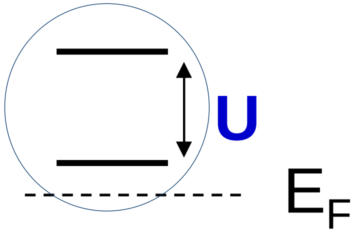
Anderson Model

Single level impurity (=atom) with local Coulomb repulsion coupled to a metal

$$\begin{aligned} H \quad = \quad & \epsilon_d \hat{n}_{d\sigma} + U \hat{n}_{d\uparrow} \hat{n}_{d\downarrow} \\ & + \sum_k \epsilon_k \hat{n}_{k\sigma} \\ & + t \sum_k c^{\dagger}_{d\sigma} c_{k\sigma} + \mathrm{h.c.} \end{aligned}$$

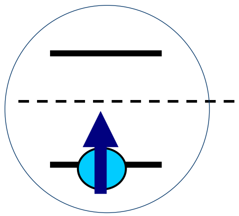


- ε$_d$: energy of the level
- U: Coulomb energy
- E$_F$: Fermi energy in the metal
- t: Hybridization with the metallic electrons
- D: metallic half-bandwidth



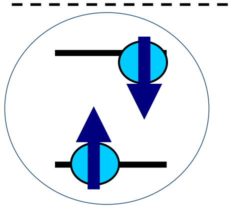
$\boxed{\langle \hat{n}_{d\sigma} \rangle \approx 0}$

$\boxed{\langle H_d \rangle \approx 0}$



$\boxed{\langle \hat{n}_{d\sigma} \rangle \approx 1}$

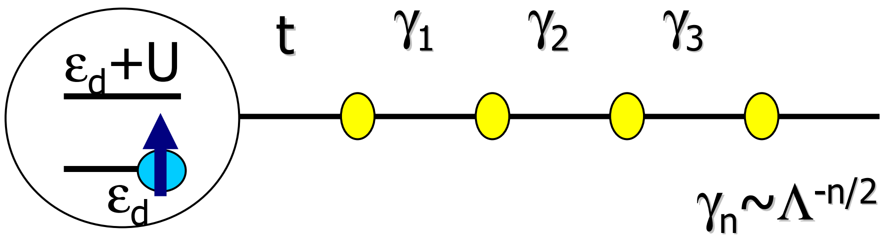
$\boxed{\langle H_d \rangle \approx \epsilon_d}$



$\boxed{\langle \hat{n}_{d\sigma} \rangle \approx 2}$

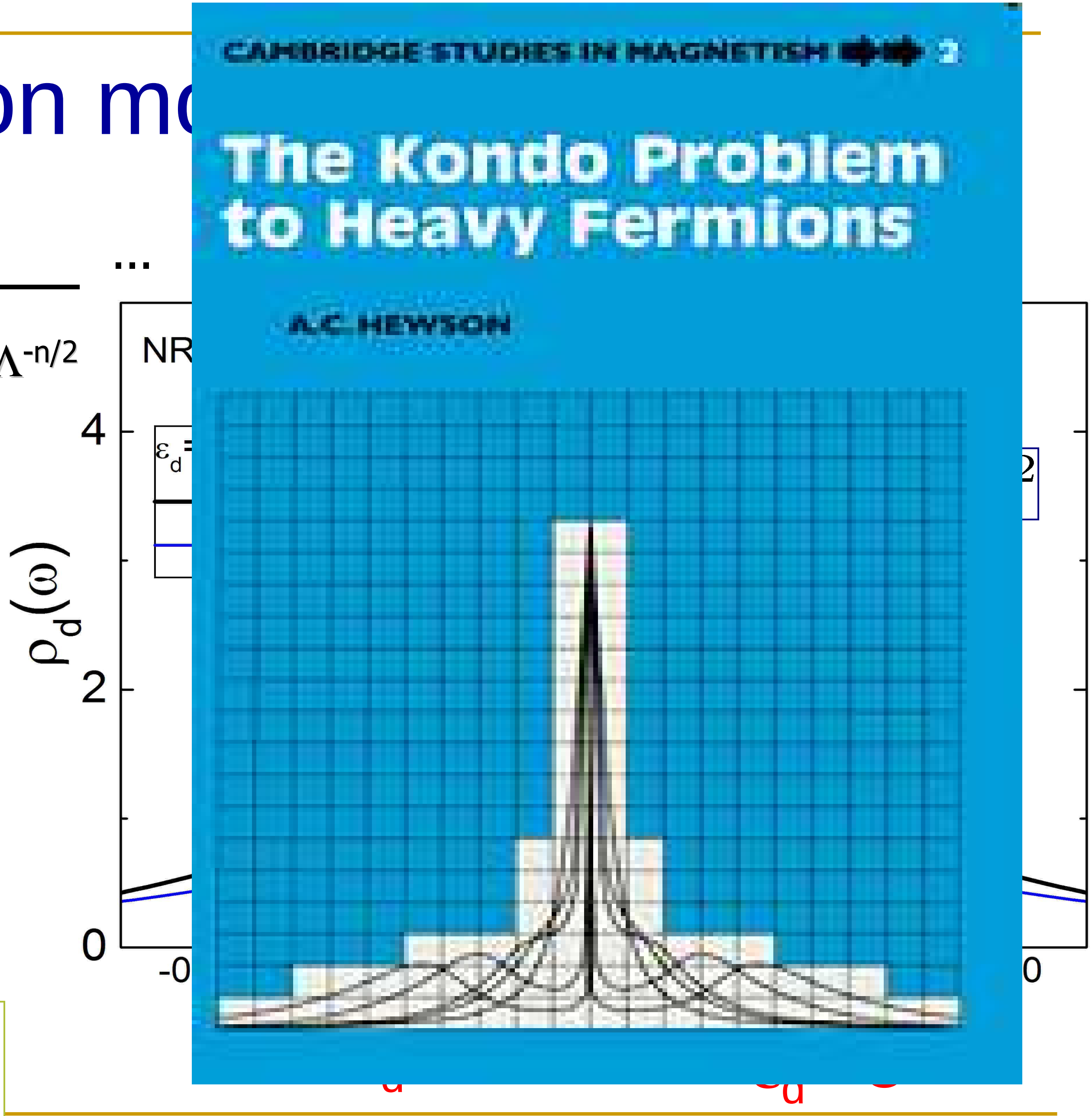
$\boxed{\langle H_d \rangle \approx 2\epsilon_d + U}$

NRG: Anderson mo



- Poles in the Green's function (GF):
  - Single-particle peaks at ε_d and ε_d+U.
  - *Many-body* peak at the Fermi energy: ***Kondo resonance*** (width ~T_K).
- NRG: good resolution at low ω

$$T_K \sim \sqrt{\frac{U\Gamma}{2}}\, e^{-\pi|\epsilon_d+U|\epsilon_d/2U\Gamma}$$



CAMBRIDGE STUDIES IN MAGNETISM 2

The Kondo Problem to Heavy Fermions

A.C. HEWSON

Brief History of Kondo Phenomena

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- Nozières Fermi liquid picture: 1974
- Bethe Ansatz solution: Andrei, Wiegmann: 1980

(Andrei, Furuya, Lowenstein, "Solution of the Kondo Problem" RMP 1983)

So, what's new about it?

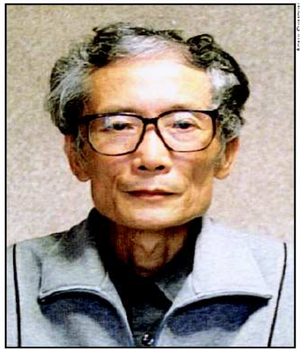
Kondo correlations observed in many different set ups:

- Transport in *quantum dots*, quantum wires, break junctions, etc.
- STM measurements of magnetic structures on metallic surfaces (e.g., single atoms, molecules. "Quantum mirage").

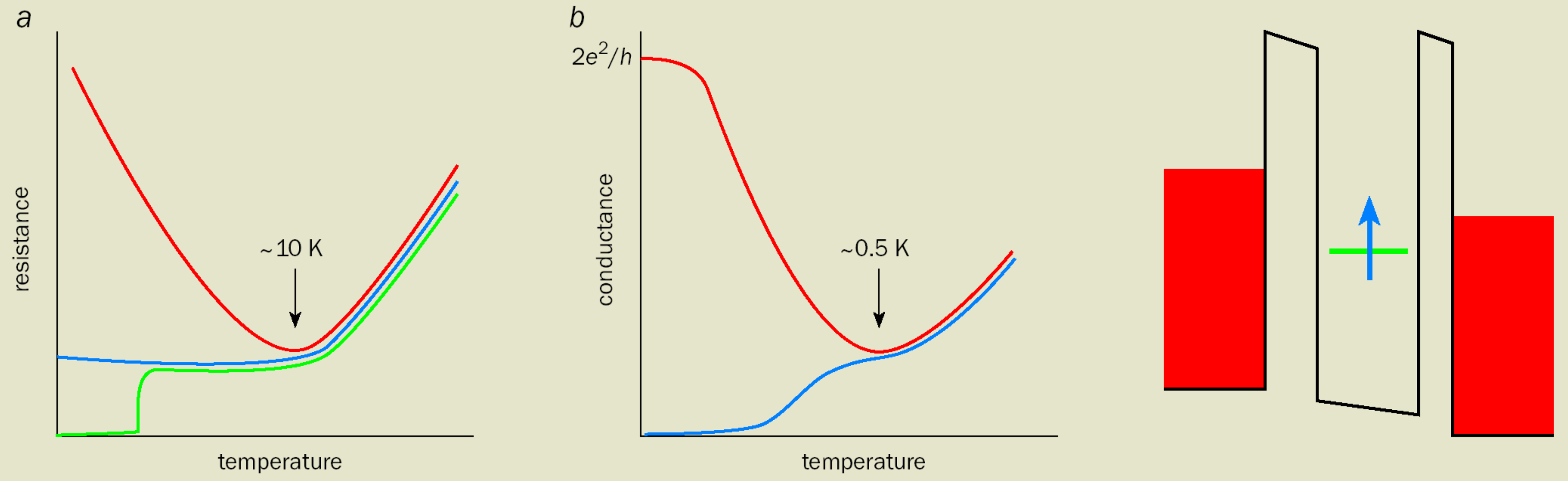
Kondo Effect in Quantum Dots

Revival of the Kondo effect

Leo Kouwenhoven and Leonid Glazman



1 The Kondo effect in metals and in quantum dots

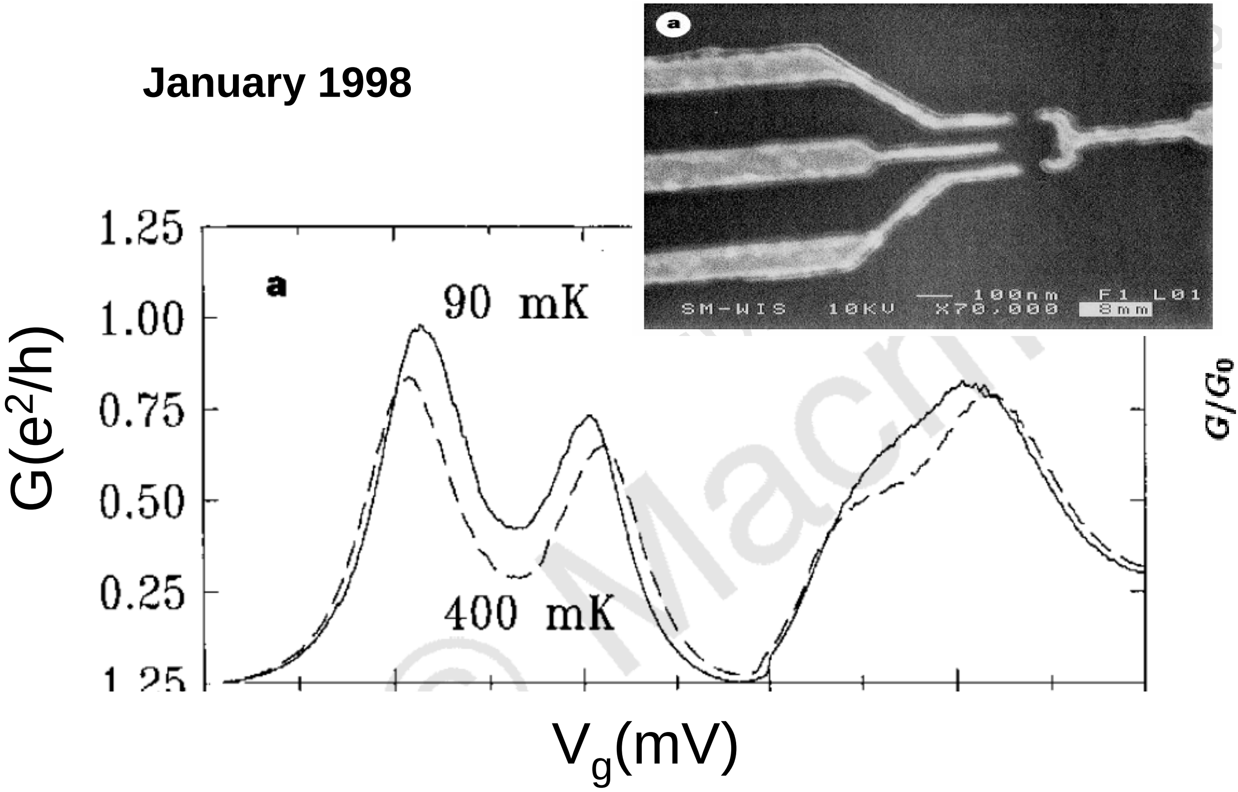


Kowenhoven and Glazman *Physics World* – Jan. 2001.

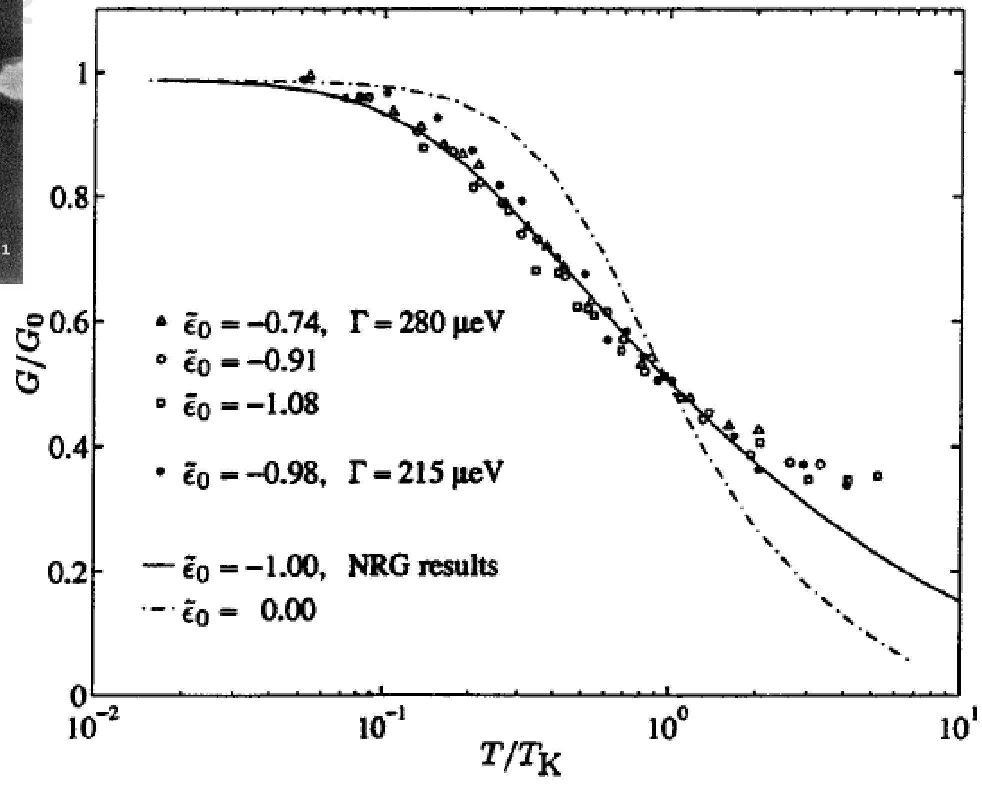
1998: “The Kondo year”

- Zero-bias conductance in semiconductor quantum dots:

January 1998



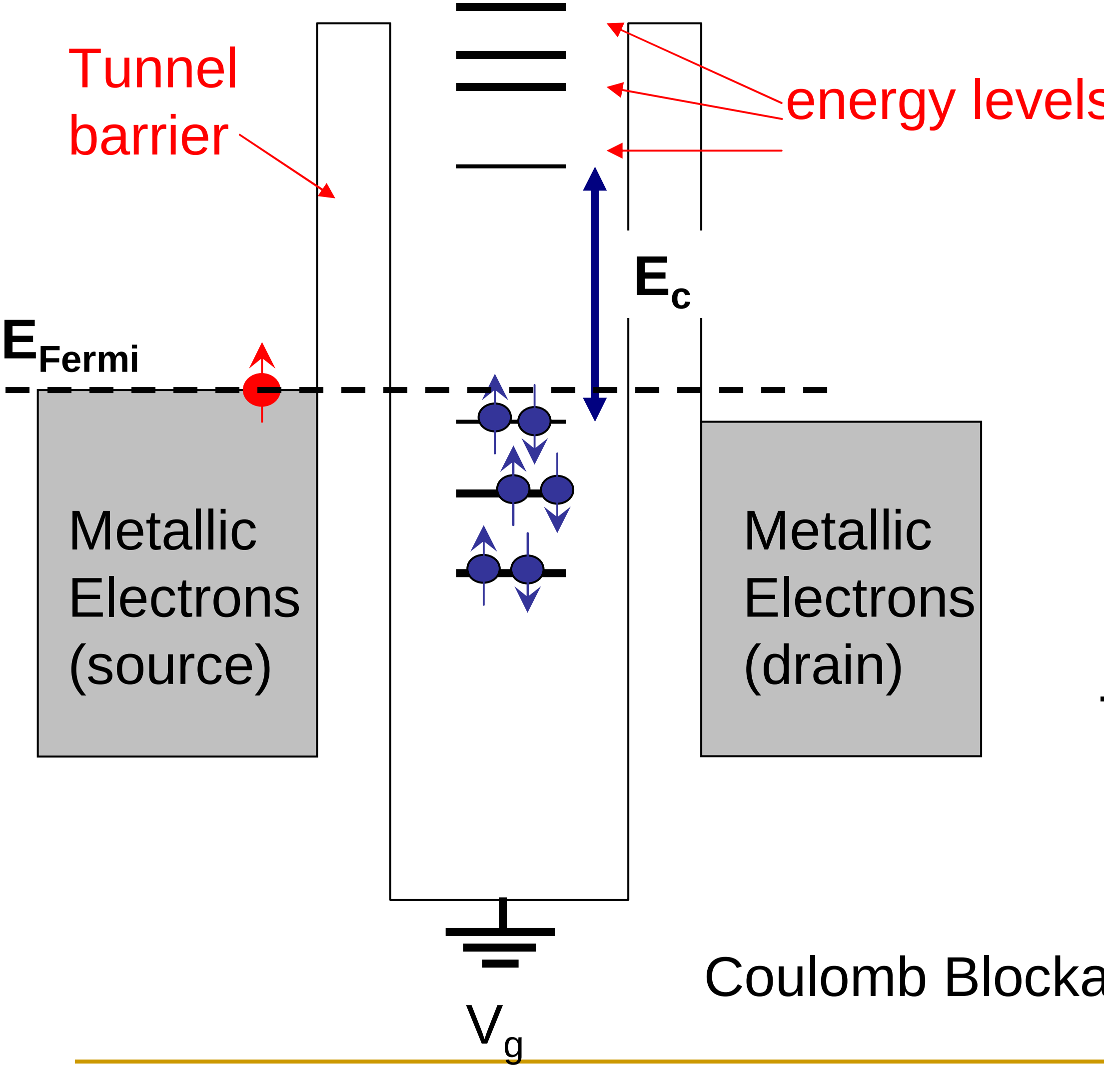
July 1998



D. Goldhaber-Gordon et al. *Nature* **391** 156 (1998)

D. Goldhaber-Gordon et al. *PRL* **81** 5225 (1998)

Coulomb Blockade in Quantum Dots



Tunnel barrier

energy levels

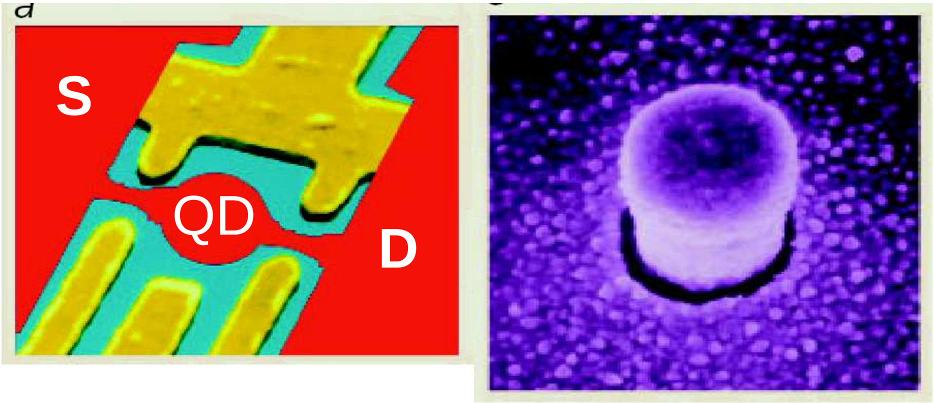
E_c

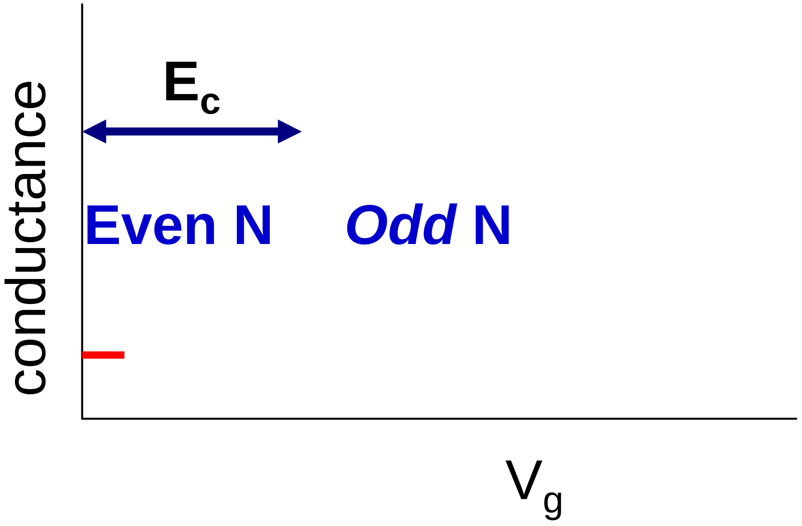
E_{Fermi}

Metallic Electrons (source)

Metallic Electrons (drain)

V_g





E_c

conductance

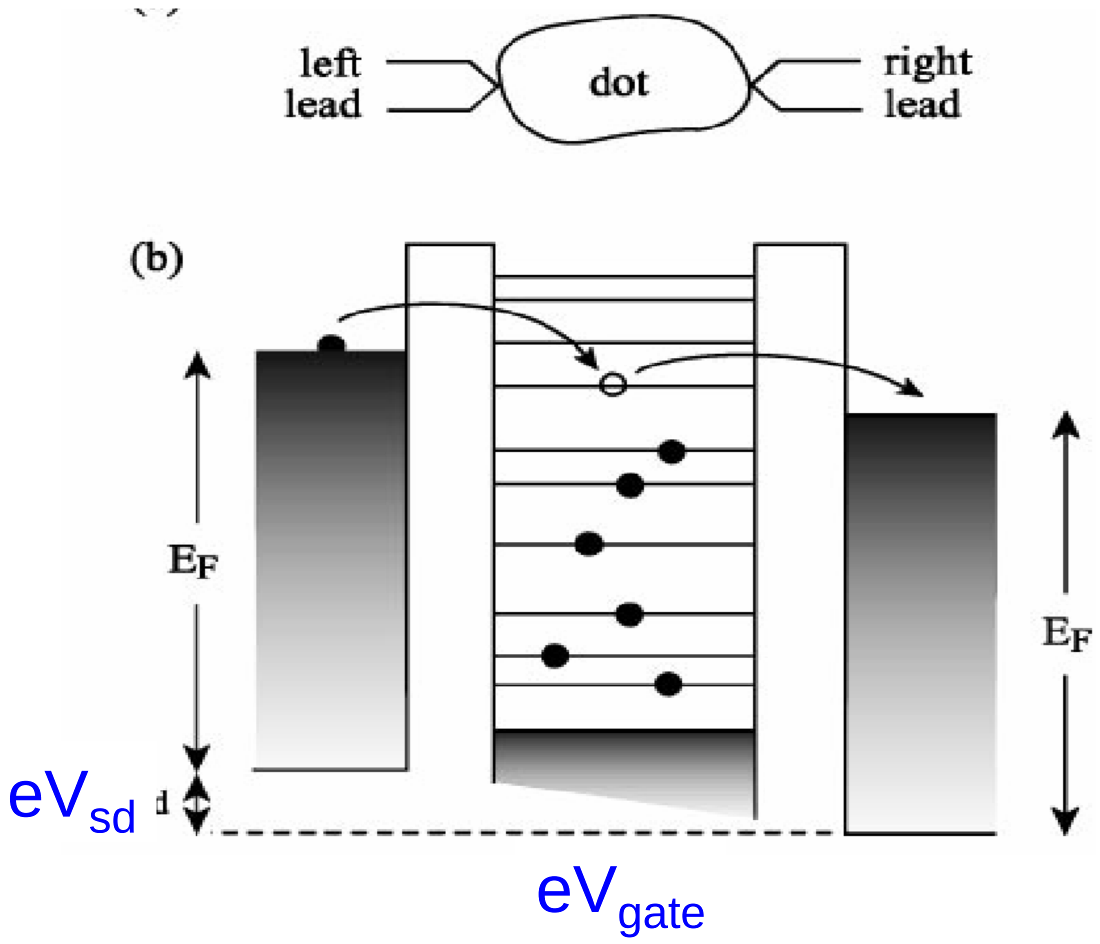
Even N ***Odd* N**

V_g

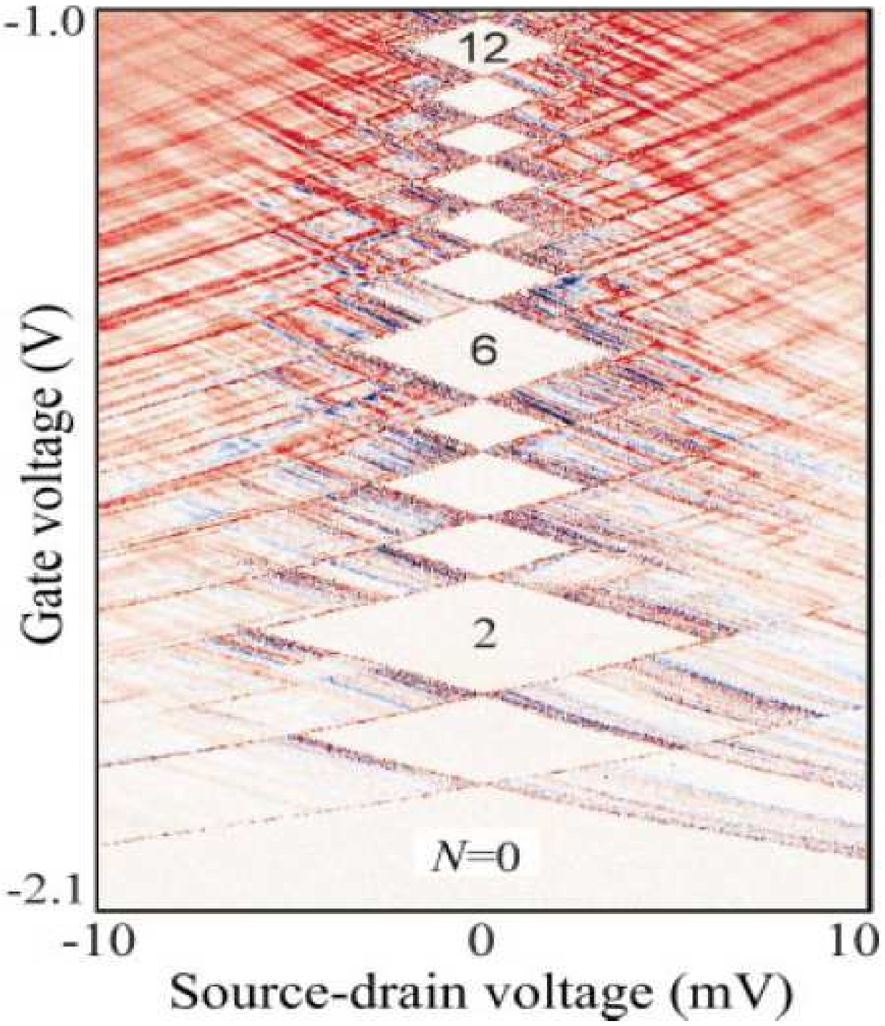
Coulomb Blockade in Quantum Dots

“Coulomb Diamonds” (Stability Diagram)

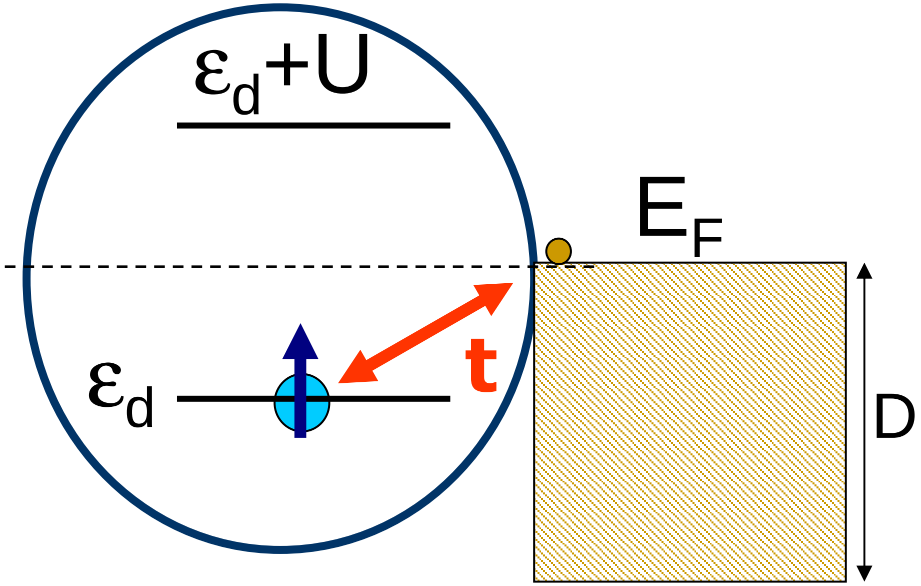
L. P. Kouwenhoven et al. *Science* **278** 1788 (1996).



Coulomb Blockade in Quantum Dots



Anderson model for quantum dots



$$
\begin{aligned}
H &= \epsilon_d \hat{n}_{d\sigma} + U \hat{n}_{d\uparrow} \hat{n}_{d\downarrow} \\
&+ \sum_k \epsilon_k \hat{n}_{k\sigma} \\
&+ t \sum_k c^\dagger_{d\sigma} c_{k\sigma} + \mathrm{h.c.}
\end{aligned}
$$

with

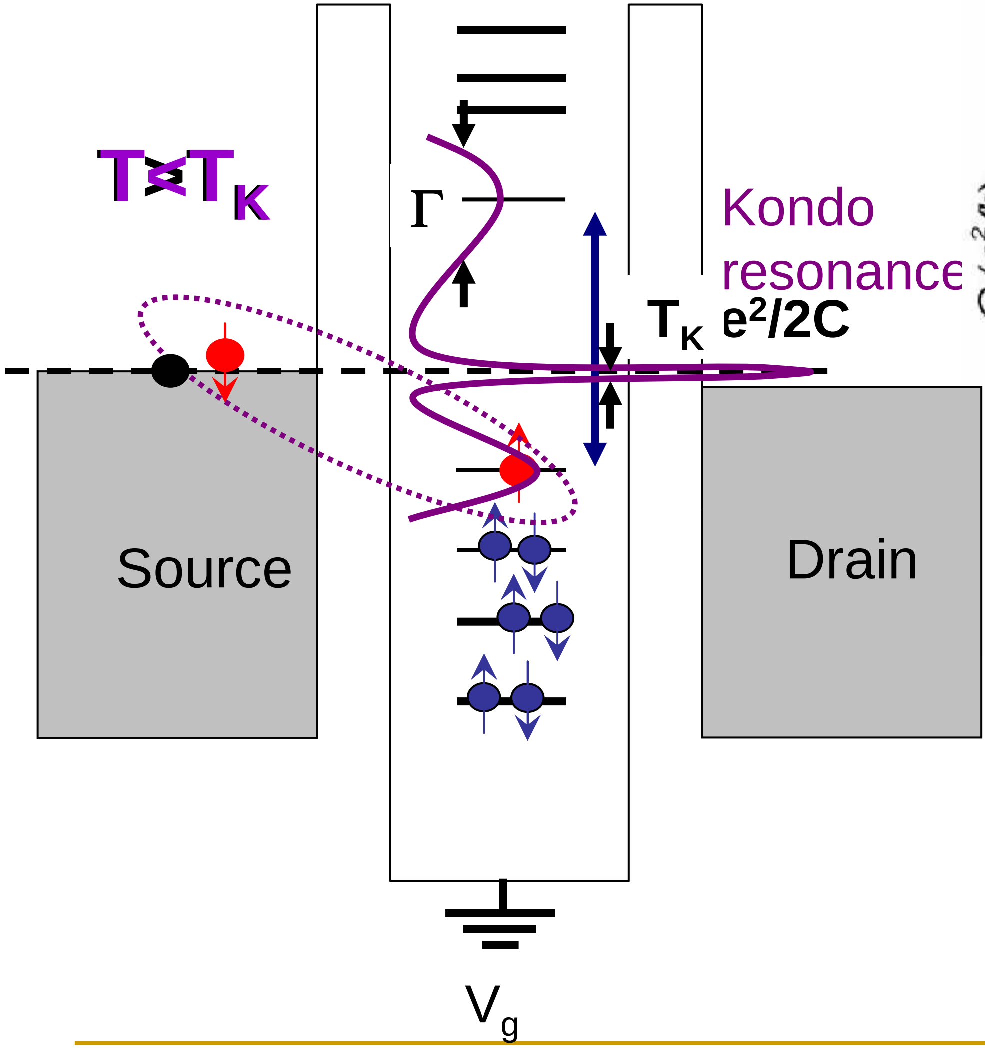
$$
\begin{aligned}
\hat{n}_{d\sigma} &= c^\dagger_{d\sigma} c_{d\sigma} \\
\hat{n}_{k\sigma} &= c^\dagger_{k\sigma} c_{k\sigma}
\end{aligned}
$$

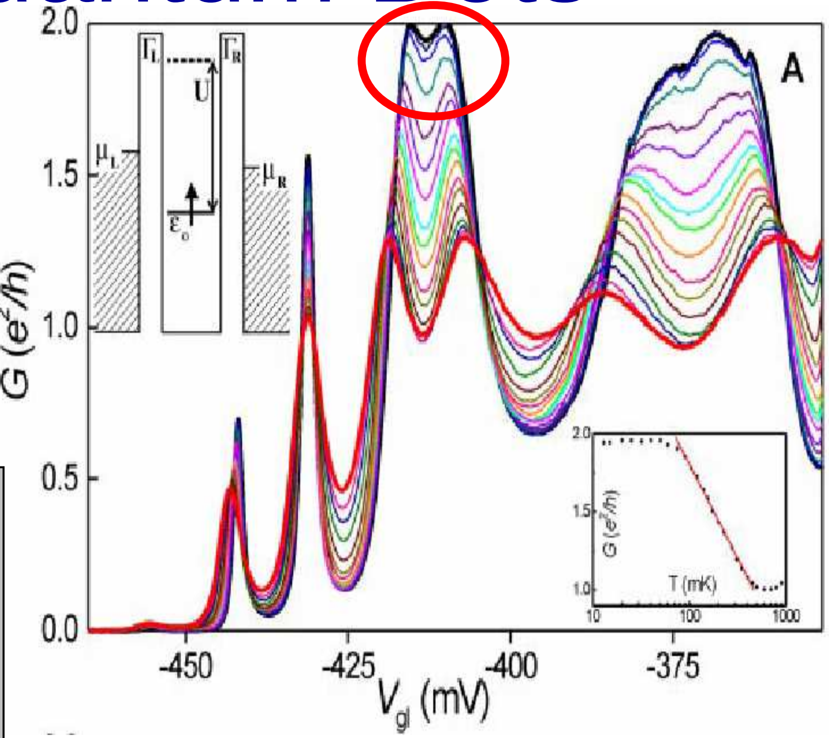
- ε_d : energy of the level
- U: Coulomb energy
- E_F: Fermi energy in the metal
- t: Hybridization w/ metal
- D: metallic half-bandwidth

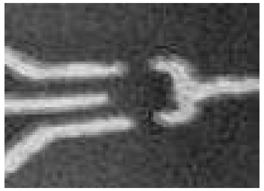
“Quantum dot language”

- ε_d : position of the level (V_g)
- U: Charging energy
- E_F: Fermi energy in the leads
- t: dot-lead tunneling
- D: bandwidth

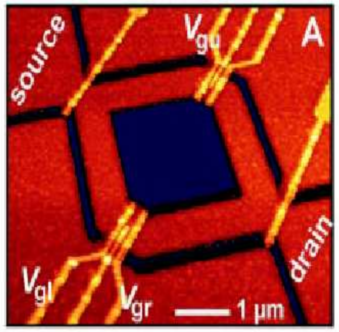
Kondo Effect in Quantum Dots







Goldhaber-Gordon et al
Nature **391** 156 (1998)



van der Wiel et al.,
Science **289** 2105
(2000).

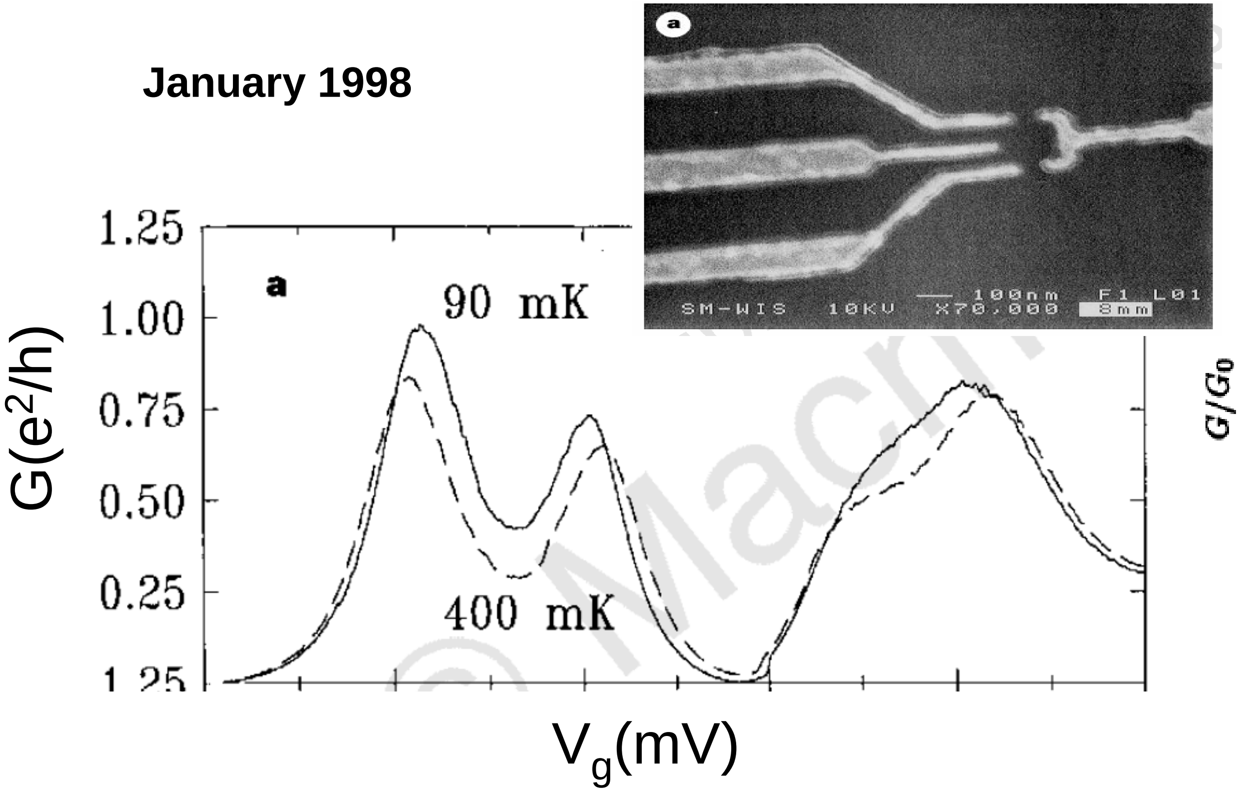
- $T>T_K$: Coulomb blockade (low G)
- $T<T_K$: Kondo singlet formation
- Kondo resonance at E_F (width T_K).
- New conduction channel at E_F:

Zero-bias enhancement of G (→2e^2/h!)

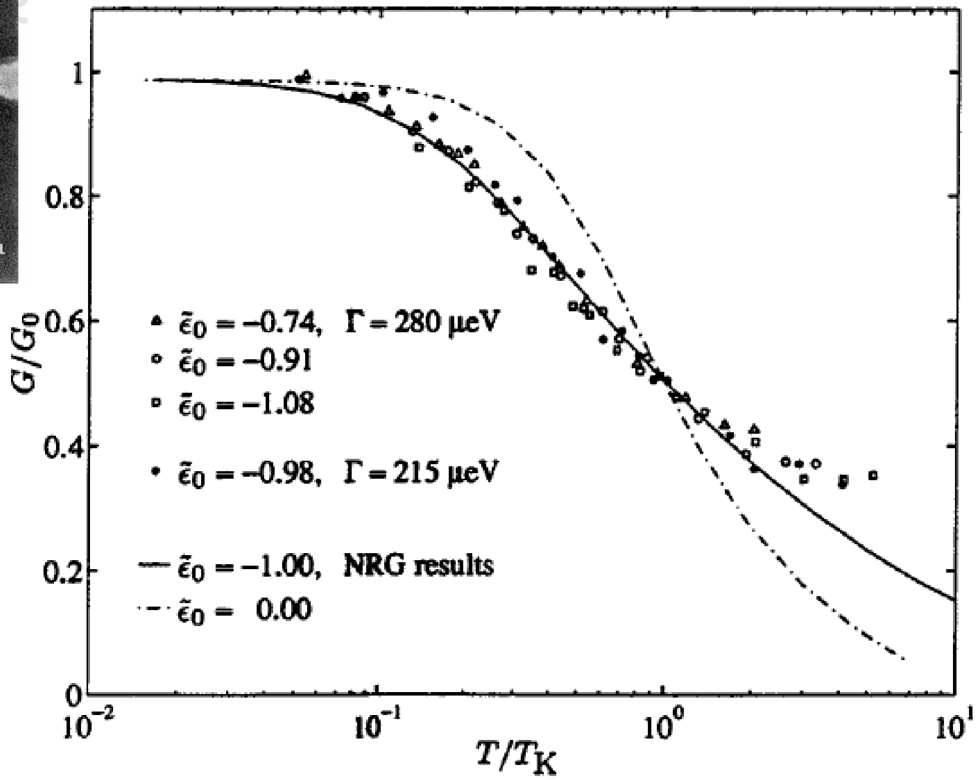
1998: “The Kondo year”

- Zero-bias conductance in semiconductor quantum dots:

January 1998



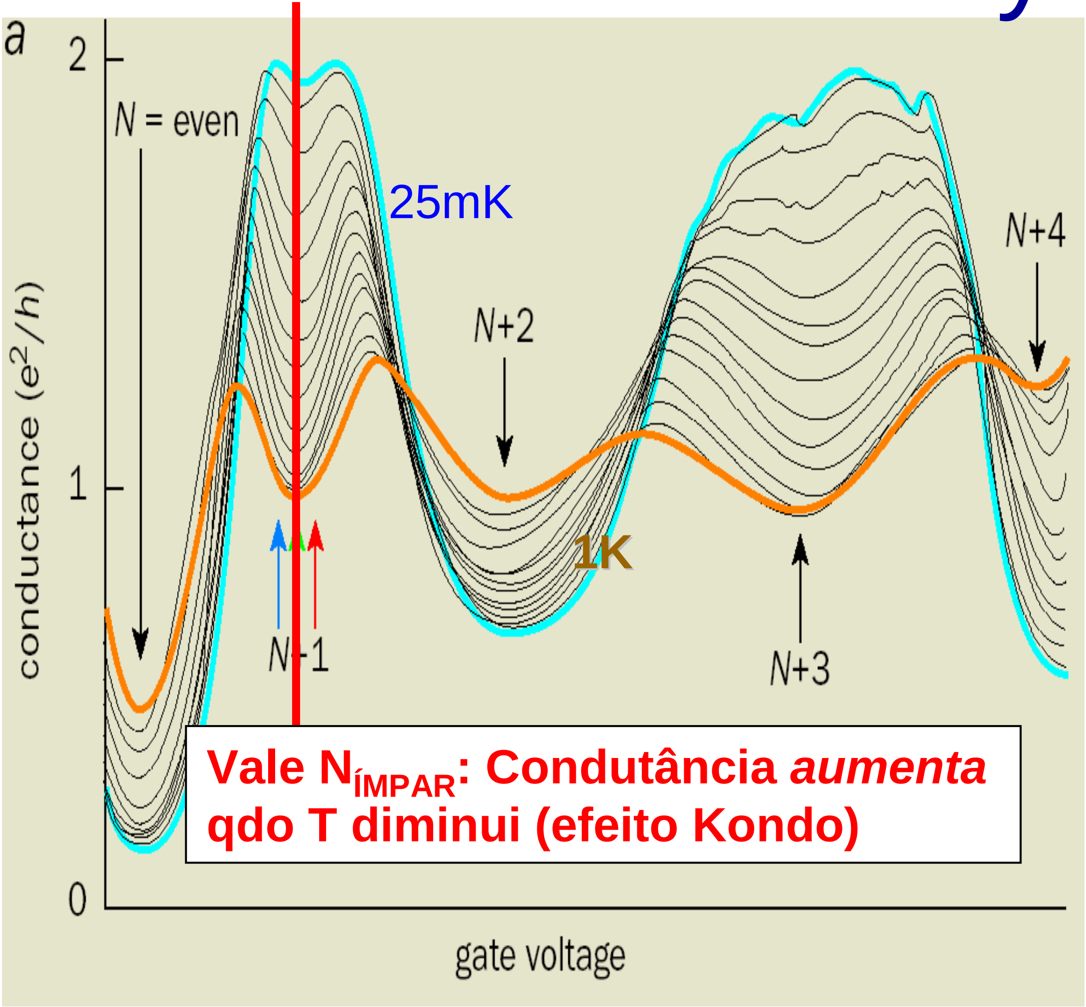
July 1998



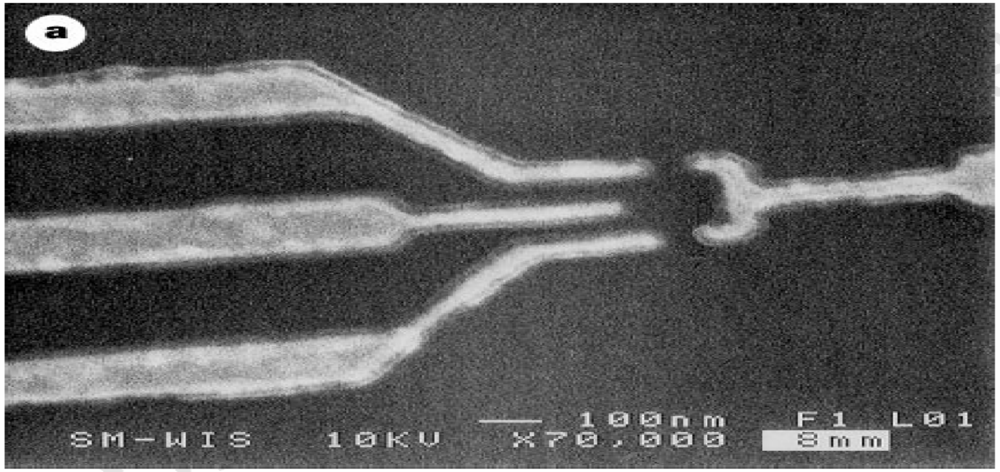
D. Goldhaber-Gordon et al. *Nature* **391** 156 (1998)

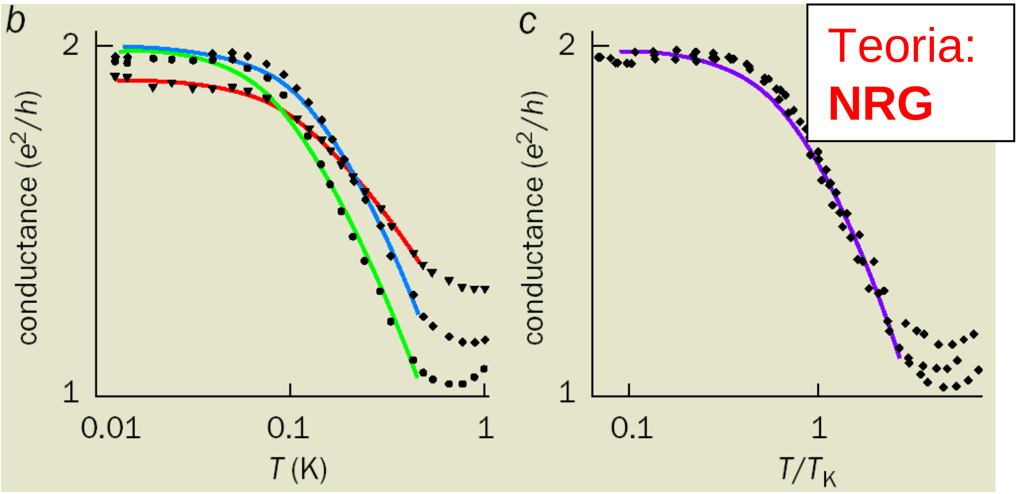
D. Goldhaber-Gordon et al. *PRL* **81** 5225 (1998)

1998: “The Kondo year”



Vale $N_{\text{ÍMPAR}}$: Condutância *aumenta* qdo T diminui (efeito Kondo)





Teoria:
NRG

Temperatura de Kondo T_k (~0.5K): um ÚNICO parâmetro escala todas as curvas G vs T

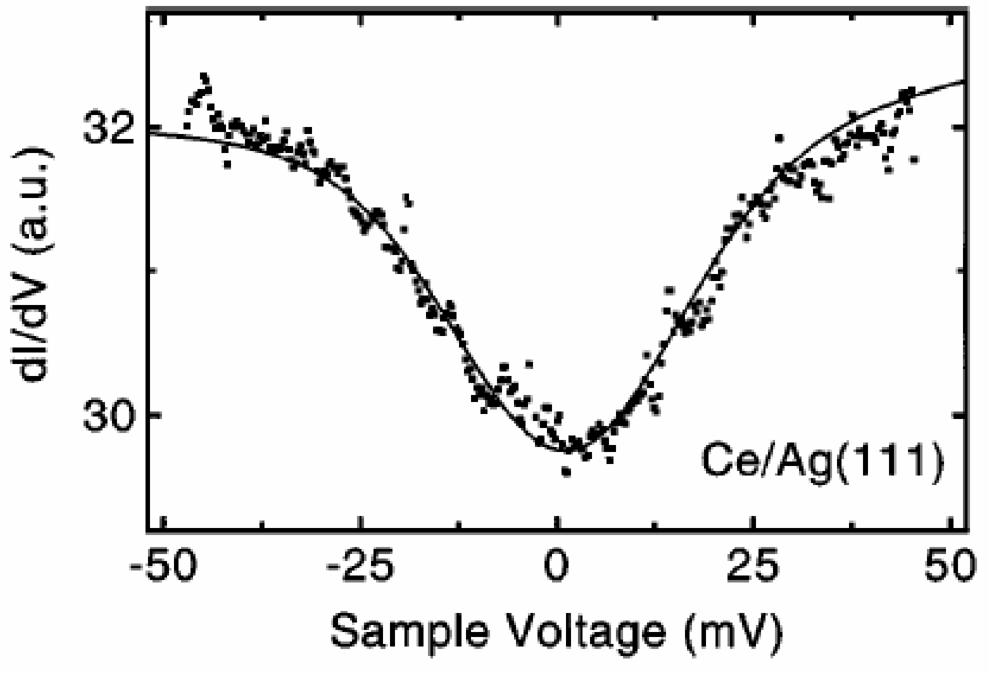
Kowenhoven and Glazman *Physics World* – Jan. 2001.

From: Goldhaber-Gordon *et al. Nature* **391** 156 (1998), *PRL* **81** 5225 (1998)

1998: “The Kondo year”

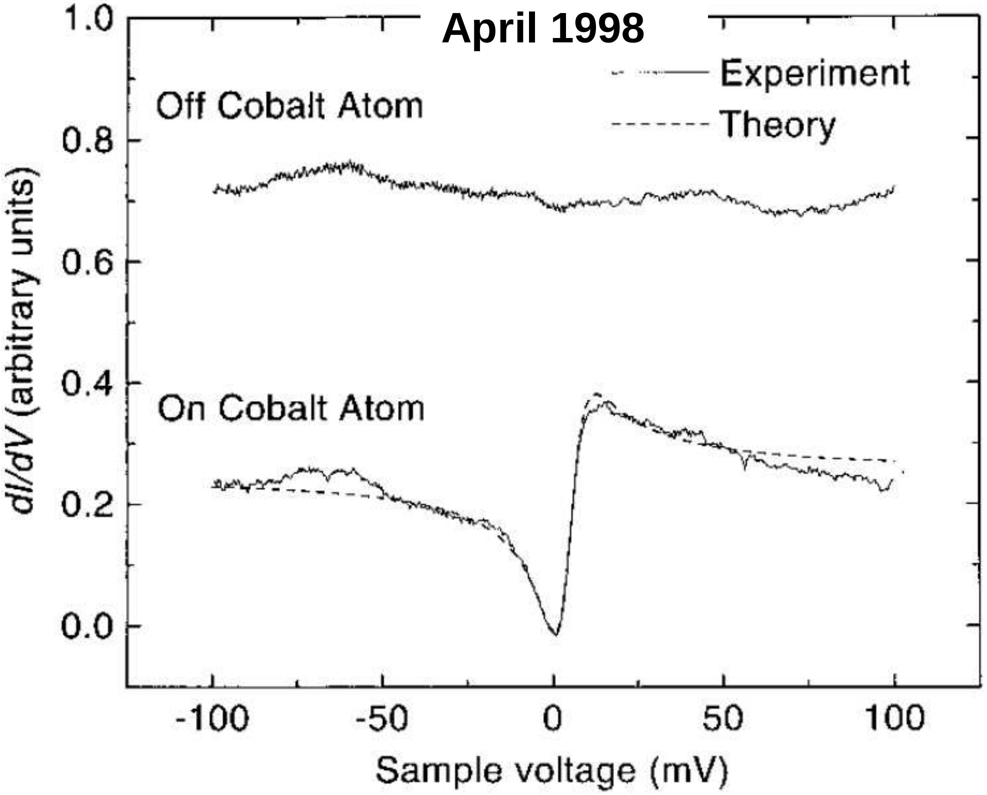
- Magnetic atoms on surfaces: zero-bias anomaly in STM dI/dV

March 1998



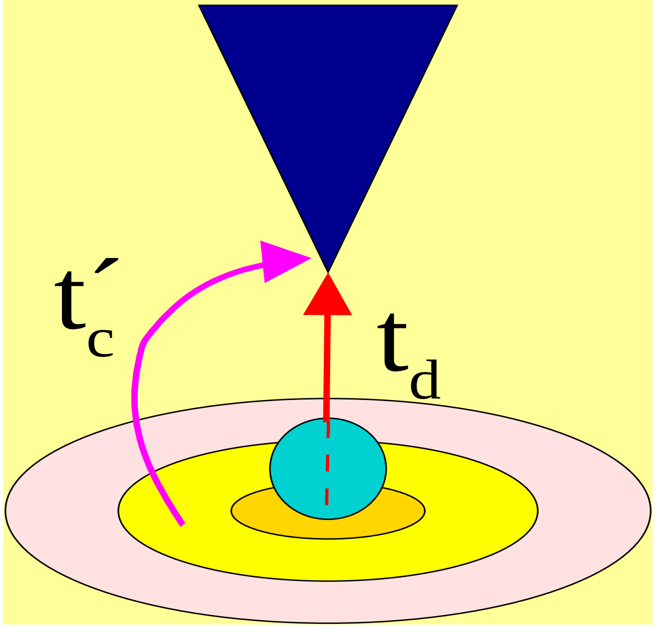
J. Li, et al. *PRL* **80** 2893 (1998)

April 1998

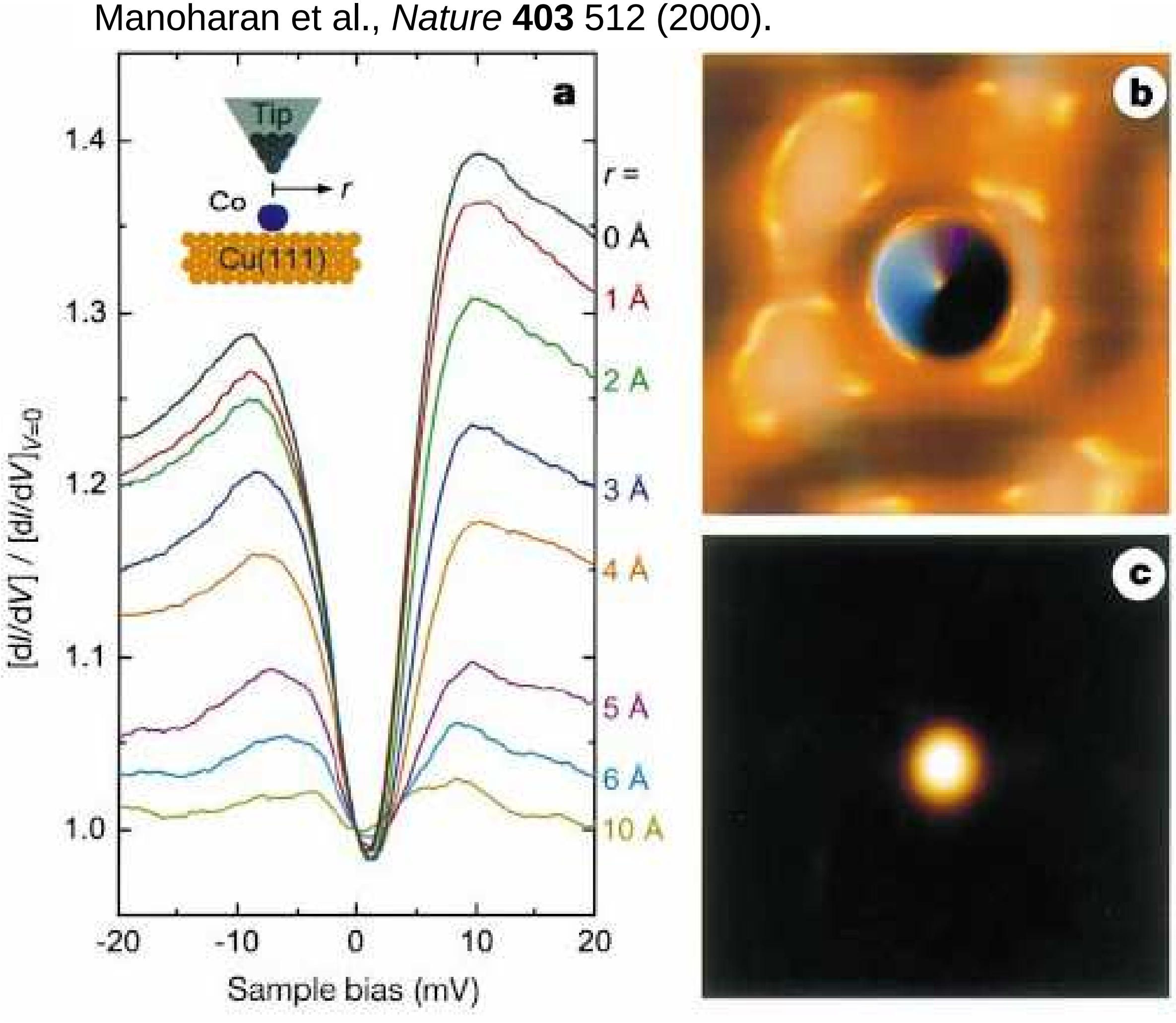


Madhavan et al., *Science* **280** 567 (1998).

Kondo effect in surfaces (STM images).



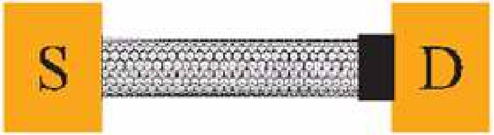
Manoharan et al., *Nature* **403** 512 (2000).

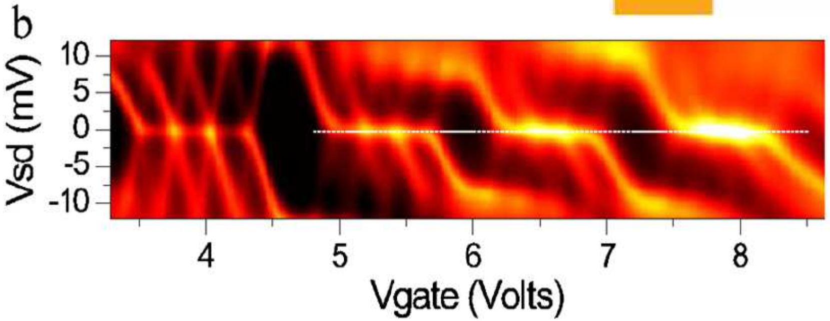


- Magnetic (Co, Fe) atoms on metallic *surfaces!*
- Right ingredients for Kondo.
- In this case, Kondo is marked by a *dip* at zero-bias conductance (dI/dV at V_{bias}=0).

Kondo everywhere!

Carbon Nanotubes

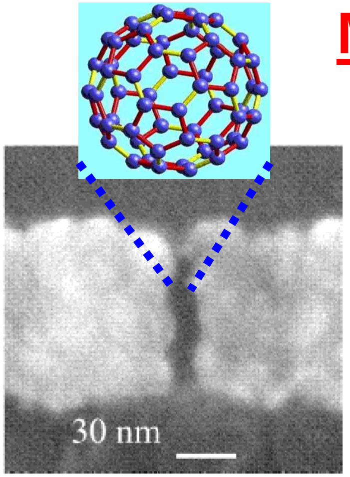




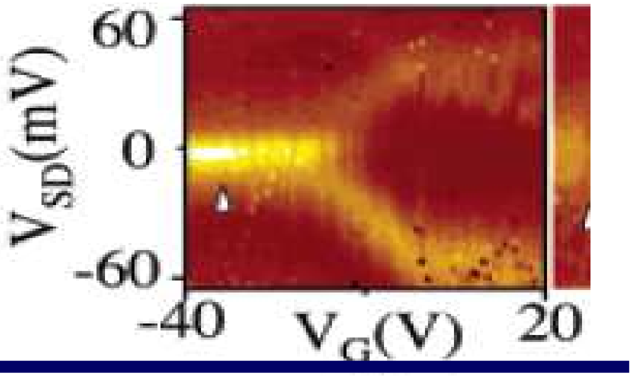
Makarovski, Liu, Filkenstein

PRL **99** 066801 (2007).

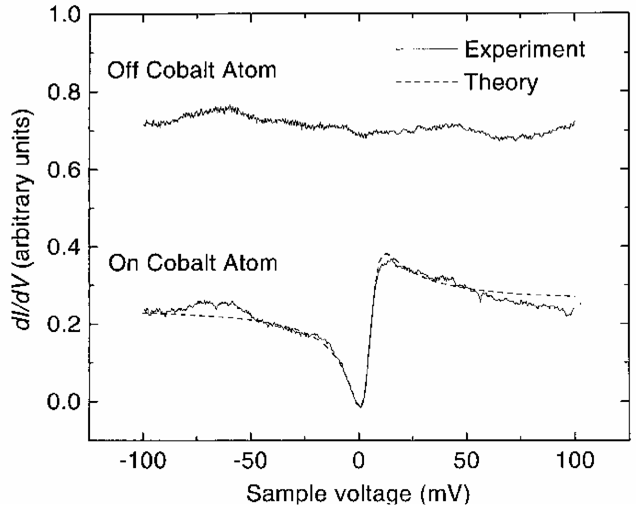
Molecular Junctions

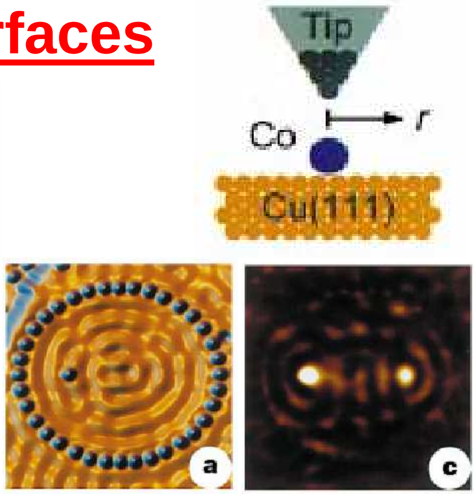


Yu, Natelson, *NanoLett.* **4** 79 (2004).



Magnetic atoms on surfaces

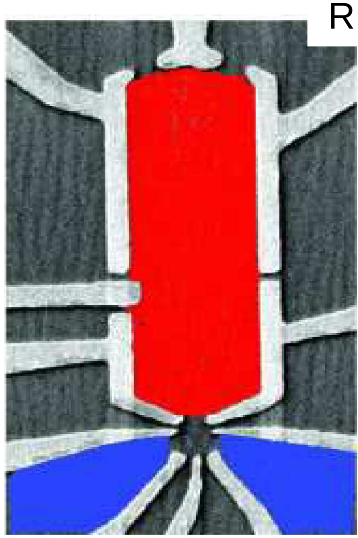




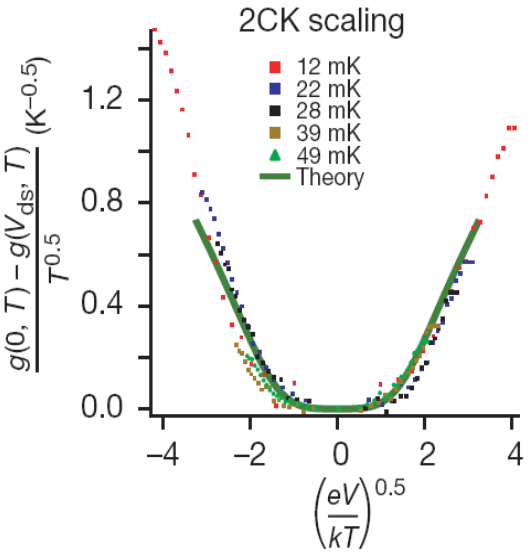
Manoharan et al., *Nature* **403** 512 (2000).

Madhavan et al., *Science* **280** 567 (1998).

Semiconductor Quantum dots

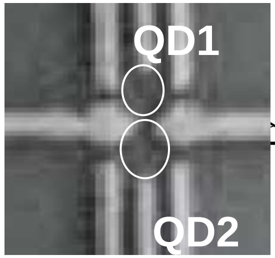


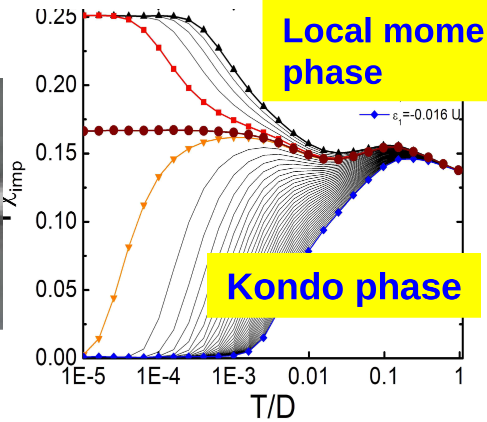
R. Potok *et al.* *Nature* **446** 167 (2007).



Kondo+NRG em nanoestruturas: exemplos

Pontos Quânticos semicondutores

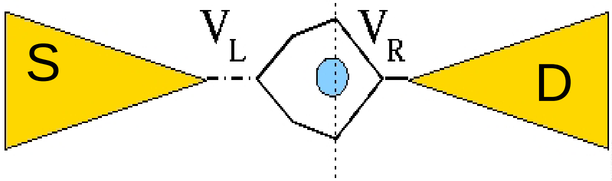


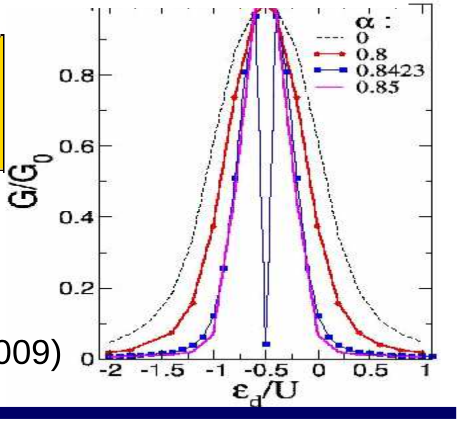


LGDS et al.
PRL **97** 096603(2006)

LGDS et al.
PRB **78** 155304 (2008).

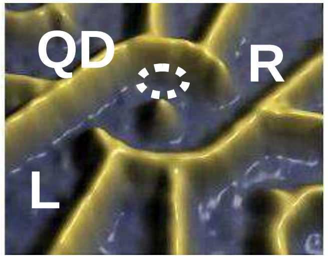
Junções moleculares c/ vibrações

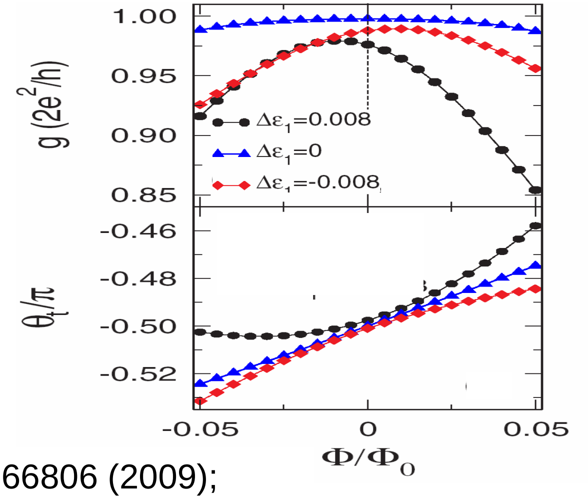




LGDS, E.Dagotto, PRB **79** 155302 (2009)

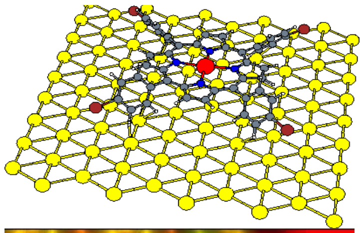
Anéis Quânticos + efeito Aharonov Bohm

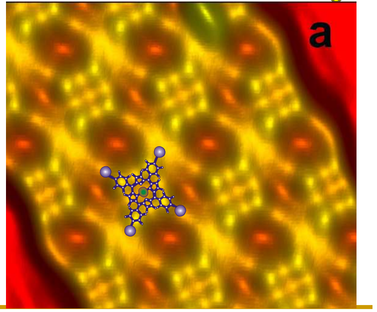


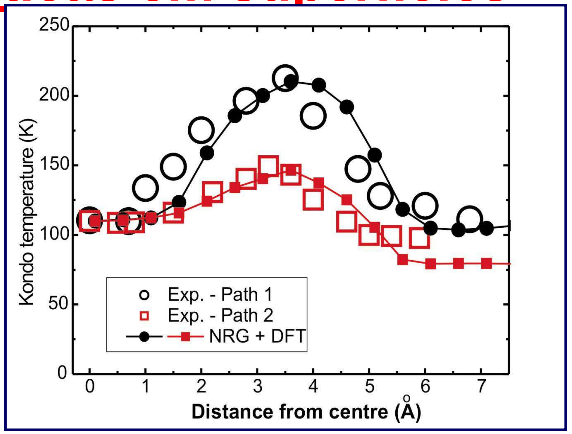


LGDS et al. PRL **102** 166806 (2009);

Moléculas magnéticas em superfícies







LGDS et al. PRB **80** 155143 (2009)

Perera et al. PRL **105** 106601 (2010)

Mapa do Seminário

◆ **15 anos do Efeito Kondo em nanoestruturas.**

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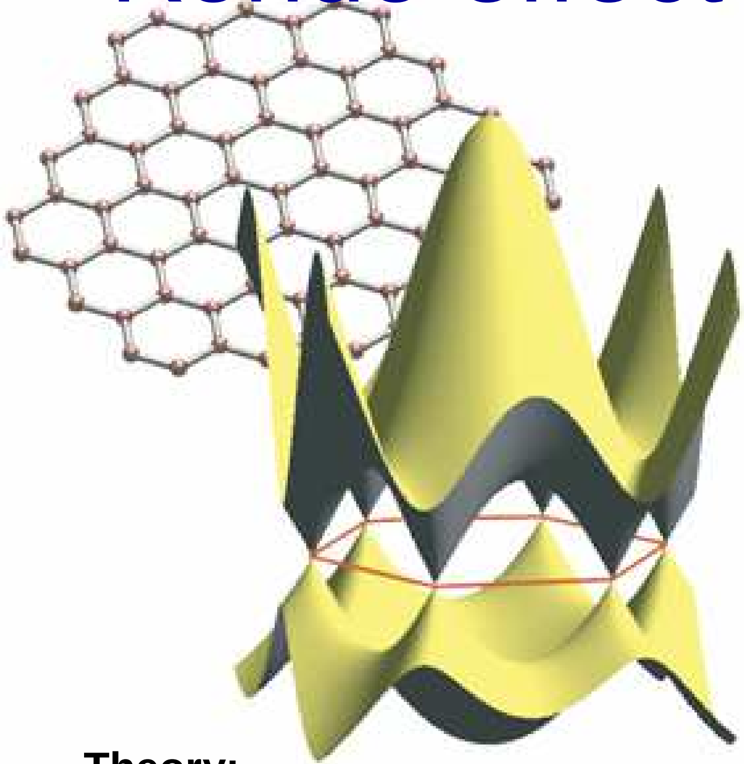
Kondo effect with Dirac Fermions



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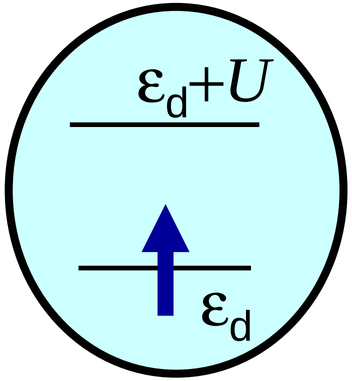
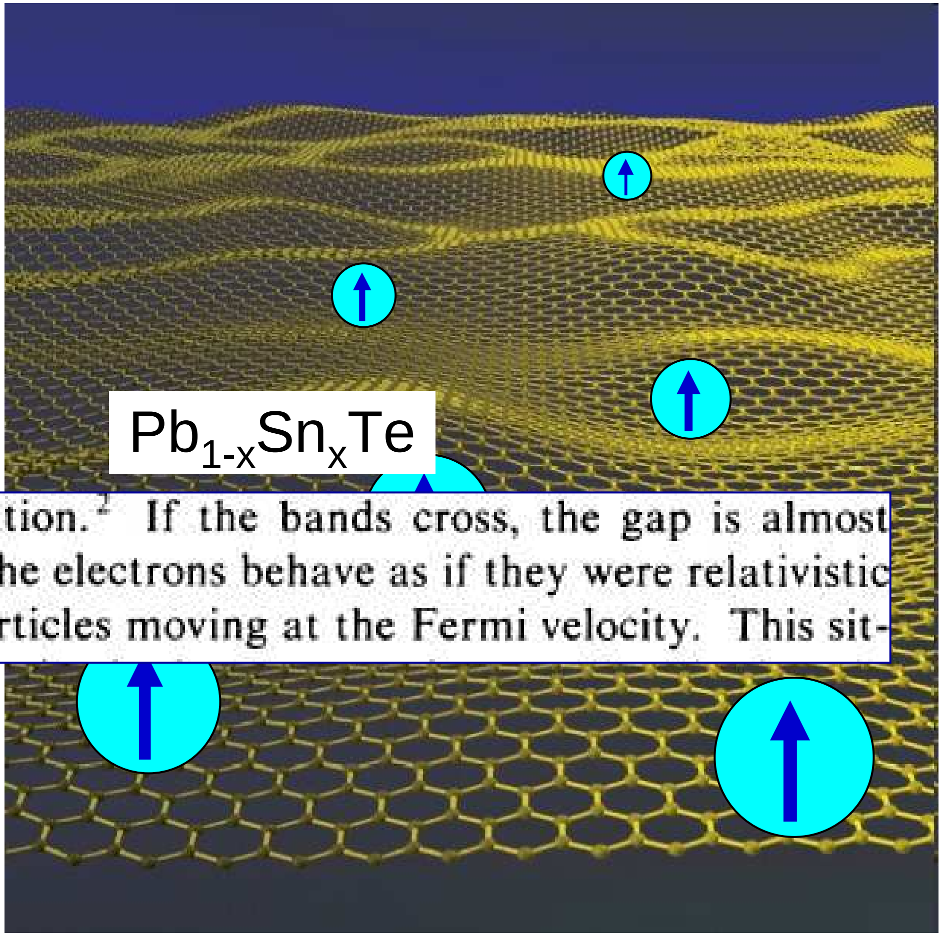
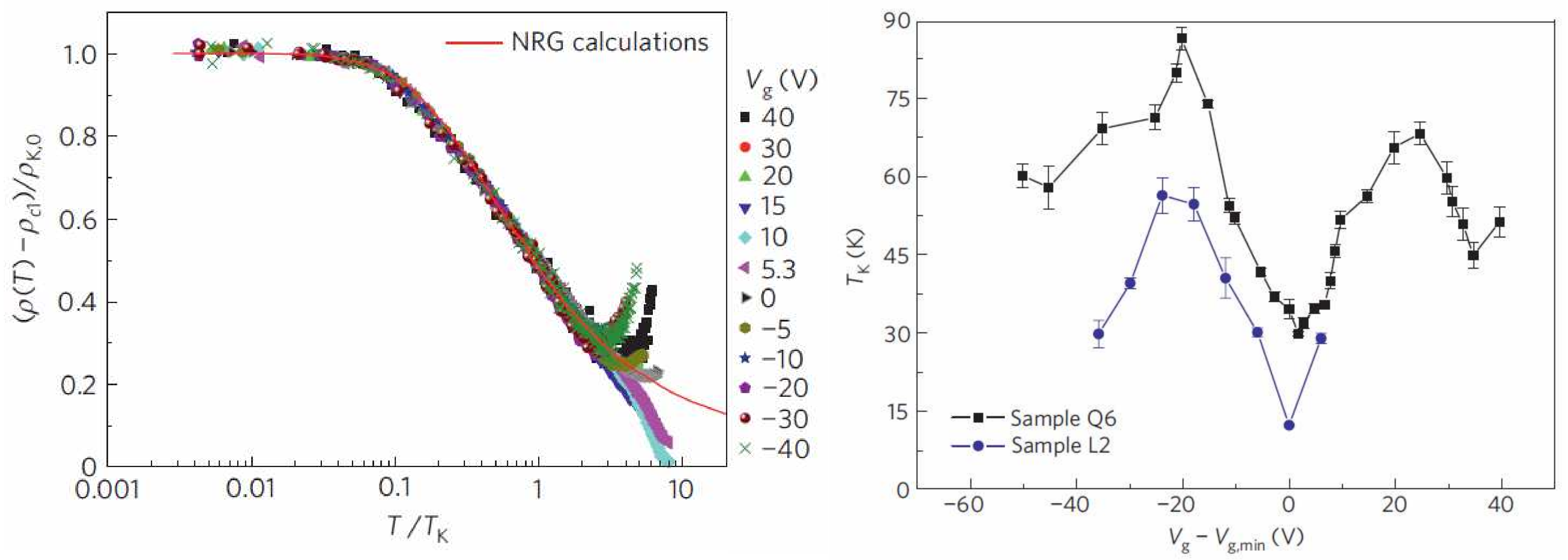




cal composition.[2] If the bands cross, the gap is almost linear and the electrons behave as if they were relativistic massless particles moving at the Fermi velocity. This sit-

Theory:
D. Withoff and E. Fradkin, PRL **64** 1835 (1990).
C. Gonzalez-Buxton, K. Ingersent, PRB **57**, 14254 (1998)
P.S. Cornaglia et al. PRL **102** 046801 (2009).
M. Vojta, et al., Europhys. Lett. **90**, 27006 (2010).
Review: L. Fritz and M. Vojta, **arXiv:1208.3113** (2012)
Experiment?

Kondo-like ρ(T) features in irradiated graphene



Jian-Hao Chen et al., *Nature Phys.* **7** 535 (2011)

- Resistivity vs Temperature measurements in disordered graphene.
- Short-range disorder intentionally caused by irradiation.
- Left: Kondo-like scaling in T/T_K. Right: T_K vs gate voltage.

Kondo effect in graphene: questions.

- Where does the localized (magnetic) state come from?

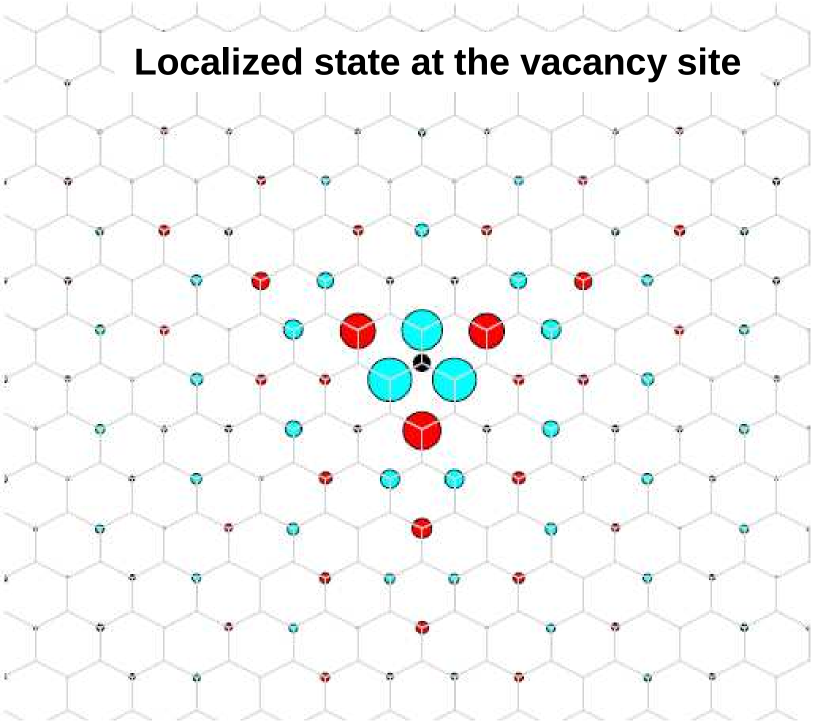
  R: Vacancies (=mid-gap states) V. M. Pereira et al., *PRB* **77** 115109 (2008)

- How does it couple to the continuous band?

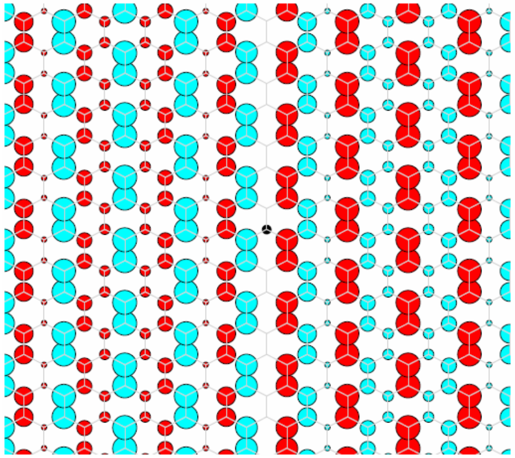
- Does this system retain features of an Anderson model coupled to Dirac fermions?

Mid-gap state in the presence of vacancies.

- Tight-binding calculations: single vacancy leads to midgap state

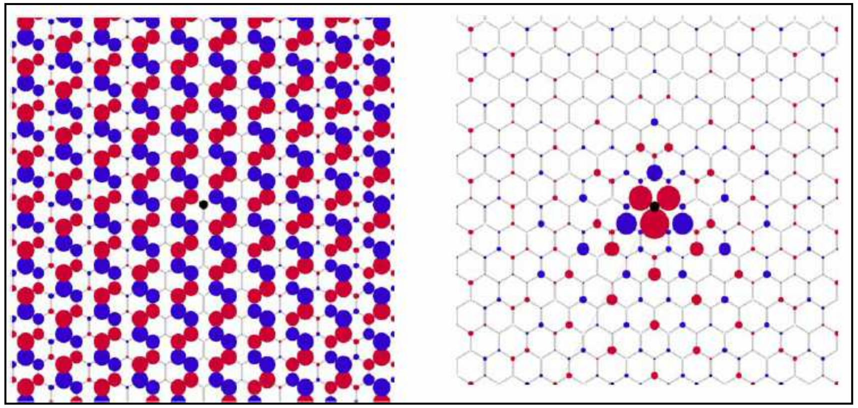


Localized state at the vacancy site



Typical delocalized state

Agreement with previous results



V. M. Pereira et al., *PRB* **77** 115109 (2008)

- Vacancy tight-binding model:

$$H_{\mathrm{v}} = -t \sum_{\langle i,j \rangle} c_i^\dagger c_j + t \sum_{\langle \mathrm{v},j \rangle} c_{\mathrm{v}}^\dagger c_j + \mathrm{H.c.}$$

$$H_{\mathrm{v}}|\nu\rangle = \begin{cases} \varepsilon_\nu |\nu\rangle & \text{for} \quad \varepsilon \neq 0, \quad |\nu\rangle \text{ is extended,} \\ 0|\nu\rangle & \text{for} \quad \nu = \mathrm{v}, \quad |\mathrm{v}\rangle \text{ is localized.} \end{cases}$$

Kondo effect in graphene: questions.

- Where does the localized (magnetic) state comes from?

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- How does it couple to the continuous band?

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R2: Long-range disorder (this work)

- Does this system retain features of an Anderson model coupled to Dirac fermions?

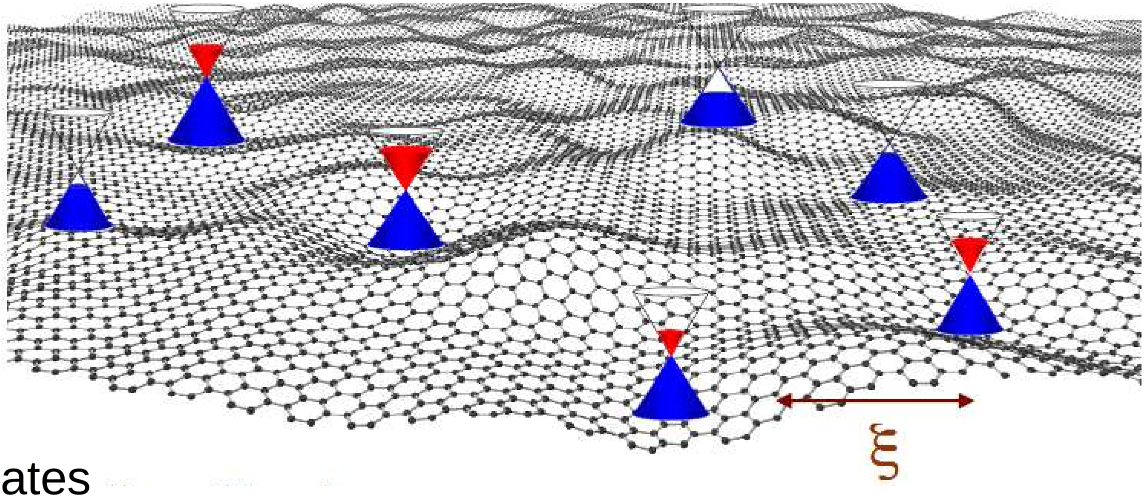
Tight-binding: long-range disorder+vacancy.

- How to couple the localized state to the graphene band?

$$U_{\rm dis}(\boldsymbol{r}_i) = \sum_{j=1}^{N_{\rm imp}} W_j \, e^{\frac{-(\boldsymbol{r}_i - \boldsymbol{R}_j)^2}{2\xi^2}}$$

$$H = H_{\rm v} + U_{\rm dis}$$

Disorder (weak)



- Our “basis”: localized and extended states

$$H_{\rm v} = -t \sum_{\langle i,j \rangle} c_i^\dagger c_j + t \sum_{\langle {\rm v},j \rangle} c_{\rm v}^\dagger c_j + {\rm H.c.}$$

$$H_{\rm v}|\nu\rangle = \begin{cases} \varepsilon_\mu |\mu\rangle & \text{and} \quad \varepsilon_\mu \neq 0, \quad |\mu\rangle \text{ is extended,} \\ \varepsilon_0 |0\rangle & \text{and} \quad \varepsilon_0 = 0 \quad |0\rangle \text{ is localized.} \end{cases}$$

- Projectors:

$$\mathbf{1} = \hat{P} + \hat{Q} \equiv \sum_\mu |\mu\rangle\langle\mu| + |0\rangle\langle 0|$$

$$H = \hat{P} H \hat{P} + \hat{Q} H \hat{P} + \hat{P} H \hat{Q} + \hat{Q} H \hat{Q}$$

Tight-binding: long-range disorder+vacancy.

- How to couple the localized state to the graphene band? Disorder (weak)

Extended Coupling Localized

$$\boxed{H = \hat{P}H\hat{P} + \hat{Q}H\hat{P} + \hat{P}H\hat{Q} + \hat{Q}H\hat{Q}}$$

$$\boxed{\hat{P}H\hat{P} = \sum_{\mu} |\mu\rangle\varepsilon_{\mu}\langle\mu| + \sum_{\mu\mu'} |\mu\rangle\langle\mu|U_{\rm dis}|\mu'\rangle\langle\mu'|} \rightarrow$$

Effective band density of states.

$$\boxed{\rho_{\rm dis}(\omega) = \sum \delta(\omega - \varepsilon_{\beta})}$$

$$\boxed{\hat{Q}H\hat{P} + \text{h.c.} = \sum_{\mu} |0\rangle\langle 0|U_{\rm dis}|\mu\rangle\langle\mu| + \text{h.c.}} \rightarrow$$

Coupling to the localized state

$$\boxed{t_{\beta 0} \equiv \langle\beta|U_{\rm dis}|0\rangle}$$

$$\boxed{\hat{Q}H\hat{Q} = \sum_{\mu} |0\rangle\varepsilon_0\langle 0| + |0\rangle\langle 0|U_{\rm dis}|0\rangle\langle 0| = |0\rangle\varepsilon_0^{\rm dis}\langle 0|} \rightarrow$$

Renormalized state energy

$$\boxed{\varepsilon_0^{\rm dis} \equiv \langle 0|U_{\rm dis}|0\rangle}$$

Hybridization function from TB calculations

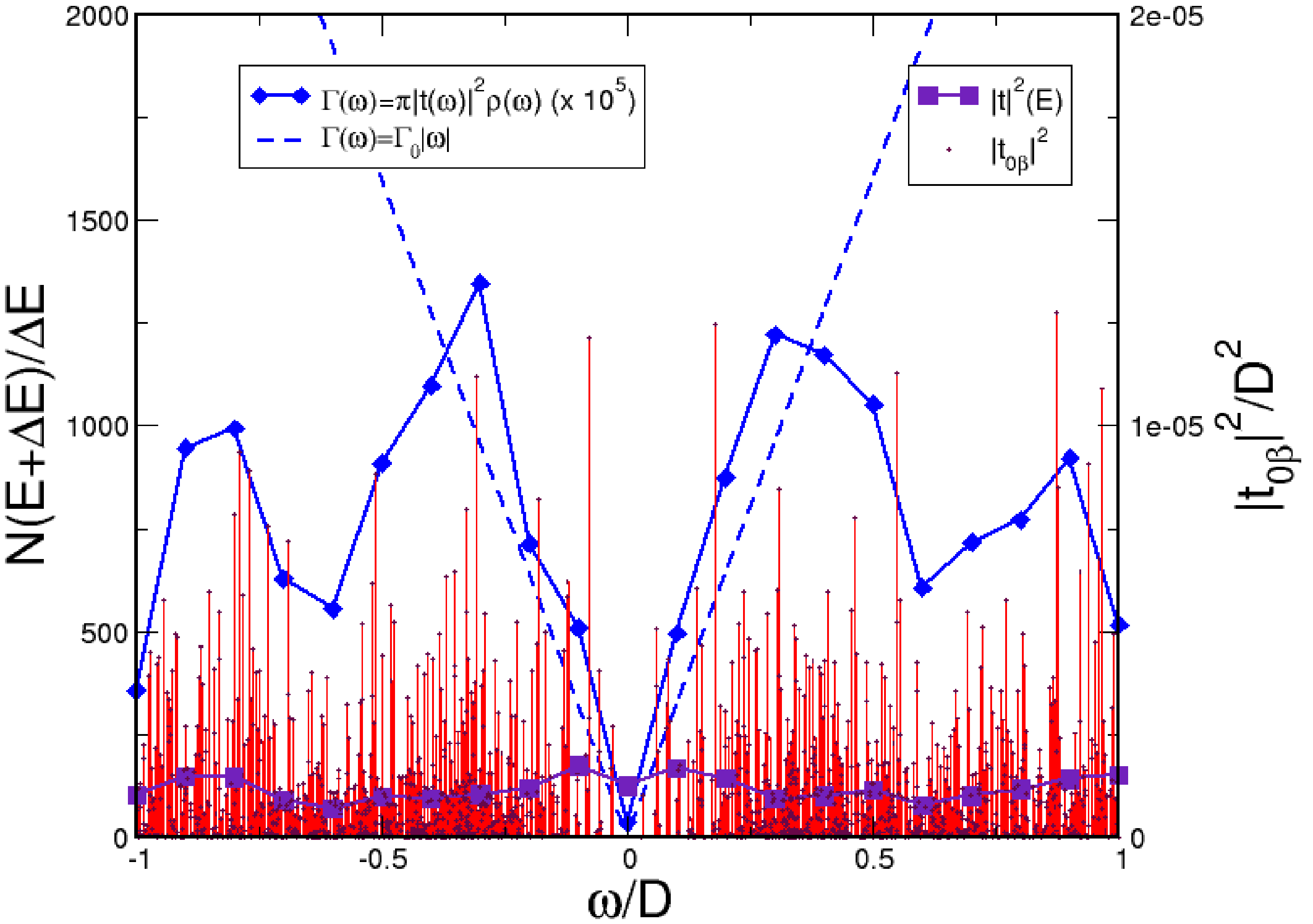
For each realization:

$$|t_{0\beta}|^2 \qquad \varepsilon_\beta$$

$$\boxed{\Gamma_{\text{dis}}(\omega) = \pi |t_\omega|^2 \rho_{\text{dis}}(\omega)}$$

And also:

$$\boxed{\varepsilon_0^{\text{dis}} \equiv \langle 0 | U_{\text{dis}} | 0 \rangle}$$



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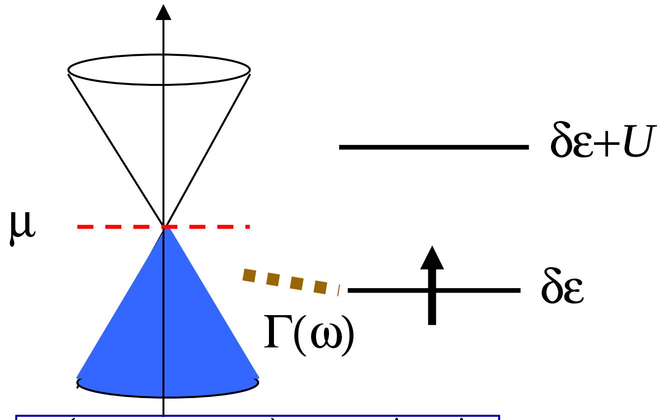
NRG calculations: TB-derived Anderson model

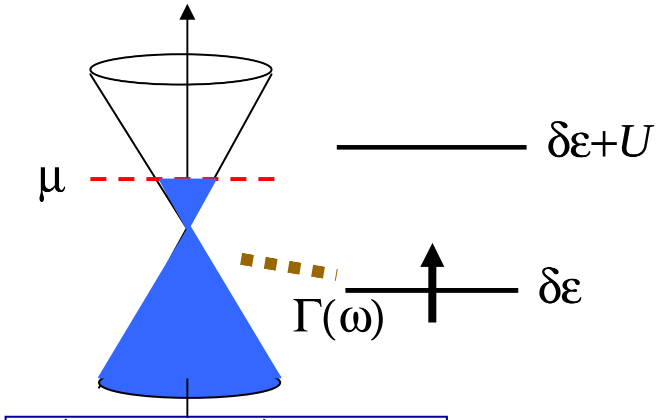
Anderson model with Dirac fermions

“Pseudogap” model



$$\boxed{\rho(\omega \rightarrow 0) \sim |\omega|^r}$$

“metallic” model

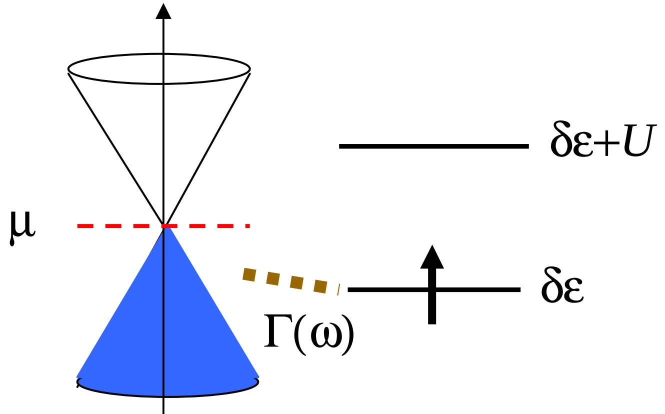


$$\boxed{\rho(\omega \rightarrow 0) = \rho_0}$$

- Anderson impurity coupled to a “Dirac band” with linear dispersion.
- δε: impurity state energy
- $\mu(V_g)$: Fermi energy (gate-dependent)
- μ(0): Fermi energy at charge neutrality
- Realization of the “pseudogap Anderson model” for V_g=0.

Anderson model with Dirac fermions.

“Pseudogap” model



$$\rho(\omega \to 0) \sim |\omega|^r$$

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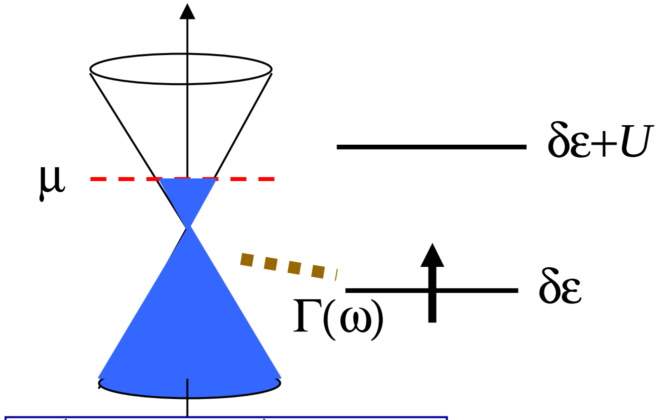

$$\rho(\omega \to 0) = \rho_0$$

$H_A = H_{\rm state} + H_{\rm band} + H_{s-b}$ where:

$$\begin{aligned} H_{\rm state} &= \delta\epsilon\, n_{d\sigma} + U n_{d\uparrow} n_{d\downarrow} \qquad (\delta\epsilon = \epsilon_0 - \mu(V_g)) \\ H_{\rm band} &= \frac{1}{D}\int_{-D}^{D} d\omega\, \omega\, c^{\dagger}_{\omega\sigma} c_{\omega\sigma} \qquad (\rho(\omega) = \rho_0 |\omega - \Delta\mu|) \\ H_{s-b} &= \frac{1}{\sqrt{D}}\int_{-D}^{D} d\omega \sqrt{\rho(\omega)}\, t_{s-b} \left(c^{\dagger}_{d\sigma} c_{\omega\sigma} + \text{h.c.} \right) . \end{aligned}$$

- Anderson impurity coupled to a “Dirac band” with linear dispersion.
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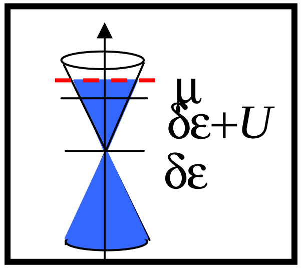
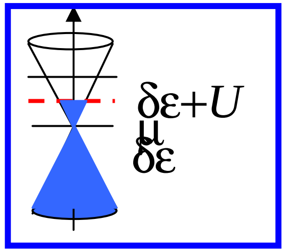
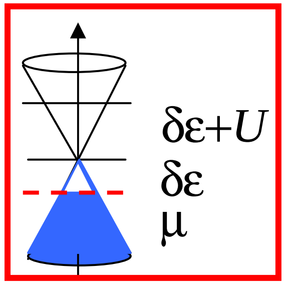
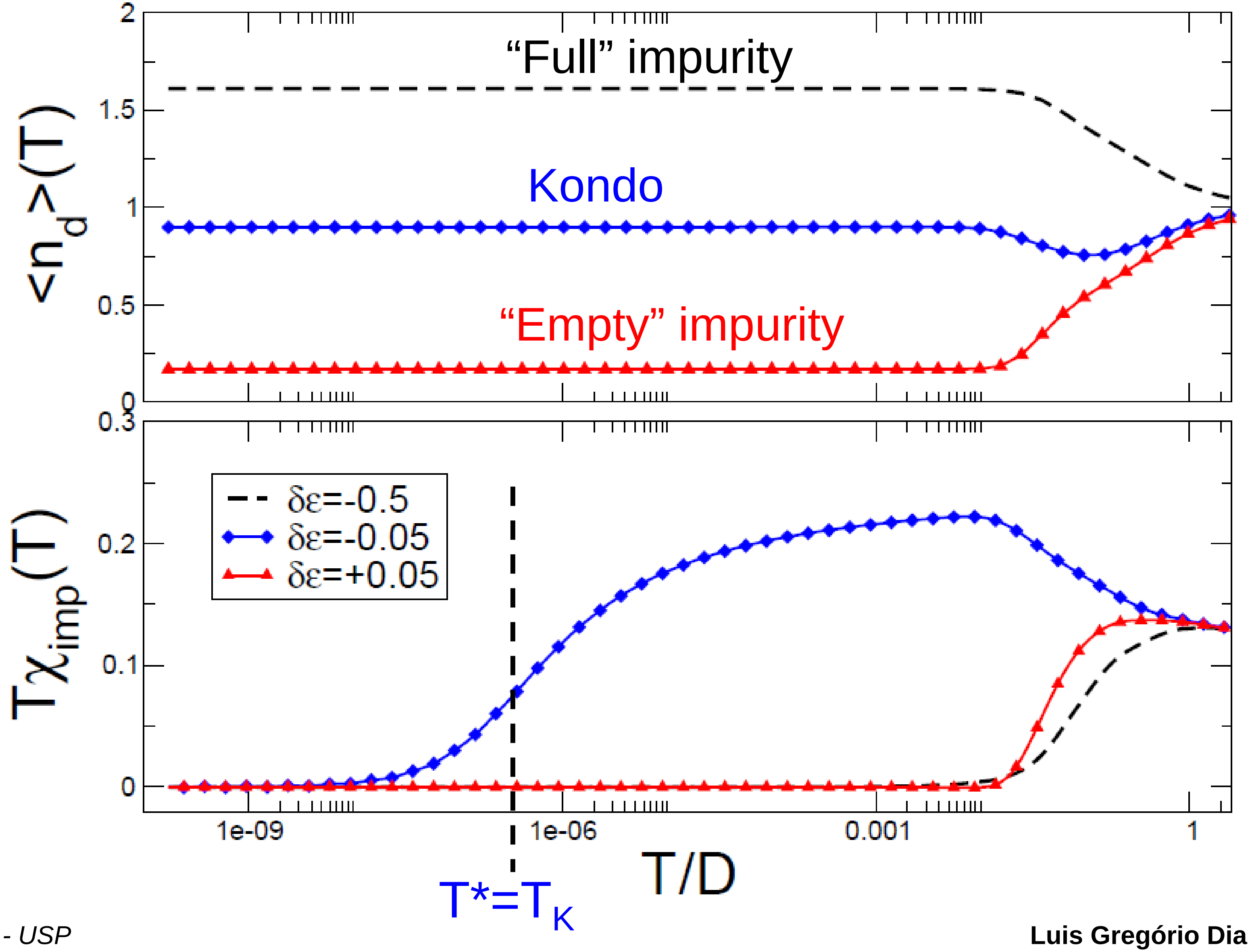
Metallic model: “vanilla” Kondo effect (NRG)

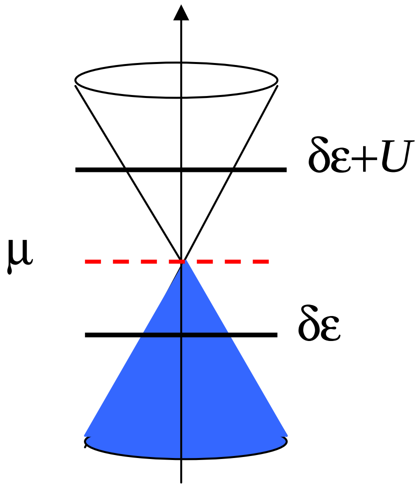


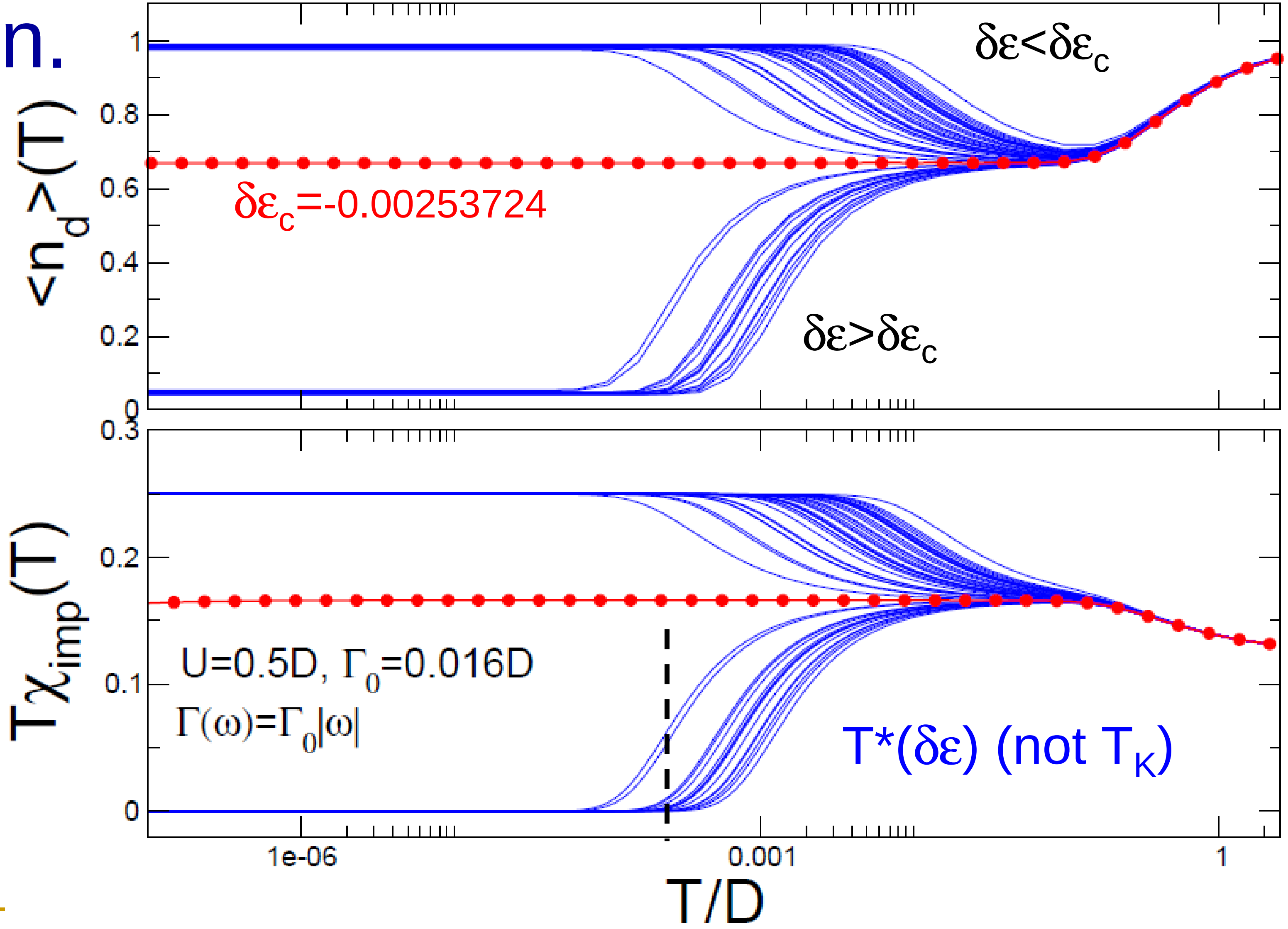






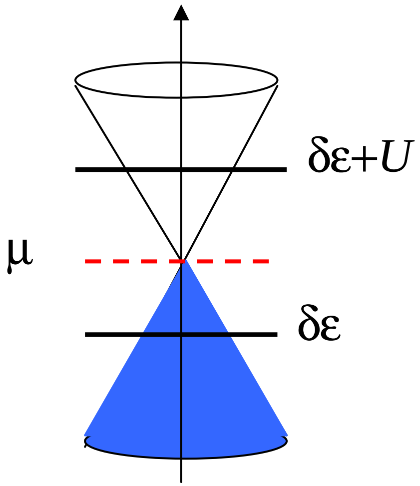
Pseudogap model: quantum phase transition.

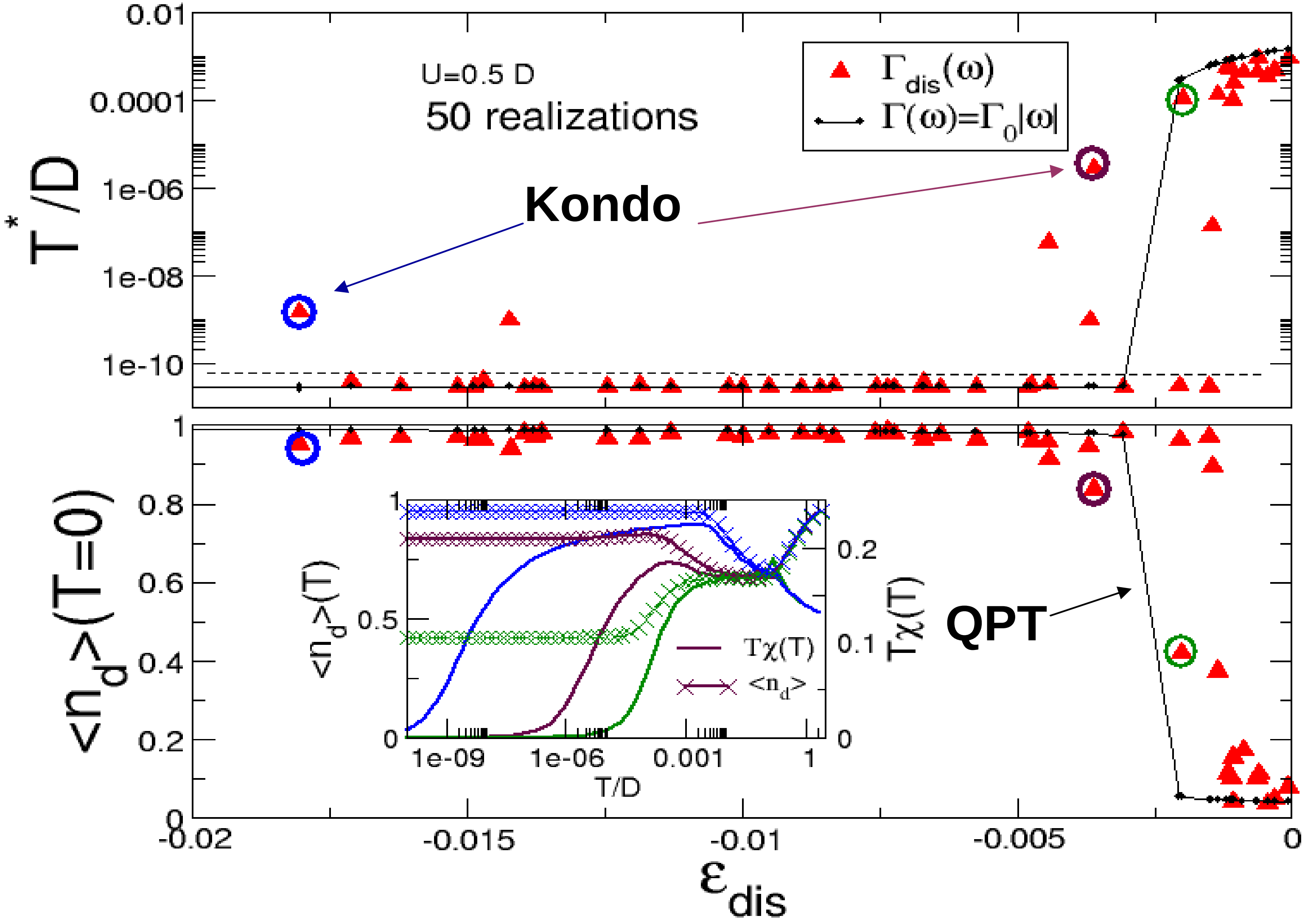




Tight-binding + NRG.

$\mu=0$





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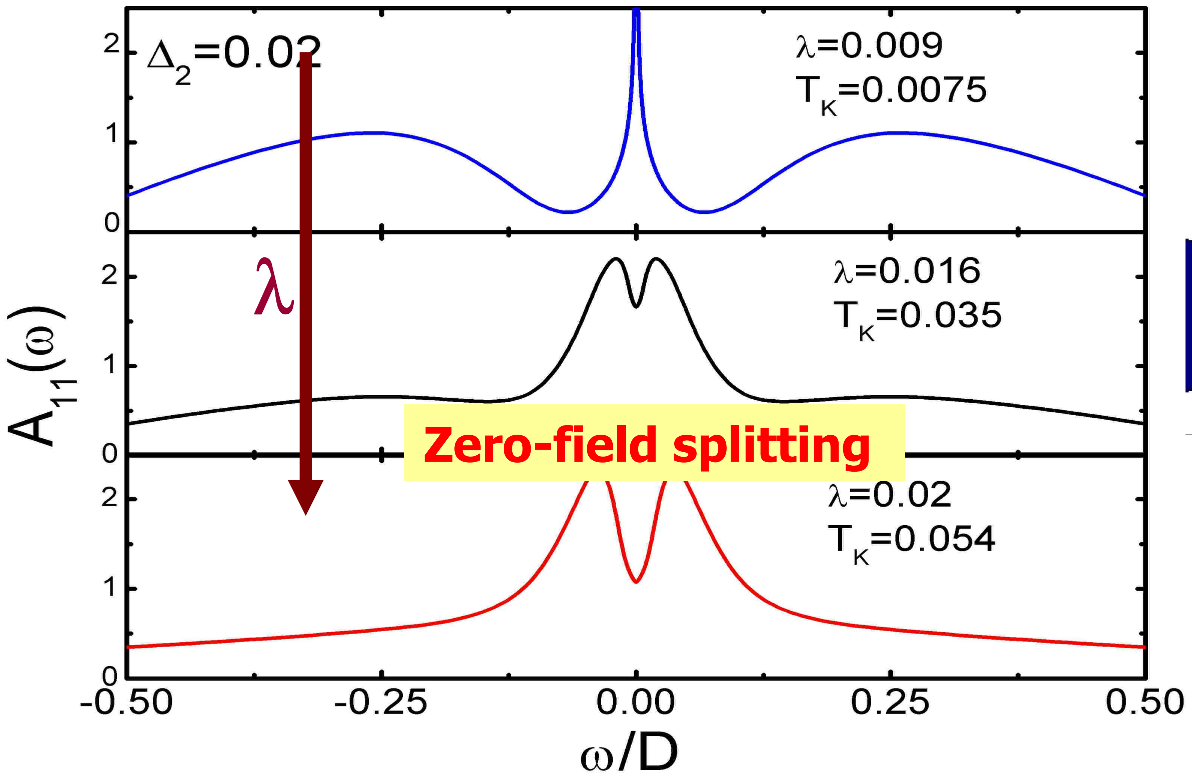
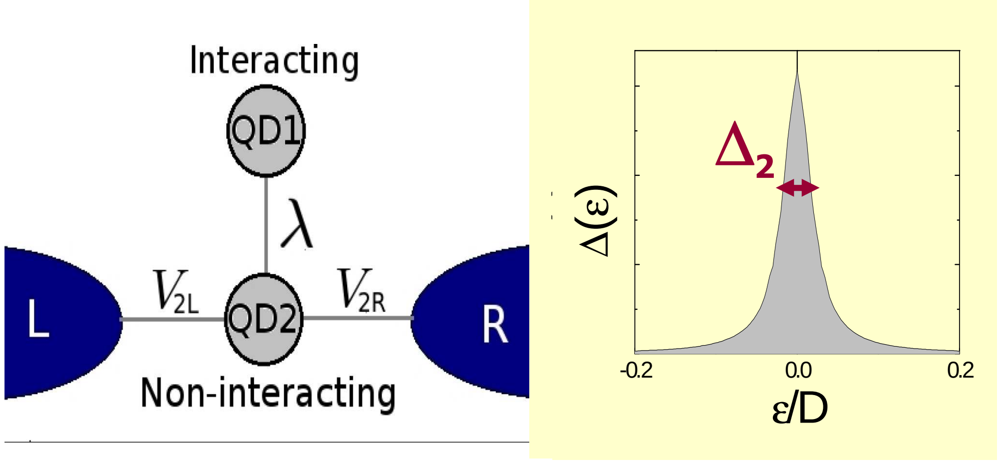
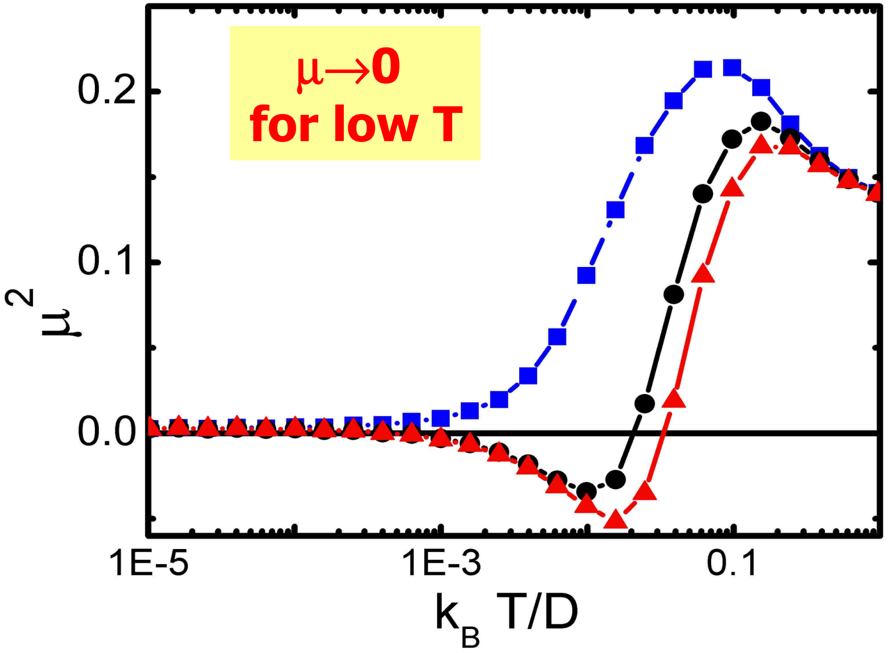
LDS, E. Vernek, K. Ingersent, N. Sandler, S. Ulloa,
PRB **87** 205313 (2013)

Zero-field splitting of the Kondo peak



Zero-field splitting





μ→0 for low T

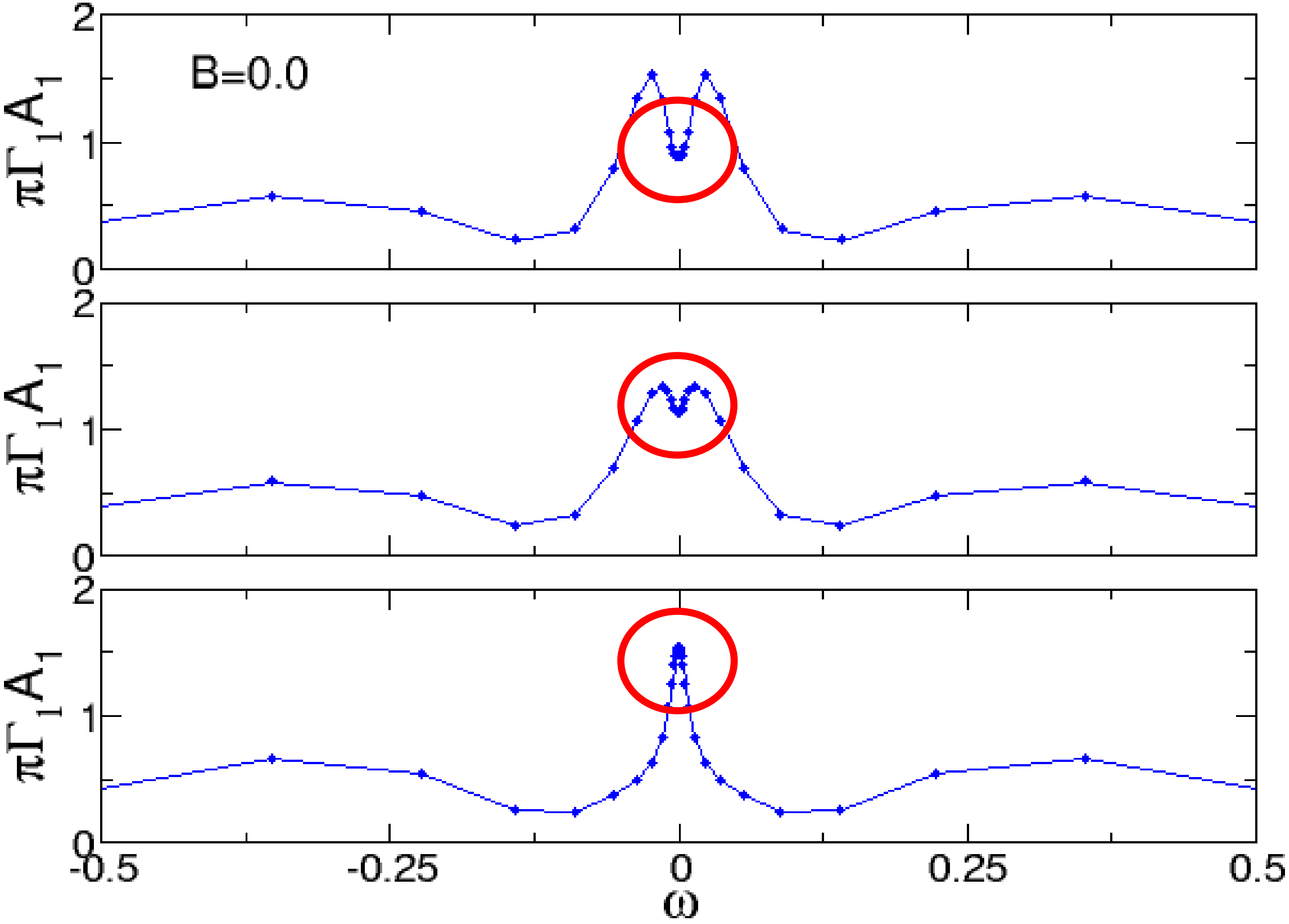
Side-coupled QDs Splitting of the Kondo peak at $T_K > \Delta_2$.

LDS et al PRL **97** 096603 (2006).
Vaugier et al. PRL **99**, 209701 (2007).
LDS et al. PRL **99**, 209702 (2007).
Vaugier et al. PRB **76**, 165112 (2007).
LDS et al PRB **78**, 153304 (2008).

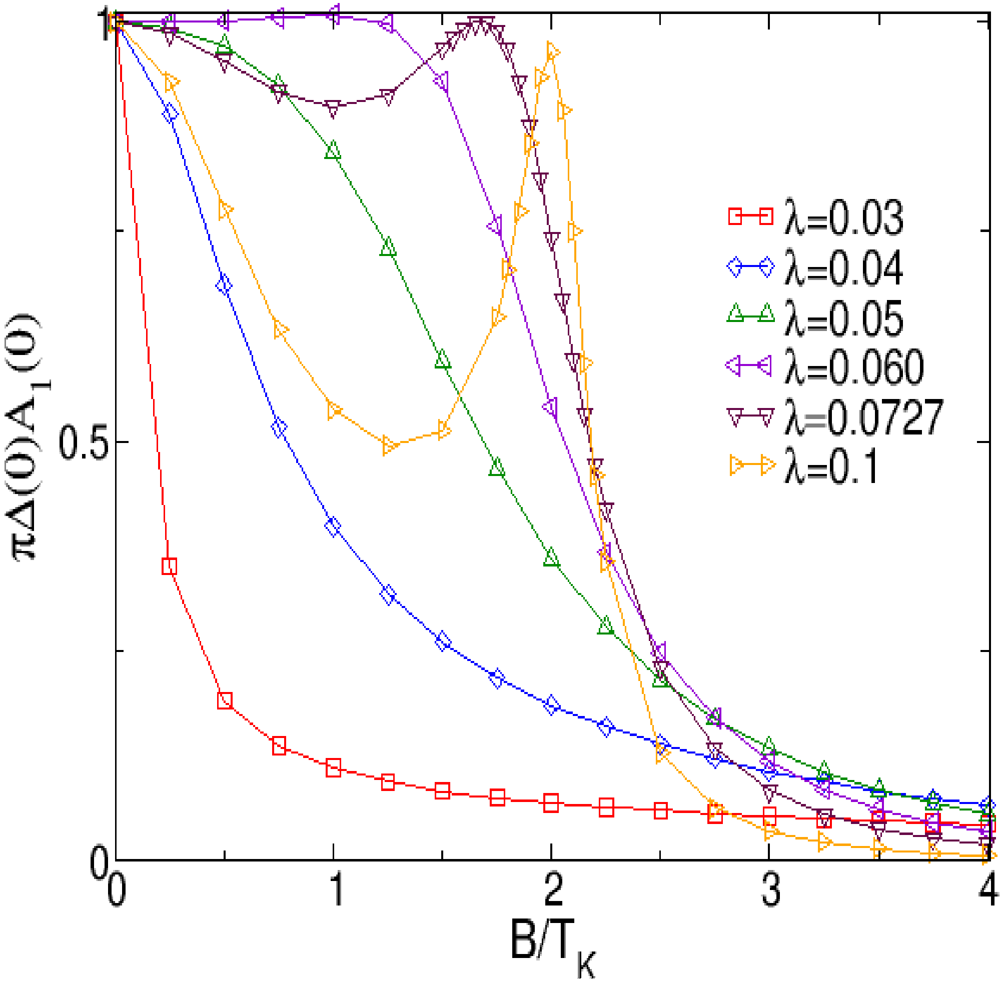
Kondo Peak Splitting: Zeeman + orbital

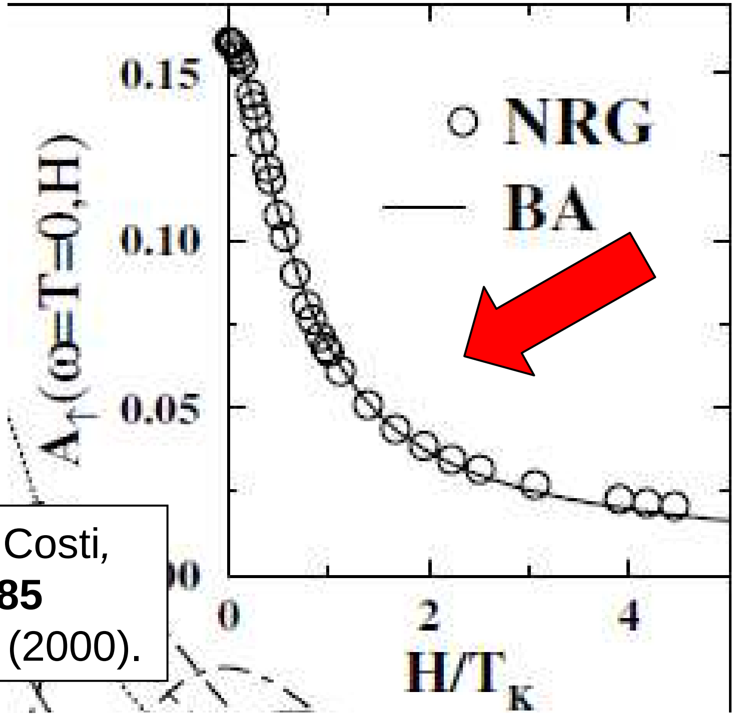
$$A_{1\sigma}(\omega) = -\frac{\mathrm{Im}\ \mathcal{G}_{1\sigma}(\omega)}{\pi}$$

$$A_1(\omega) = \frac{A_{1\uparrow}(\omega) + A_{1\downarrow}(\omega)}{2}$$



A(0) vs field: non-universal decay?





T. A. Costi,
PRL **85**
1504 (2000).

p-h-s: ***monotonic decay with h!***

$$\pi\Gamma A_d(0,h) = \cos^2\left(\pi M_d\right)$$

D.E. Logan, N.L. Dickens,
JPCM **13** 9713 (2001)

$$M_d(h) = \frac{1}{2}\left(\langle n_{d\uparrow}\rangle - \langle n_{d\downarrow}\rangle\right)$$

Friedel Sum Rule revisited

$$\left.\begin{aligned}
&\mathcal{G}_{1\sigma}(\omega,T) \rightarrow \text{ Fully interacting GF in dot 1}\\
&\Sigma^0_{1\sigma}(\omega) \rightarrow \text{ Non-interacting self-energy}\\
&A_{1\sigma}(\omega) = -\frac{1}{\pi}\mathrm{Im}\mathcal{G}_{1\sigma}(\omega)\\
&\Delta_\sigma(\omega) = -\mathrm{Im}\ \Sigma^0_{1\sigma}(\omega)
\end{aligned}\right\}$$

Langreth, PR **150** 516 (1966).
Vaugier et al. PRB **76**, 165112 (2007).

$$\boxed{\pi\Delta_\sigma(0)A_{1\sigma}(0,0) = \sin^2\left(\pi\langle n_{1\sigma}\rangle + \varphi_\sigma\right)}$$

where

$$\boxed{\varphi_\sigma = \mathrm{Im}\int_{-\infty}^{0}\frac{\partial\Sigma^0_{1\sigma}(\omega)}{\partial\omega}\,\mathcal{G}_{1\sigma}(\omega)\ d\omega}$$

"Usual" case:

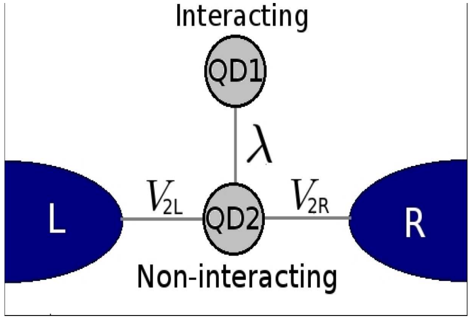
$$\boxed{\Delta_\sigma(\omega) = \Delta_0}$$

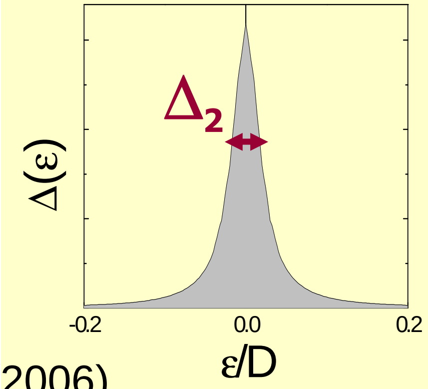
$$\boxed{\varphi_\sigma = 0}$$

DQD mapping:

$$\boxed{\Delta_\sigma(\omega) = \frac{\lambda^2\Delta_2}{(\omega-\varepsilon_{2\sigma})^2+\Delta_2^2}}$$

$$\boxed{\varphi_\sigma \neq 0}$$





LDS et al PRL **97** 096603 (2006).

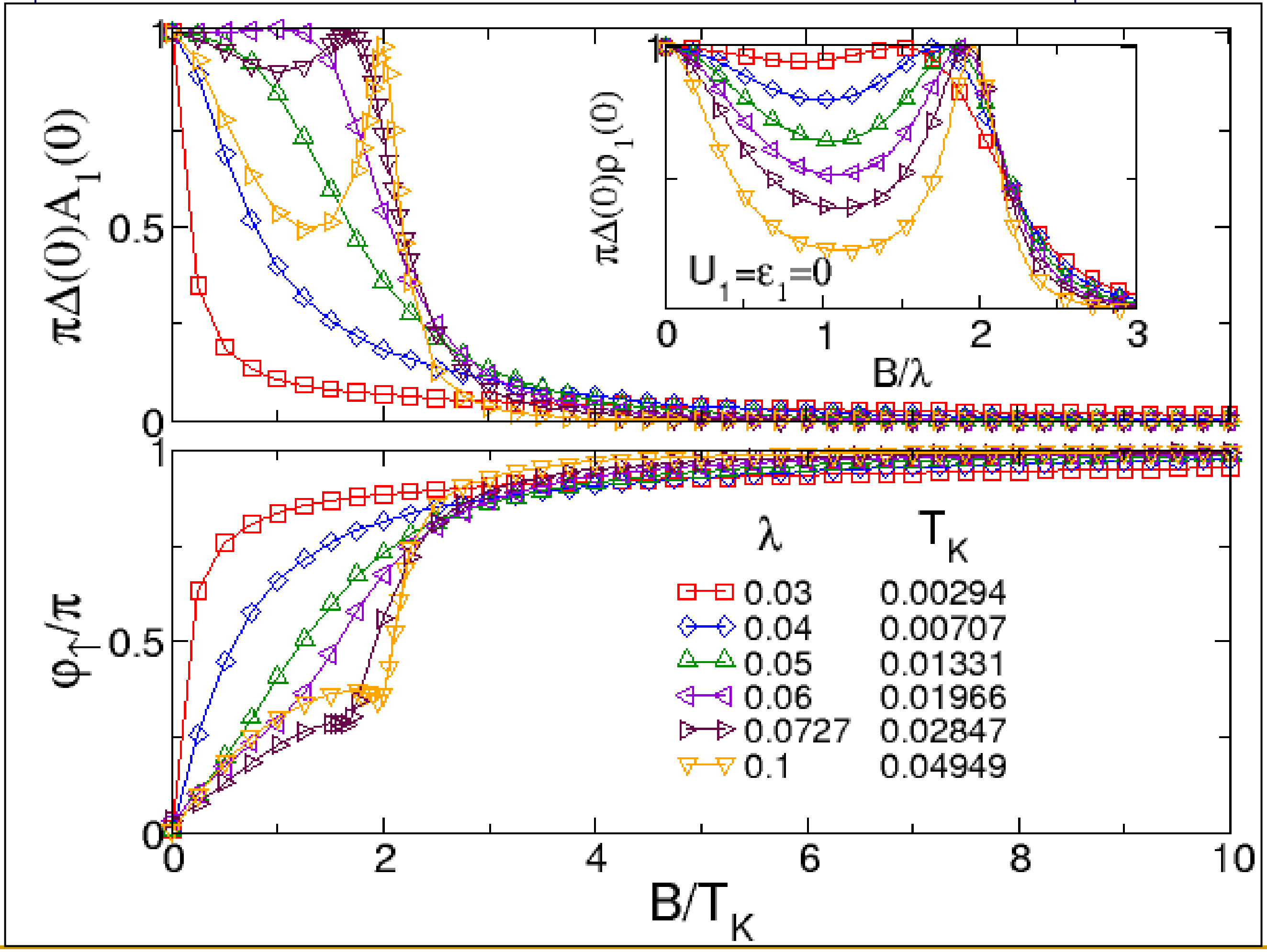
Magnetic Field dependence at the Fermi energy:

$$\boxed{\frac{\pi}{4}\sum_\sigma \Delta_\sigma(0)\,A_{1\sigma}(0,h) = \cos^2(\pi M_1 + \varphi_{1\uparrow})}$$

$$\boxed{M_1(h) = \frac{1}{2}\left(\langle n_{1\uparrow}\rangle - \langle n_{1\downarrow}\rangle\right)}$$

FSR revisited: *B*-dependence

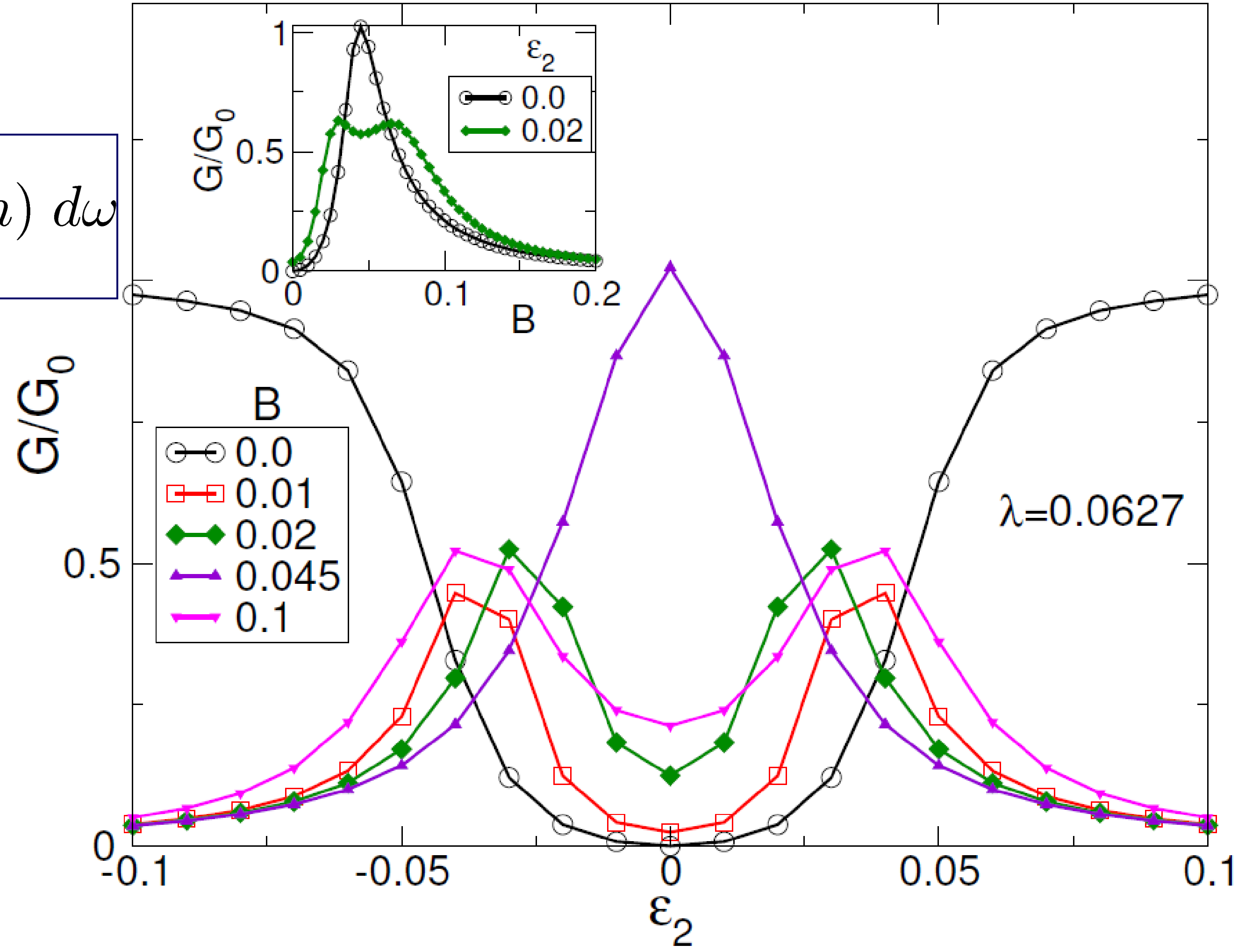
$$\frac{\pi}{4}\sum \Delta_{\sigma}(0)\,A_{1\sigma}(0,h) = \cos^{2}(\pi M_1 + \varphi_{1\uparrow})$$



Conductance

$$G_\sigma = \frac{\pi e^2}{4h} \sum_\sigma \int \Delta_\sigma(0)\, A_{1\sigma}(0,h)\, d\omega$$

$$G = \frac{G_\uparrow + G_\downarrow}{2}$$

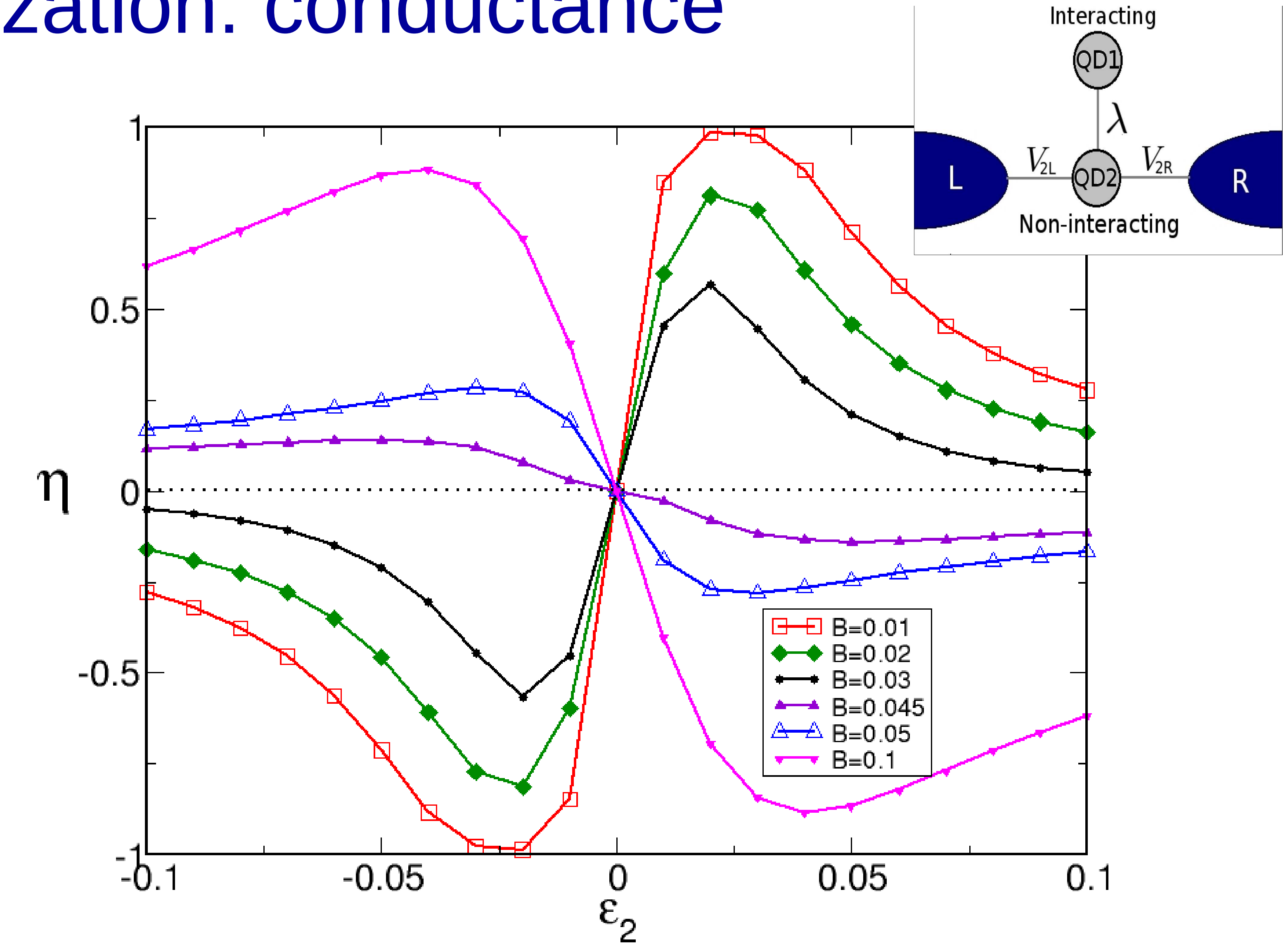


Spin-polarization: conductance

$$\eta = \frac{G_\uparrow - G_\downarrow}{G_\uparrow + G_\downarrow}$$

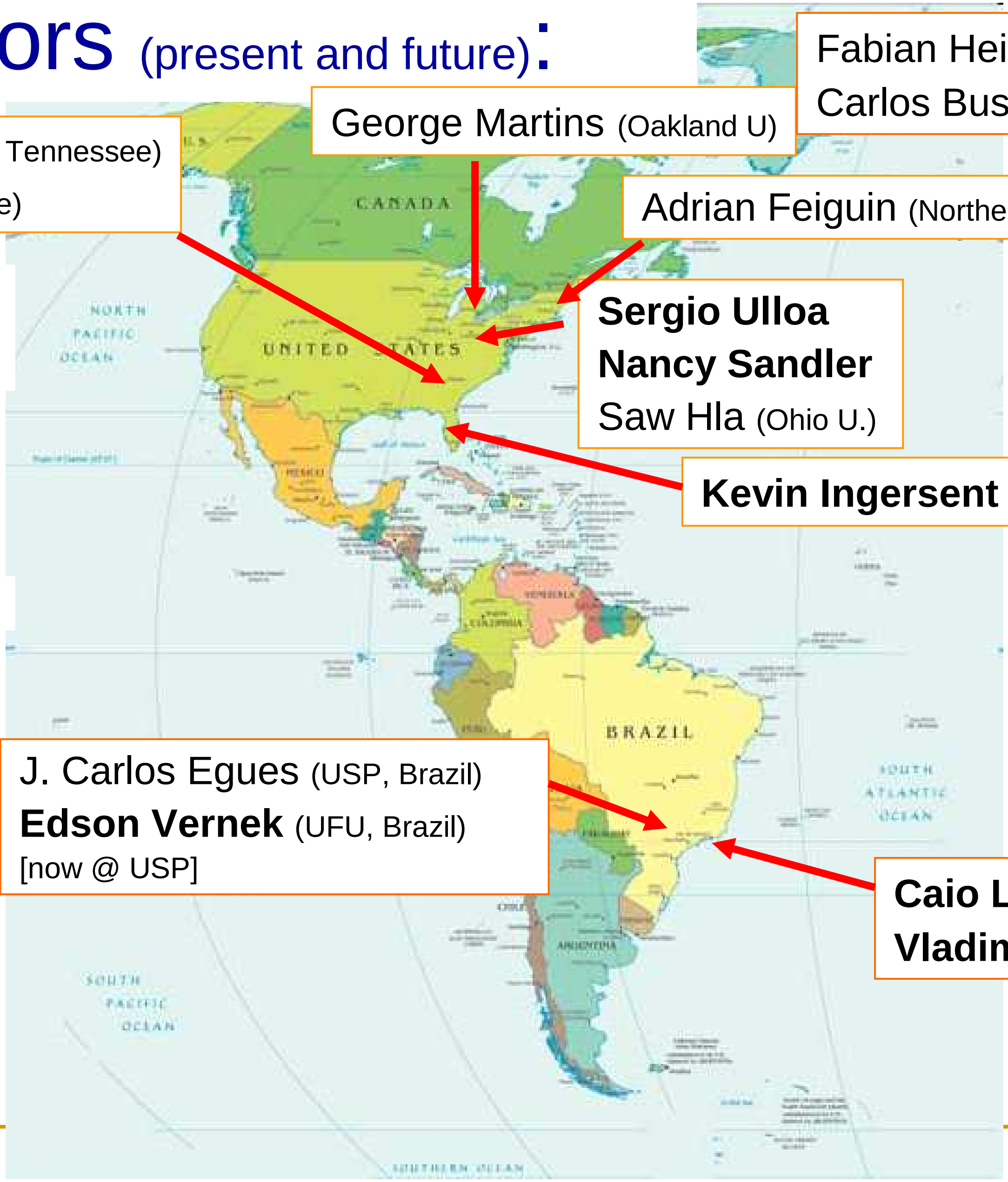
Wire + side-coupled QD (non-Kondo)

Aligia and Salguero,
PRB **70**, 075307 (2004).
M. E. Torio, et al.
EPJ B **37**, 399 (2004).



LDS, E. Vernek, K. Ingersent, N. Sandler, S. Ulloa,
PRB **87** 205313 (2013)

Collaborators (present and future):



Elbio Dagotto (Oak Ridge,U. Tennessee)
Gonzalo Alvarez (Oak Ridge)

George Martins (Oakland U)

Fabian Heidrich-Meisner (Münich)
Carlos Busser (Münich)

Adrian Feiguin (Northeastern)

Sergio Ulloa
Nancy Sandler
Saw Hla (Ohio U.)

Kevin Ingersent (UF)

J. Carlos Egues (USP, Brazil)
Edson Vernek (UFU, Brazil)
[now @ USP]

Caio Lewenkopf (UFF, Brazil)
Vladimir Miranda (UFF, Brazil)

Support (USA):
DOE: DE-AC05-00OR22725 (ORNL)
NSF: DMR-0706020 (TN);

Brazil: CNPq, FAPESP, PRP-USP

CNPq

FAPESP

USP
PRÓ-REITORIA
DE PESQUISA

NAP Q-Nano