Surface Code Threshold in the Presence of Correlated Errors

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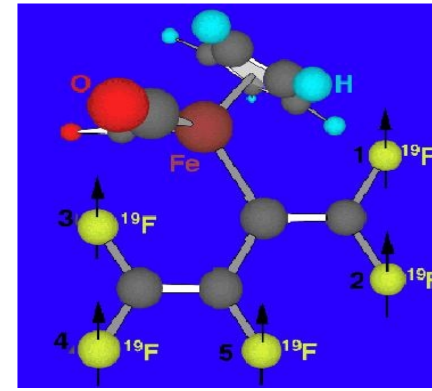
Quantum Computers



"But, we are going to be even more ridiculous later and consider bits written on **one atom** instead of the present 10^{11} atoms. Such **nonsense** is very **entertaining** to **professors** like me. I hope you will find it interesting and entertaining also."

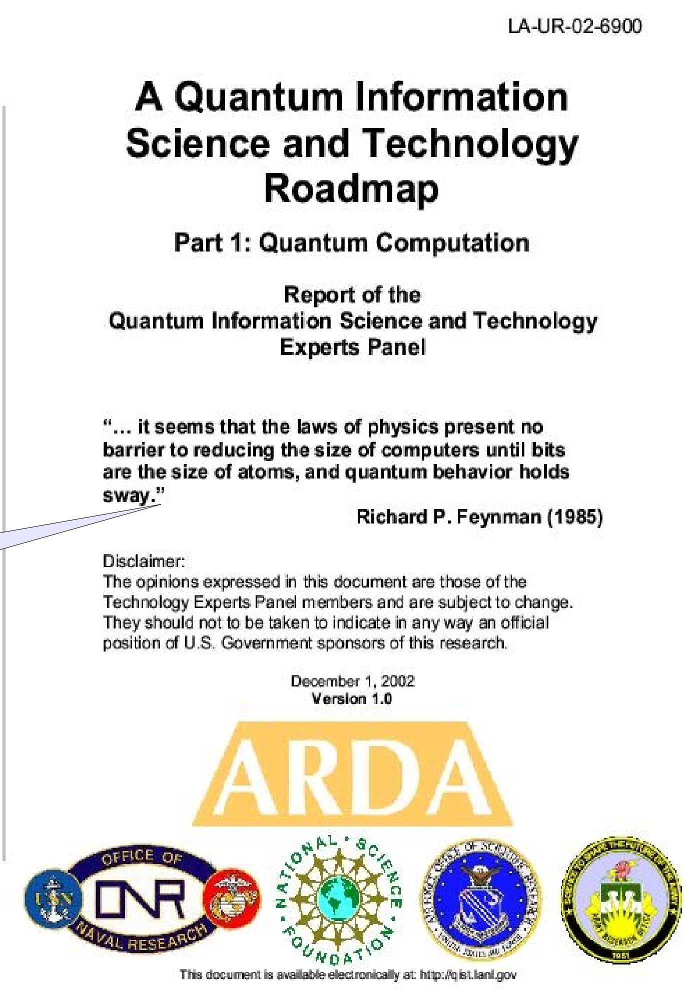
Richard Feynman em
"The Feynman Lectures on Computations".

The "Road Map to Quantum Computing"

Decoherence and Scalability are the main problems to the development of quantum computing.

"It Seems that the laws of physics present no barrier to reducing the size of computers until bits are the size of atoms, and quantum behavior holds sway."

Richard P. Feynman.



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A Quantum Information Science and Technology Roadmap

Part 1: Quantum Computation

Report of the Quantum Information Science and Technology Experts Panel

"… it seems that the laws of physics present no barrier to reducing the size of computers until bits are the size of atoms, and quantum behavior holds sway."

Richard P. Feynman (1985)

Disclaimer:
The opinions expressed in this document are those of the Technology Experts Panel members and are subject to change. They should not to be taken to indicate in any way an official position of U.S. Government sponsors of this research.

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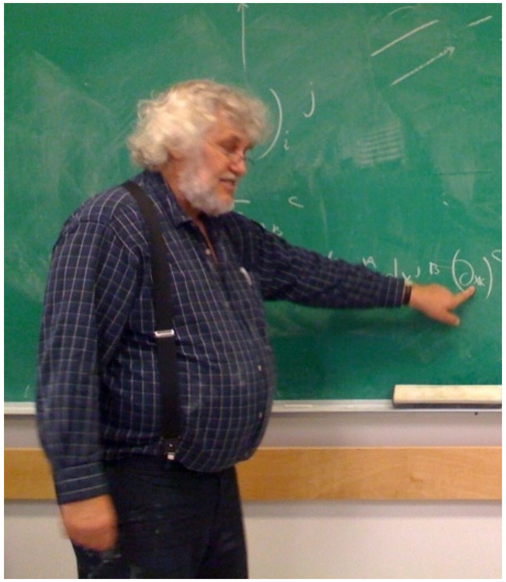
The big problem with Q.C.

Quantum coherence is fragile.

Tony Leggett

Amir Caldeira



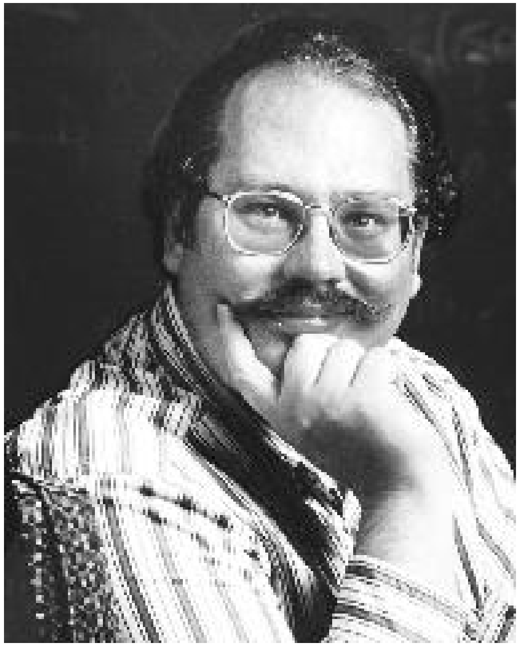
My paraphrasing of the papers

W. Unruh in 1995

“I do not believe it is possible
to maintain coherence.”

R. Alicki in 2007

Then, in 1996, came QEC



Peter Shor in an e-mail to Leonid Levin.

“I’m trying to say that you don’t need a machine which handles amplitudes with great precision. If you can build quantum gates with accuracy of 10^4, and put them together in the right fault-tolerant way, quantum mechanics says that you should be able to factor large numbers.”

Quantum Error Correction: the best hope for success.

There are other methods to reduce decoherence, but it is believed that QEC will be present in any QC.

QC and Quantum Communications became theoretically possible.

R. Alicki

My paraphrasing:

"I still do not believe it is possible."

Wojciech Zurek

Quantum error correction is the most versatile. Will be used some way or another.

Classical Error Correction

Majority vote:

$$|\bar{\uparrow}\rangle = |\uparrow\uparrow\uparrow\rangle$$
$$|\bar{\downarrow}\rangle = |\downarrow\downarrow\downarrow\rangle$$

No-Cloning Theorem:
It is impossible to duplicate an unknown quantum state.

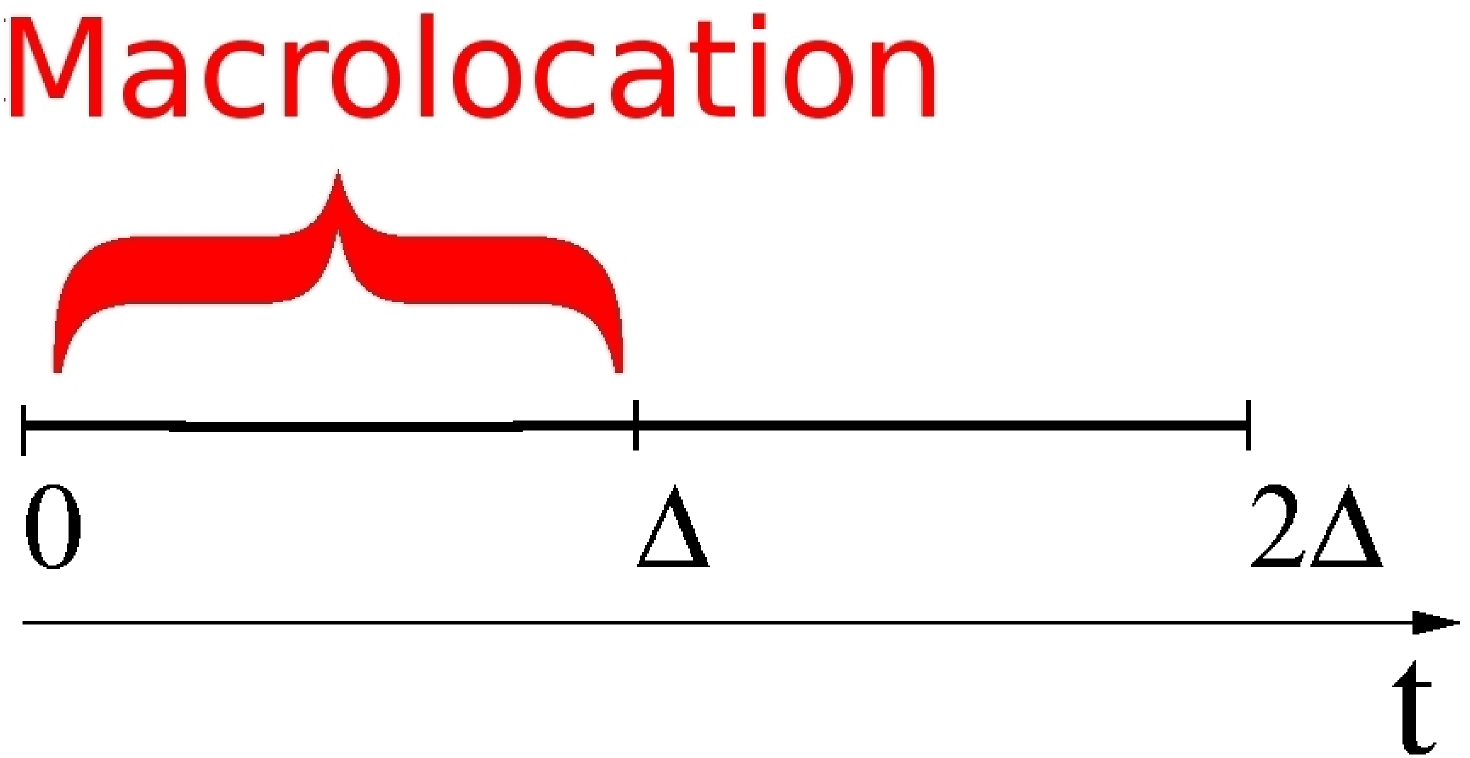
QEC

Entanglement was the solution

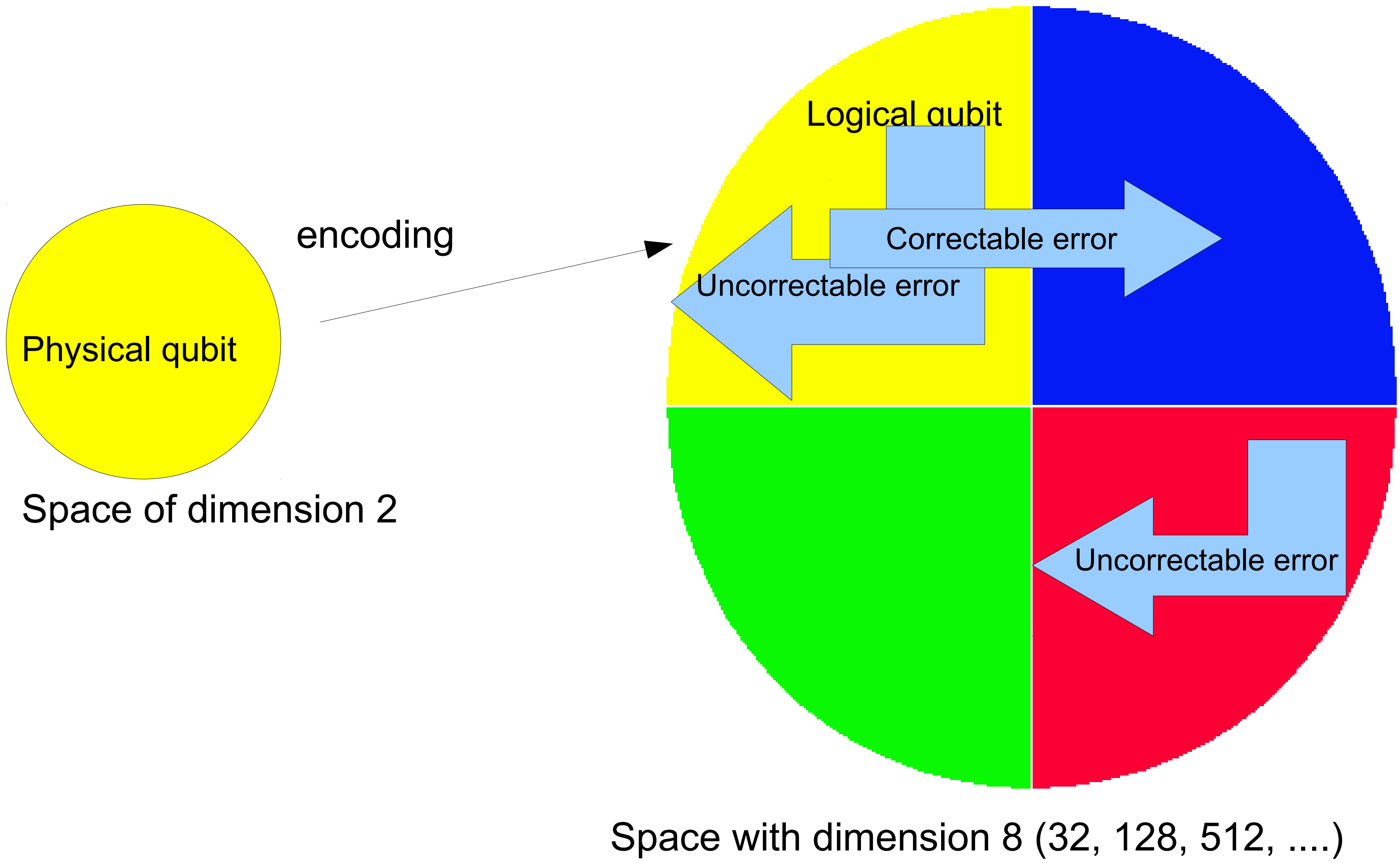
$$
\begin{aligned}
|\bar{\uparrow}\rangle &= \left(|\uparrow\uparrow\uparrow\rangle + |\uparrow\downarrow\downarrow\rangle + |\downarrow\uparrow\downarrow\rangle + |\downarrow\downarrow\uparrow\rangle\right)/2 \\
|\bar{\downarrow}\rangle &= \left(|\downarrow\downarrow\downarrow\rangle + |\downarrow\uparrow\uparrow\rangle + |\uparrow\downarrow\uparrow\rangle + |\uparrow\uparrow\downarrow\rangle\right)/2
\end{aligned}
$$

We cannot clone a
quantum state,
but we can hide it.

QEC



QEC


encoding
Physical qubit
Space of dimension 2
Logical qubit
Correctable error
Uncorrectable error
Uncorrectable error
Space with dimension 8 (32, 128, 512, ...)

The smallest QEC code

5-qubits stabilizer code:

$$\begin{aligned} g_1 &= xzzx1 \\ g_2 &= 1xzzx \\ g_3 &= x1xzz \\ g_4 &= zx1xz \\ \bar{X} &= xxxxx \\ \bar{Y} &= yyyyy \\ \bar{Z} &= zzzzz \end{aligned}$$

All this operators keep the qubit in its logical Hilbert space.

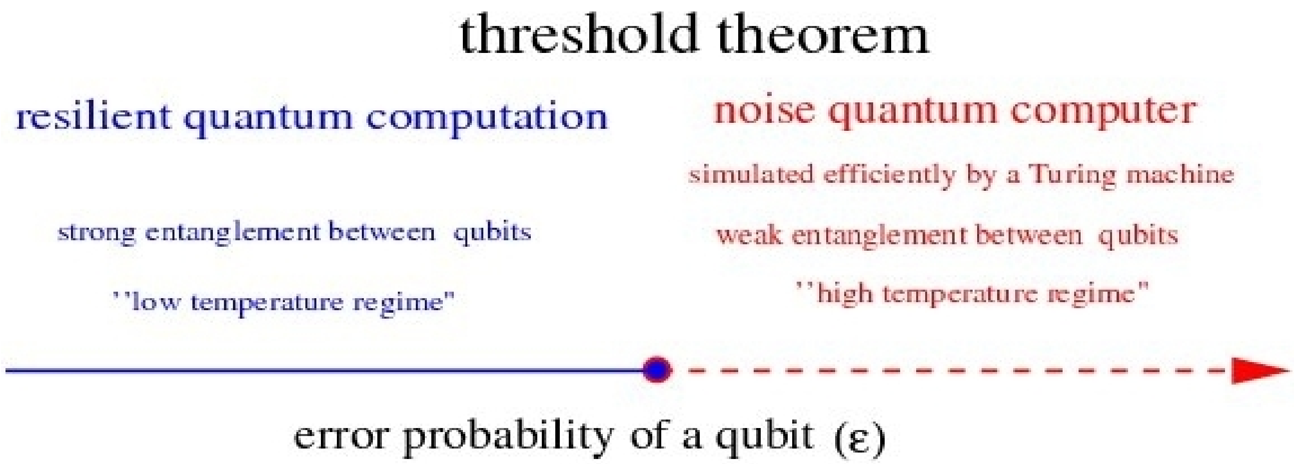
Quantum Error Correction

- "threshold theorem"

  Provided the noise strength is below a critical value, quantum information can be protected for arbitrarily long times. Hence, the computation is said to be fault tolerant or resilient.

Quantum Error Correction

- Dorit Aharonov, Phys. Rev. A 062311 (2000). Quantum to classical phase transition in a noisy QC.



QEC

- Usual assuptions in the traditional QEC theory
  - fast measurements(not fundamental-Aliferes-DiVincenzo 07);
  - fast/slow gates (not fundamental – my opinion);
  - error models (add probabilities instead of amplitudes).

What if we would like to start from a microscopic model?

The standard prescription:

1. quantum master equation and
2. dynamical semi-groups.

This is a natural approach: *The computer is the object of interest; hence one starts the discussion by integrating out the environmental degrees of freedom.*

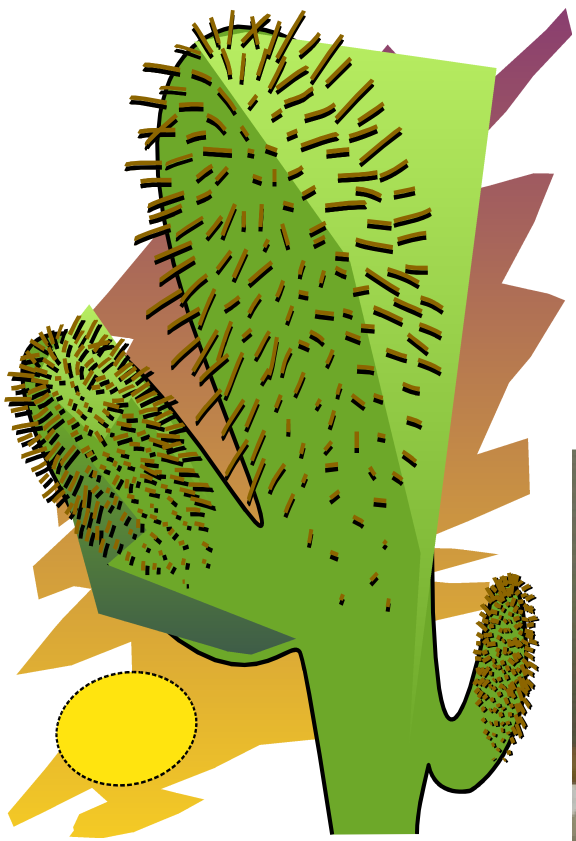
The price that we usually pay:

3. First, Born approximation;
4. then, the Markov approximation.

QEC is a perturbative method!

There are still some thorns...

All theories have hypothesis. QEC has some that are hard to fulfill in a physical system.



R. Alicki, Daniel A. Lidar and Paolo Zanardi, PRA 73 052311 (2006). I*nternal Consistency of Fault-Tolerant Quantum Error Correction in Light of Rigorous Derivations of the Quantum Markovian Limit.*

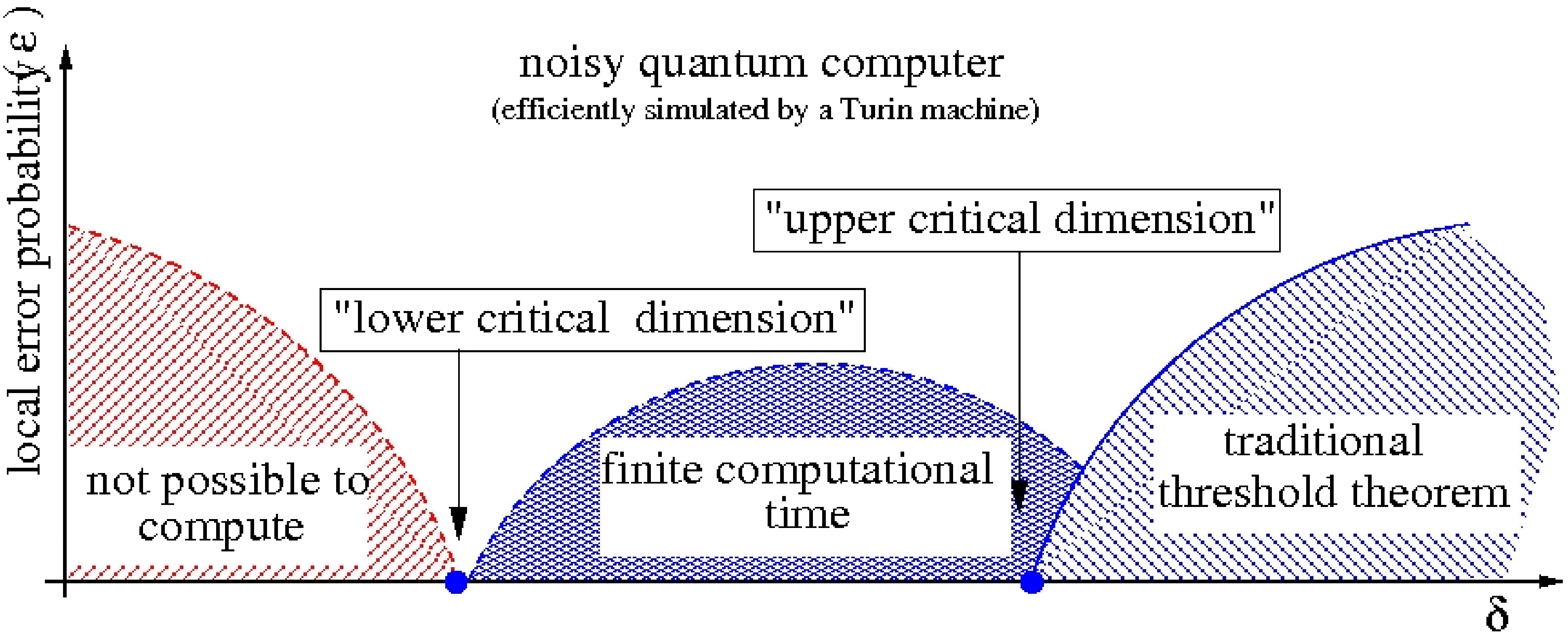
“... These assumptions are: fast gates, a constant supply of fresh cold ancillas, and a Markovian bath. We point out that these assumptions may not be mutually consistent in light of rigorous formulations of the Markovian approximation. ...”

What is the problem with the B-M approximation?

Ulrich Weiss in Quantum Dissipative Systems:

While the Markov assumption can easily be dropped, the more severe limitation of this method is the Born approximation for the kernel. In conclusion, the Born-Markov quantum master equation method provides a reasonable description in many cases, such as in NMR, inn laser physics, and in a variety of chemical reactions. However, this method turned out to be not useful in most problems of solid state physics at low temperatures for which neither the Born approximation is valid nor the Markov assumption holds.

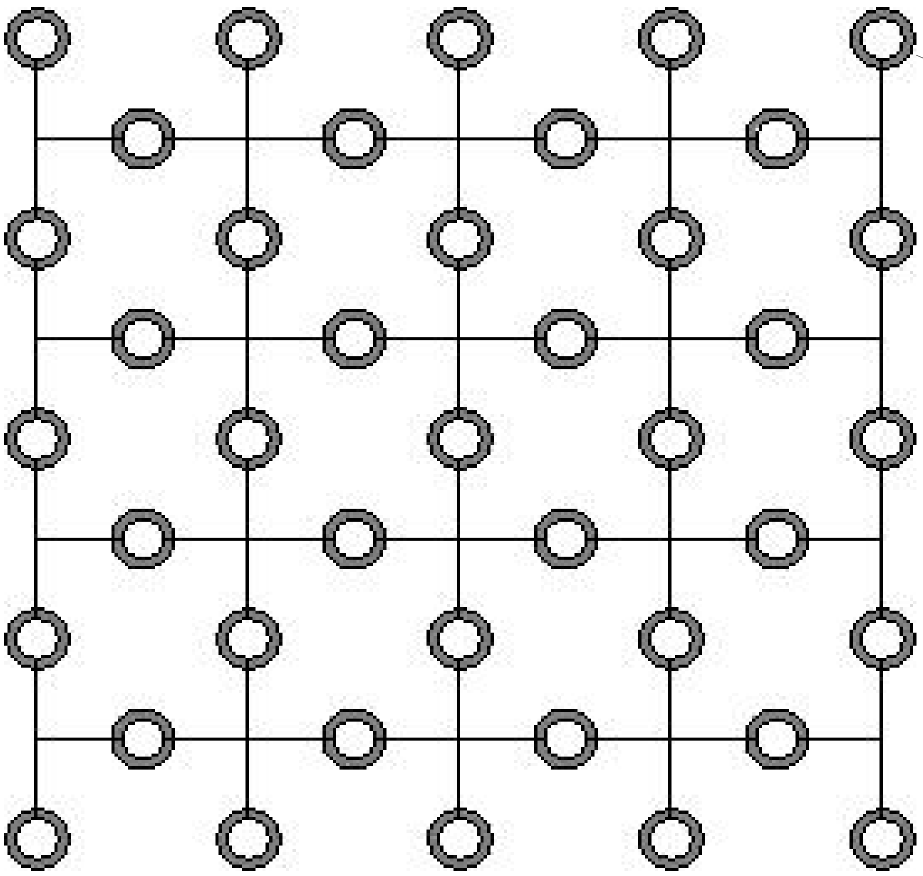
A schematic phase diagram



Surface Code

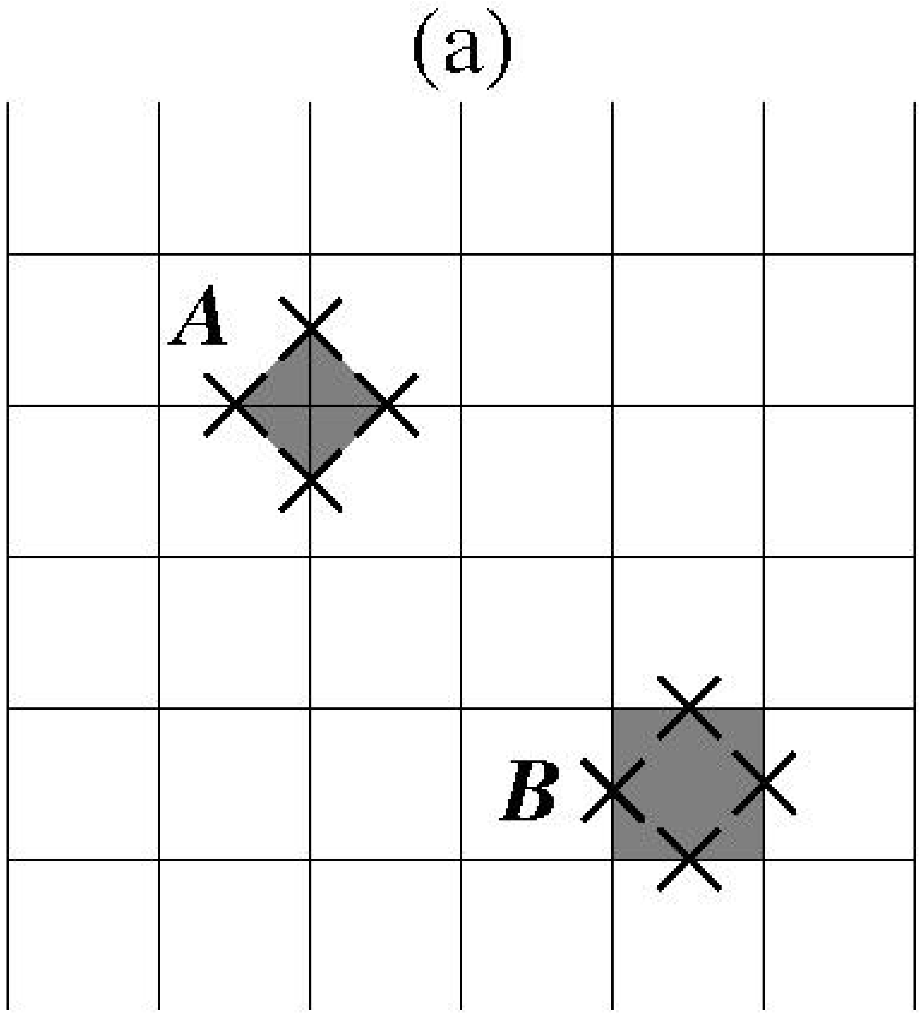
- considered one of the best quantum error correction codes
  - all syndromes and operations can be performed with spatially local operators
  - threshold estimates show that, for sufficiently large lattices, the error threshold is the highest known for two-dimensional architectures with only nearest-neighbor interactions

Surface Codes



Qubits in a square lattice

Surface Code



Star Operators:

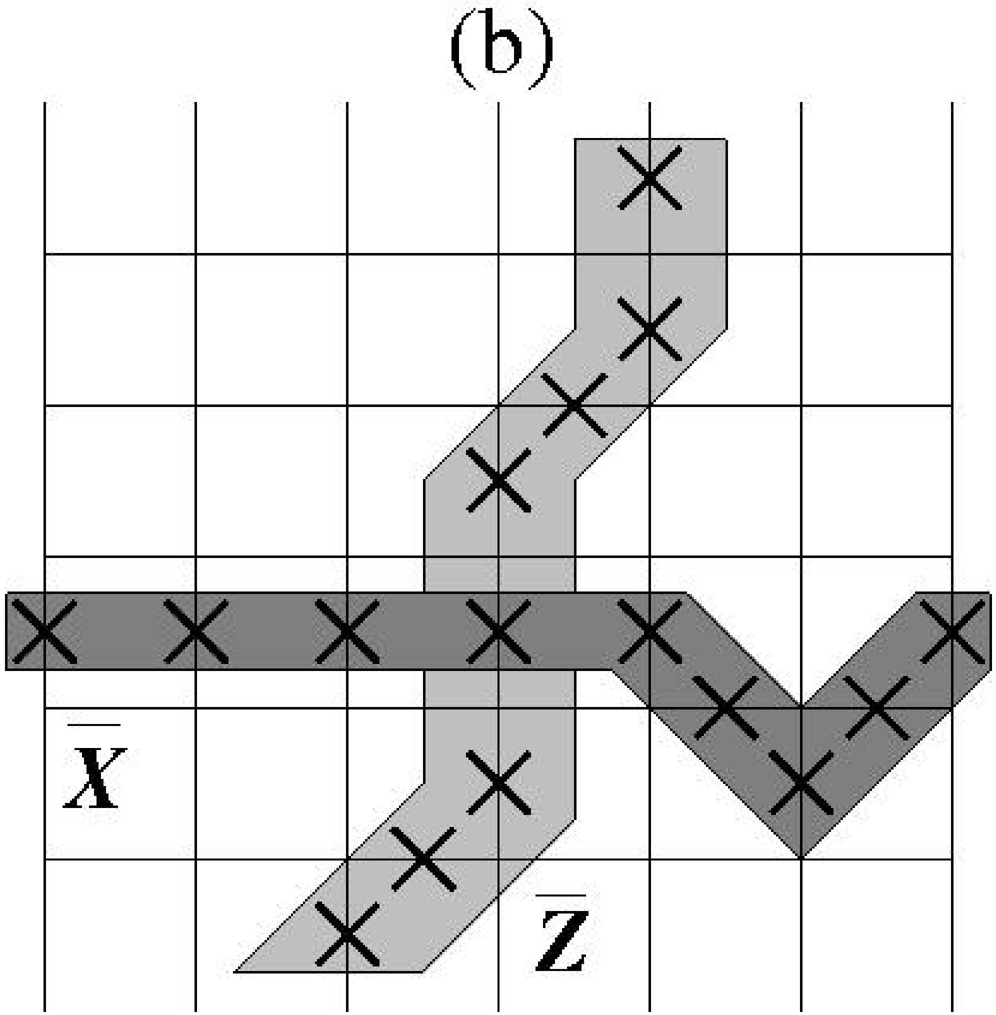
$$A_{\Diamond} = \prod_{i\in\Diamond} \sigma_i^x$$

Plaquette Operators:

$$B_{\Box} = \prod_{i\in\Box} \sigma_i^z$$

Stabilizers of the code are all possible products of star and plaquettes

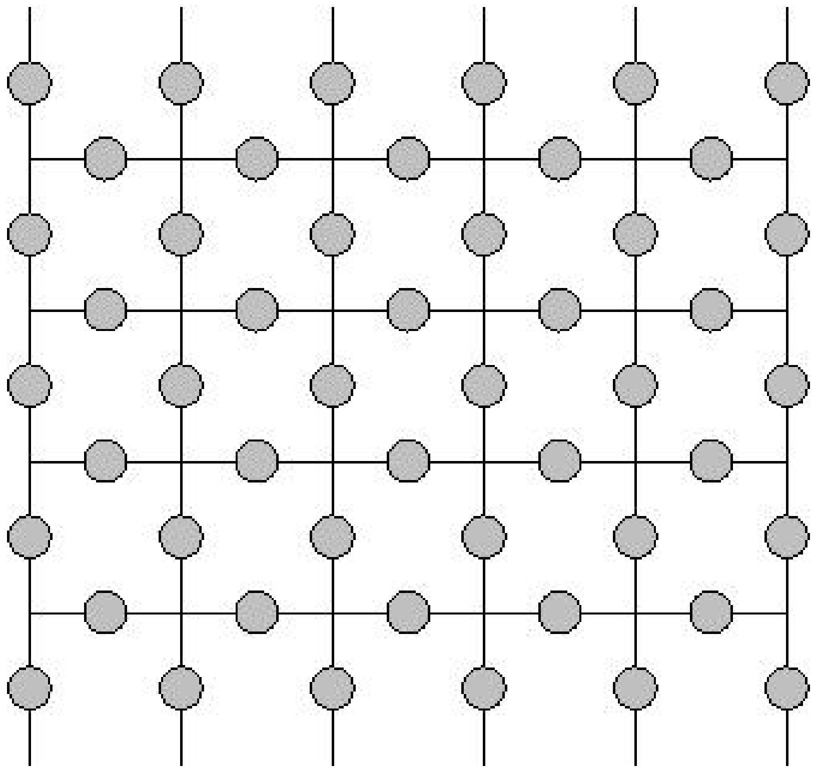
Surface Code



$$\bar{X} = \prod_{i \in \Gamma} \sigma_i^x$$

$$\bar{Z} = \prod_{i \in \bar{\Gamma}} \sigma_i^z$$

Surface Code



Logical States:

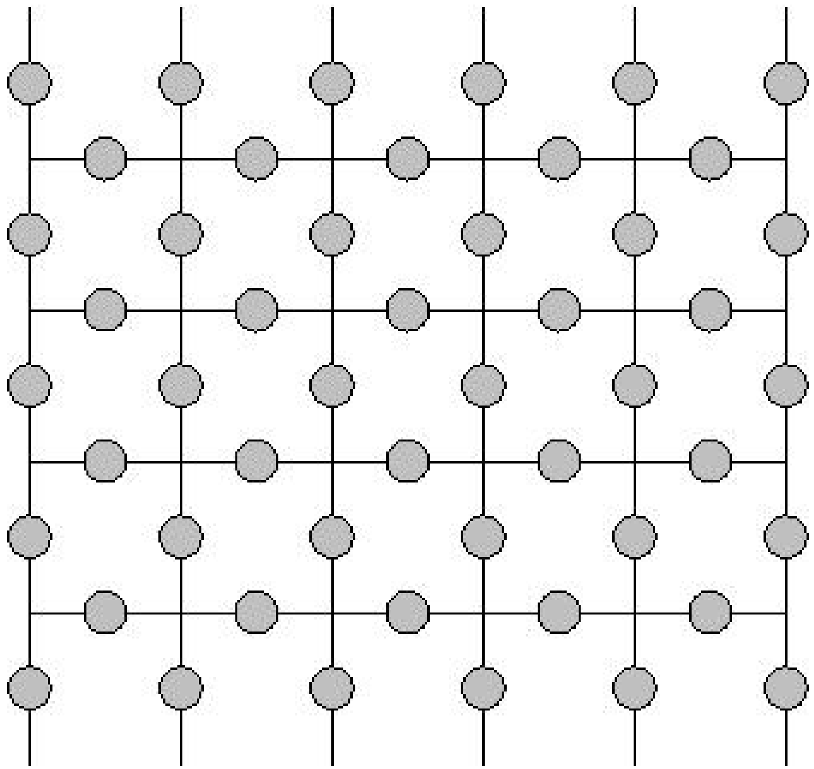
$$\{|\bar{\uparrow}\rangle = G|F_z\rangle, |\bar{\downarrow}\rangle = \bar{X}|\bar{\uparrow}\rangle\},$$

where:

$$G = \frac{1}{\sqrt{2^{N_A}}} \prod_{\Diamond} (1 + A_{\Diamond})$$

$$|F_z\rangle = \prod_{i=1}^{N} |\uparrow\rangle_{i,z}$$

Surface Code



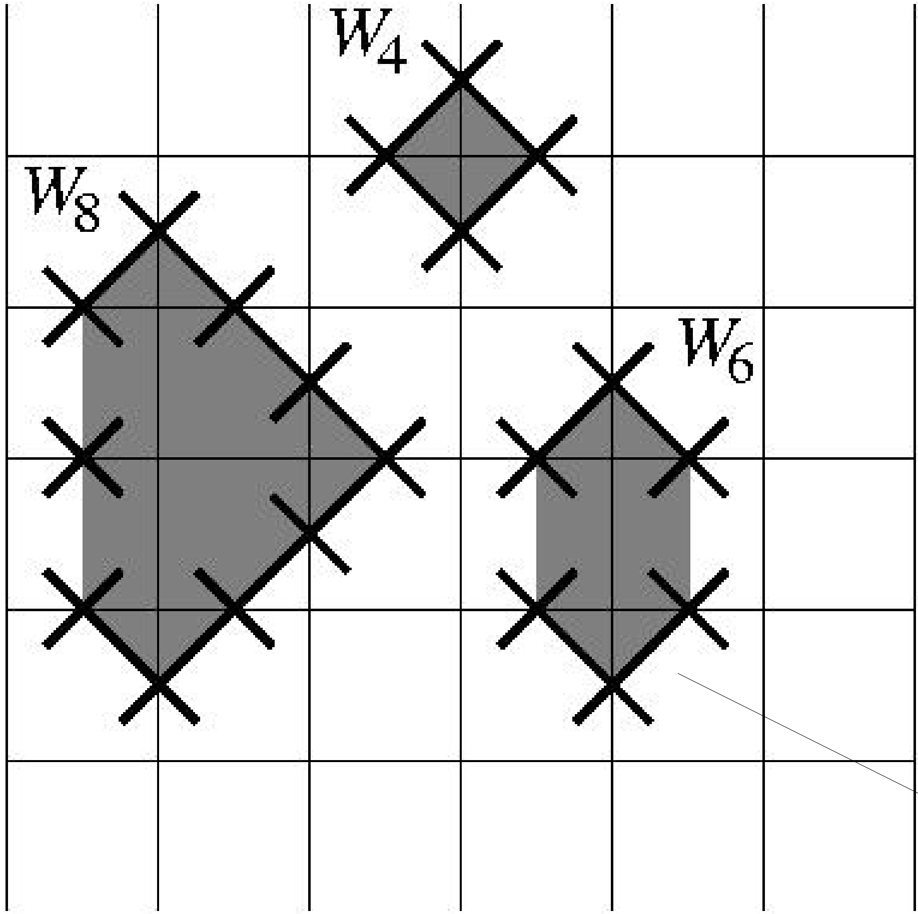
$$\left\{|\bar{\uparrow}\rangle = G|F_z\rangle, |\bar{\downarrow}\rangle = \bar{X}|\bar{\uparrow}\rangle\right\},$$

$$G = \frac{1}{\sqrt{2^{N_A}}}\prod_{\diamondsuit}(1 + A_{\diamondsuit})$$

$$|F_z\rangle = \prod_{i=1}^{N} |\uparrow\rangle_{i,z}$$

Logical words are: equal weight superpositions of all gauges.

Surface Code



X are qubit flips.

$$\{|\bar{\uparrow}\rangle = G|F_z\rangle, |\bar{\downarrow}\rangle = \bar{X}|\bar{\uparrow}\rangle\},$$

$$G = \frac{1}{\sqrt{2^{N_A}}} \prod_{\diamondsuit} (1 + A_{\diamondsuit})$$

$$|F_z\rangle = \prod_{i=1}^{N} |\uparrow\rangle_{i,z}$$

Wilson Loops – gauge invariant objects

Modeling the environment

$$\omega_{\alpha,k} = \omega_0 \left(|\mathbf{k}|/k_0\right)^{z_\alpha} \qquad \left[a_{\alpha,\mathbf{k}}, a^\dagger_{\beta,\mathbf{q}}\right] = \delta_{\mathbf{k},\mathbf{q}}\delta_{\alpha,\beta}$$

qubits

$$H = \sum_{\alpha=\{x,z\}} \sum_{|\mathbf{k}|\neq 0} \boxed{\omega_{\alpha,k}}\, a^\dagger_{\alpha,\mathbf{k}} \boxed{a_{\alpha,\mathbf{k}}} + \sum_{\alpha=\{x,z\}} \sum_{\mathbf{x}} \lambda_\alpha \boxed{:f^\alpha(\mathbf{x}):}\ \boxed{\sigma^\alpha_{\mathbf{x}}}$$

$$:f^\alpha(\mathbf{x}): = (2\pi/L)^{D/2} \sum_{\mathbf{k}\neq 0} \left(\boxed{u_{\alpha,k}} e^{i\mathbf{k}\cdot\mathbf{x}} a^\dagger_{\alpha,\mathbf{k}} + \mathrm{H.c.}\right),$$

$$|u_{\alpha,k}|^2 = \kappa_0^{-D} \left(|\mathbf{k}|/k_0\right)^{2s_\alpha}$$

Time evolution in the interaction picture

$$\hat{U}(\Delta, 0) = T_t \exp\left[-i \int_0^\Delta dt\, H_I(t)\right]$$

$$\begin{aligned}\hat{U}(\Delta) \quad &= \quad e^{-\frac{\lambda^2}{2} N \mathcal{G}_{\mathbf{rr}}(\Delta)}\, e^{-\frac{\lambda^2}{2}\sum_{\mathbf{r}\neq\mathbf{s}} \Phi_{\mathbf{rs}}(\Delta)\sigma_{\mathbf{r}}^x \sigma_{\mathbf{s}}^x} \\ &\times \;\; : e^{-i\lambda \sum_{\mathbf{r}} F_{\mathbf{r}}(\Delta)\sigma_{\mathbf{r}}^x} :,\end{aligned}$$

$$\Phi_{\mathbf{rs}}(\Delta) = \mathcal{G}_{\mathbf{rs}}(\Delta) + \int_0^\Delta dt_1 \int_0^{t_1} dt_2\, [f(\mathbf{r}, t_1), f(\mathbf{s}, t_2)]$$

$$\mathcal{G}_{\mathbf{rs}}(\Delta) = \langle 0 | F_{\mathbf{r}}(\Delta)\, F_{\mathbf{s}}(\Delta) | 0 \rangle \qquad F_{\mathbf{r}}(\Delta) = \int_0^\Delta dt\, f(\mathbf{r}, t),$$

An example: an ohmic bath

$$
\begin{aligned}
\hat{U}(\Delta) &= e^{-\frac{\lambda^2}{2} N \mathcal{G}_{\mathbf{rr}}(\Delta)}\, e^{-\frac{\lambda^2}{2}\sum_{\mathbf{r}\neq\mathbf{s}} \Phi_{\mathbf{rs}}(\Delta)\sigma^x_{\mathbf{r}}\sigma^x_{\mathbf{s}}} \\
&\times \; : e^{-i\lambda \sum_{\mathbf{r}} F_{\mathbf{r}}(\Delta)\sigma^x_{\mathbf{r}}} :,
\end{aligned}
$$

$$
\Phi_{\mathbf{rs}}(\Delta) = \left(\frac{\omega_0}{v}\right)^2 \begin{cases} \operatorname{arcosh}\left(\frac{v\Delta}{|\mathbf{r}-\mathbf{s}|}\right) + \frac{i\pi}{2}, & 0 < |\mathbf{r}-\mathbf{s}| < v\Delta, \\ i\arcsin\left(\frac{v\Delta}{|\mathbf{r}-\mathbf{s}|}\right), & 0 < v\Delta < |\mathbf{r}-\mathbf{s}|. \end{cases}
$$

Fictitious Temperature

$$\hat{U}(\Delta) = e^{-\frac{\lambda^2}{2} N \mathcal{G}_{\mathbf{rr}}(\Delta)} e^{-\frac{\lambda^2}{2} \sum_{\mathbf{r} \neq \mathbf{s}} \Phi_{\mathbf{rs}}(\Delta) \sigma_{\mathbf{r}}^x \sigma_{\mathbf{s}}^x} \times :e^{-i\lambda \sum_{\mathbf{r}} F_{\mathbf{r}}(\Delta) \sigma_{\mathbf{r}}^x}:,$$

$$e^{-\beta \sum_{\mathbf{r} \neq \mathbf{s}} J_{\mathbf{rs}} \sigma_{\mathbf{r}}^x \sigma_{\mathbf{s}}^x}$$

$$\beta = \frac{1}{2} \left(\frac{\lambda \omega_0}{v} \right)^2$$

Initial State

$$|\psi_0\rangle = (G|F_z\rangle) \otimes |0\rangle.$$

Logical state $\bar{\uparrow}$

Bosonic ground state

Syndrome Extraction: the "nonerror evolution"

Assuming that we can reset the environment at the end of a QEC step, we can rewrite the syndrome extraction as the projector:

$$P' = |\psi_0\rangle\langle\psi_0| + \bar{X}|\psi_0\rangle\langle\psi_0|\bar{X}.$$

Evolution without a logical error

Evolution with a logical error

The fidelity after the QEC procedure

$$\mathcal{F} = \frac{|\langle\psi_0|P'\hat{U}(\Delta)|\psi_0\rangle|}{|\langle\psi_0|\hat{U}^\dagger(\Delta)P'\hat{U}(\Delta)|\psi_0\rangle|}$$

$$\mathcal{F} \quad = \quad \frac{|\mathcal{A}|}{\sqrt{|\mathcal{A}|^2 + |\mathcal{B}|^2}},$$

$$\mathcal{A} = \langle\psi_0|\hat{U}(\Delta)|\psi_0\rangle$$

Evolution with no logical error

$$\mathcal{B} = \langle\psi_0|\bar{X}\hat{U}(\Delta)|\psi_0\rangle$$

Evolution with a logical error

A "Statistical Mechanics" Problem

$$\mathcal{A} = \chi \left\langle F_z \left| e^{-\beta\mathcal{H}} G^2 \right| F_z \right\rangle , \qquad \mathcal{B} = \chi \left\langle F_z \left| \bar{X} e^{-\beta\mathcal{H}} G^2 \right| F_z \right\rangle ,$$

$$\mathcal{H} = \sum_{\mathbf{r} \neq \mathbf{s}} J_{\mathbf{rs}} \, \sigma^x_{\mathbf{r}} \sigma^x_{\mathbf{s}} ,$$

Projectors

If $\beta = 0$ then the Fidelity is unity.

The "low temperature" expansion

$$|F_z\rangle = \prod_{i=1}^{N}\left(\frac{|\uparrow\rangle_{i,x} + |\downarrow\rangle_{i,x}}{\sqrt{2}}\right).$$

$$E_s = \langle S|\mathcal{H}|S\rangle$$

$$\mathcal{A} = \frac{\chi}{2^N}\sum_S e^{-\beta E_s}\langle S|G^2|S\rangle,$$

$$\mathcal{B} = \frac{\chi}{2^N}\sum_S e^{-\beta E_s}\langle S|\bar{X}G^2|S\rangle,$$

Looks like a Partition Function of the 2d-Ising model

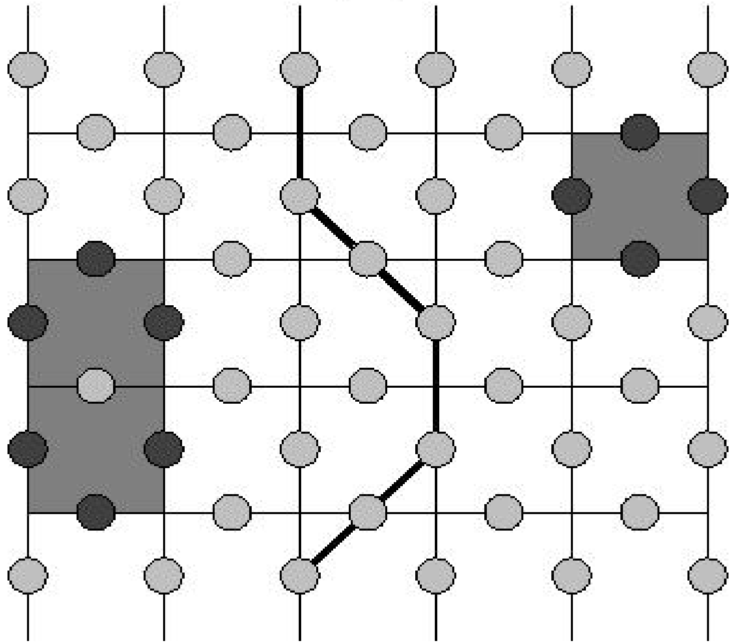
Elements of the x basis

How to Evaluate with the Projectors?

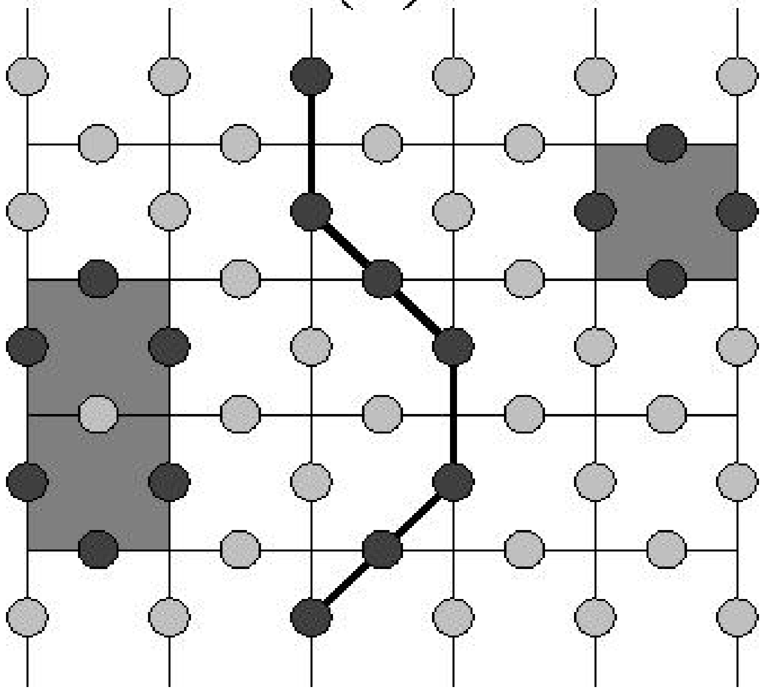
Define 2 families of states label by a logical Z

$$|S_+\rangle = \prod_j B_{\square_j}|F_x\rangle \qquad \text{and} \qquad |S_-\rangle = \bar{Z}_\gamma|S_+\rangle.$$

(a)



(b)



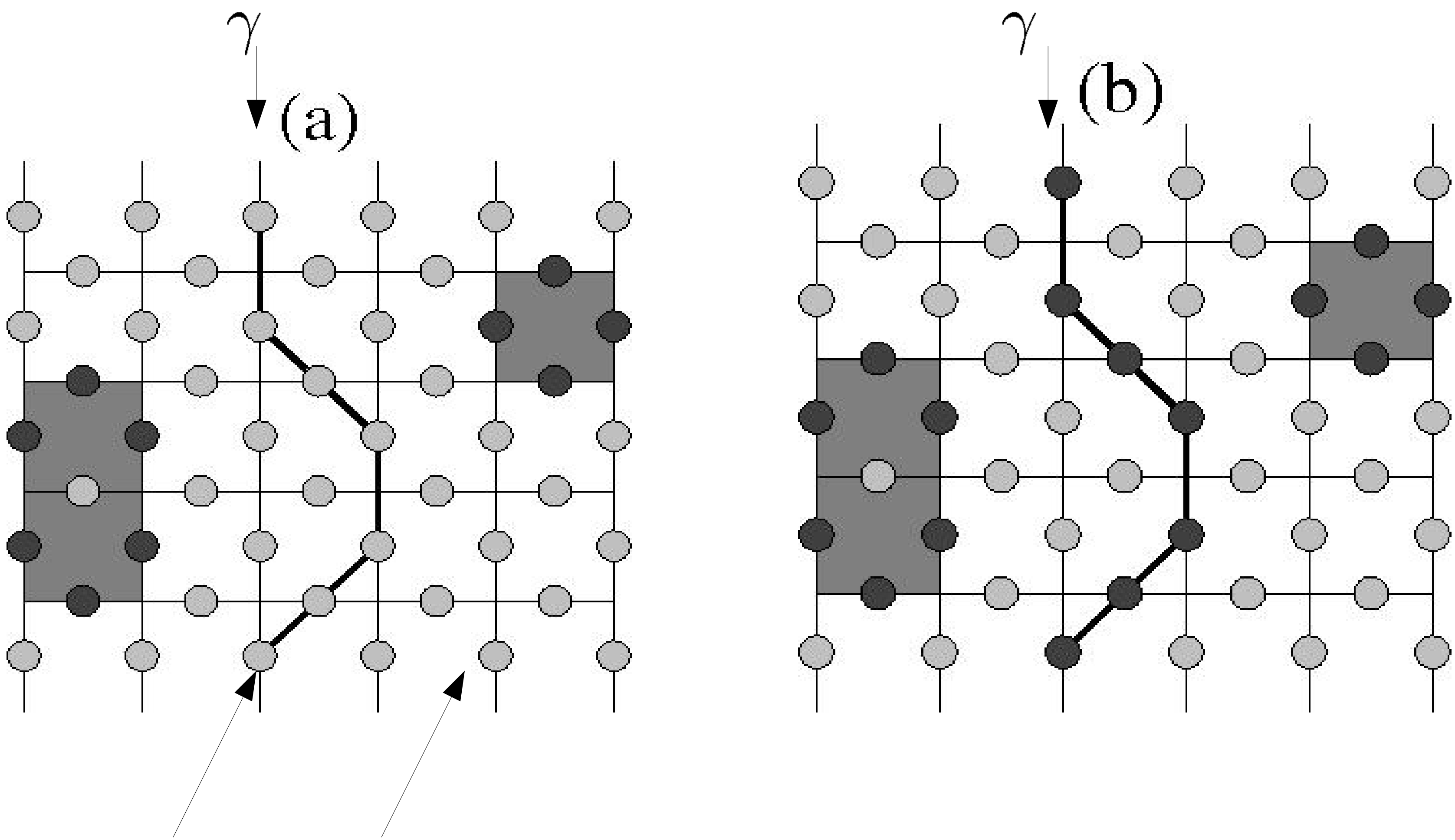
How to Evaluate with the Projectors?

Using these Families

$$\mathcal{A} = \frac{\chi}{2^N}(\mathcal{T}_+ + \mathcal{T}_-) \qquad \mathcal{B} = \frac{\chi}{2^N}(\mathcal{T}_+ - \mathcal{T}_-)$$

$$\mathcal{T}_\pm = \sum_{S_\pm} \left\langle S_\pm \left| e^{-\beta\mathcal{H}} \right| S_\pm \right\rangle .$$

How to Evaluate with the Projectors?



$$\{\mathbf{r}\} = \{\mathbf{t}_\gamma\} \oplus \{\mathbf{u}_\gamma\}$$

$$\{\mathbf{s}\} = \{\mathbf{v}_\gamma\} \oplus \{\mathbf{w}_\gamma\}$$

$$\sum_{S_\pm} = \sum_\gamma \sum_{S^\gamma_\pm}$$

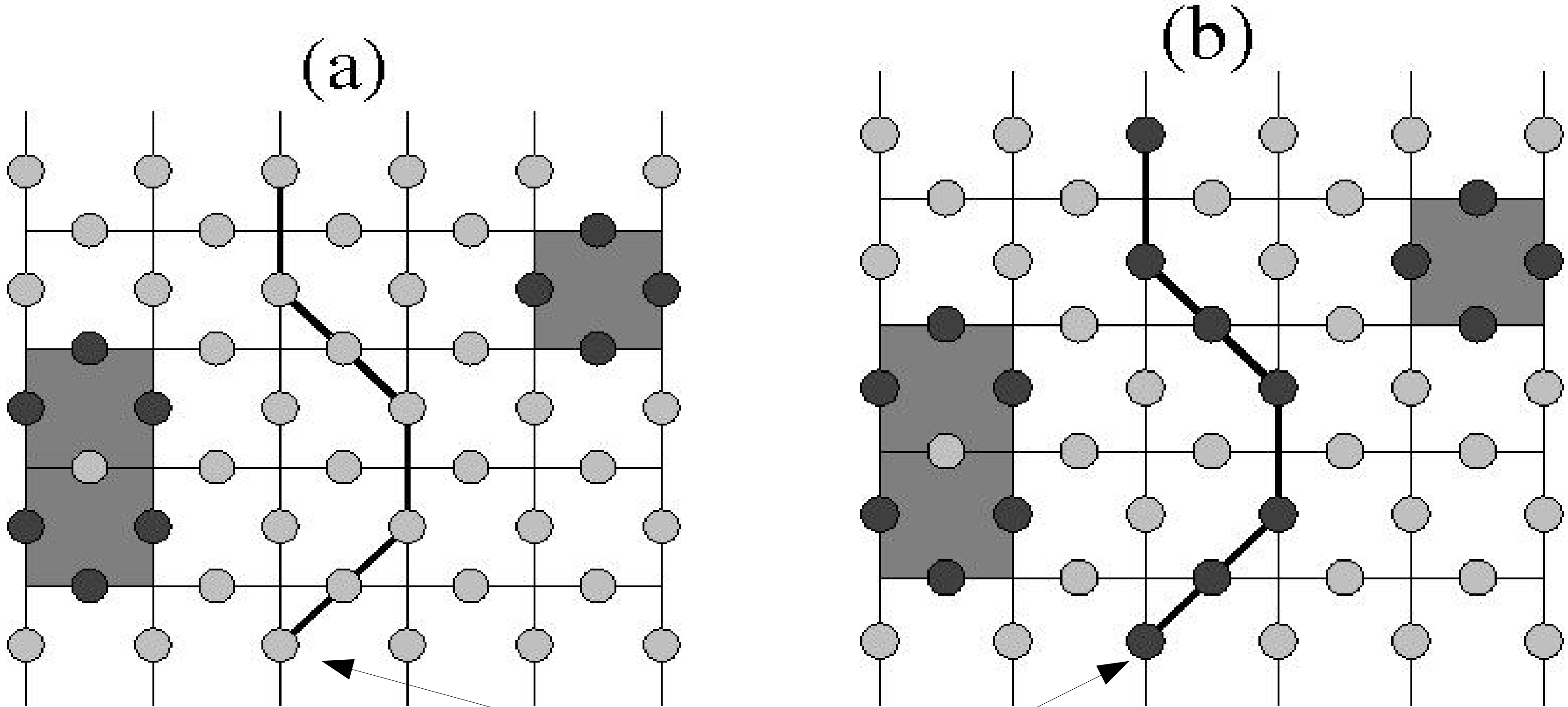
How to Evaluate with the Projectors?

$$\mathcal{T}_{\pm} = \sum_{\gamma} e^{-\beta \epsilon_{\gamma}} \sum_{S_{\pm}^{\gamma}} z_{\pm}^{\gamma}, \text{ where } \epsilon_{\gamma} = \left\langle S_{\pm}^{\gamma} \middle| \sum_{\mathbf{tv}} J_{\mathbf{tv}}\, \sigma_{\mathbf{t}}^{x} \sigma_{\mathbf{v}}^{x} \middle| S_{\pm}^{\gamma} \right\rangle$$

$$z_{\pm}^{\gamma} = \left\langle S_{+}^{\gamma} \middle| e^{-\beta \sum_{\mathbf{u} \neq \mathbf{w}} J_{\mathbf{uw}}\, \sigma_{\mathbf{u}}^{x} \sigma_{\mathbf{w}}^{x} \mp \beta \sum_{\mathbf{w}} h_{\mathbf{w}}^{\gamma}\, \sigma_{\mathbf{w}}^{x}} \middle| S_{+}^{\gamma} \right\rangle,$$

$$h_{\mathbf{w}}^{\gamma} = \sum_{\mathbf{t}} J_{\mathbf{tw}}$$

How to Evaluate with the Projectors?

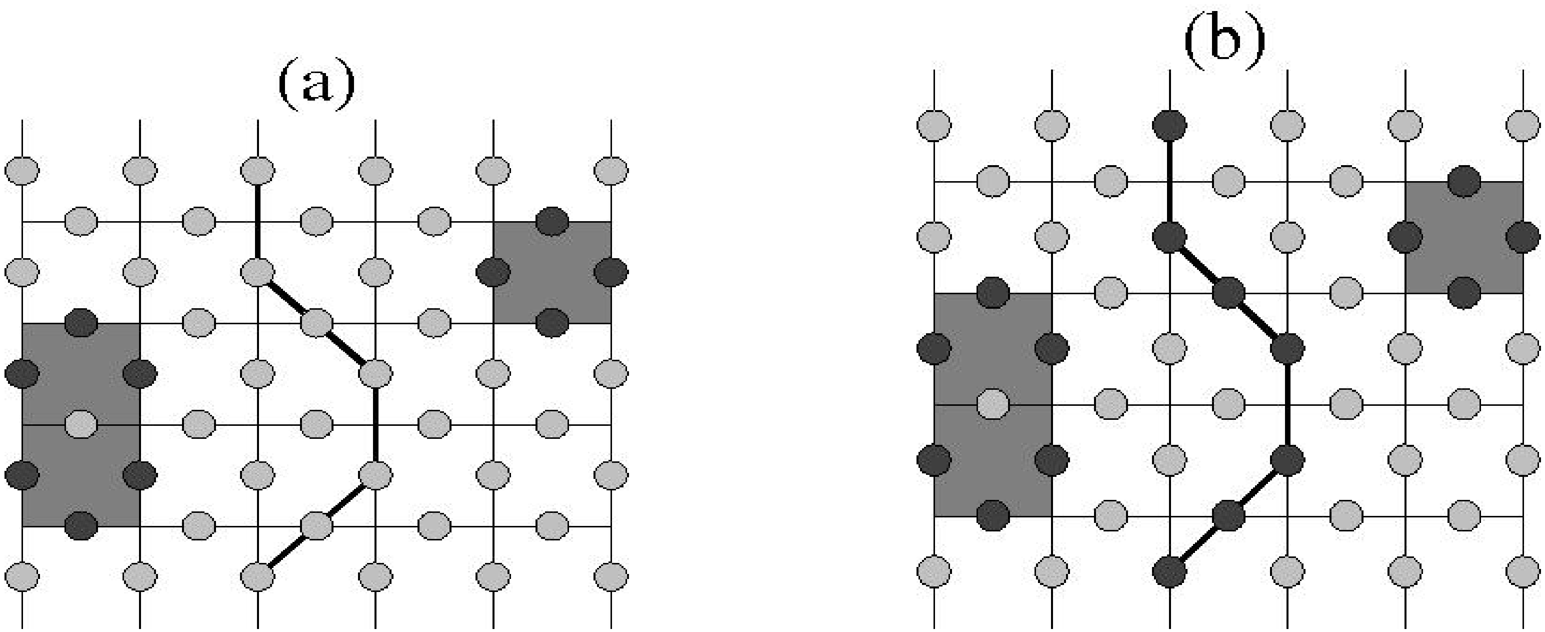


$$z^{\gamma}_{\pm} = \left\langle S^{\gamma}_{+} \left| e^{-\beta \sum_{\mathbf{u} \neq \mathbf{w}} J_{\mathbf{uw}} \, \sigma^x_{\mathbf{u}} \sigma^x_{\mathbf{w}} \mp \beta \sum_{\mathbf{w}} h^{\gamma}_{\mathbf{w}} \, \sigma^x_{\mathbf{w}}} \right| S^{\gamma}_{+} \right\rangle ,$$

The logical error path works like a magnetic field on the other sites.

Phase Transition in a simpler problem

$$J_{\mathbf{rs}} = \begin{cases} J, & \mathbf{r}, \mathbf{s} \text{ nearest neighbors and J is real,} \\ 0 & \text{otherwise,} \end{cases}$$



$$z_{\pm}^{\gamma} = \left\langle S_{+}^{\gamma} \left| e^{-\beta \sum_{\mathbf{u} \neq \mathbf{w}} J_{\mathbf{uw}} \, \sigma_{\mathbf{u}}^{x} \sigma_{\mathbf{w}}^{x} \mp \beta \sum_{\mathbf{w}} h_{\mathbf{w}}^{\gamma} \, \sigma_{\mathbf{w}}^{x}} \right| S_{+}^{\gamma} \right\rangle ,$$

Phase Transition in a simpler problem

$$z_{S^\gamma_\pm} = \left\langle F_x \left| \prod_j B_{\Box_j} e^{\beta \sum_{\mathbf{u}\neq\mathbf{w}} J_{\mathbf{uw}}\, \sigma^x_{\mathbf{u}} \sigma^x_{\mathbf{w}}} \prod_j B_{\Box_j} \right| F_x \right\rangle,$$

Maps into a ferromagnetic problem:

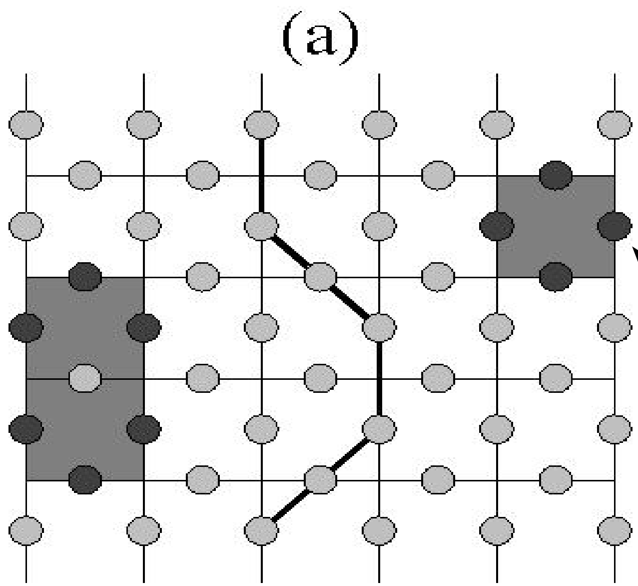
$$z_{S^\gamma_\pm} = e^{\beta \sum_{\mathbf{a}\neq\mathbf{b}} J_{\mathbf{ab}}}\, e^{\beta \sum_{\mathbf{c}\neq\mathbf{d}} J_{\mathbf{cd}}}\, e^{-\beta \sum_{\mathbf{a}\neq\mathbf{c}} J_{\mathbf{ac}}}.$$

$$\sum_{S^\gamma_\pm} z^\gamma = e^{\beta E_F} \left(1 + \sum_{\text{loops}} e^{-2\beta \sum_{\mathbf{a}\in \text{loop}, \mathbf{c}\notin\text{loop}} J_{\mathbf{ac}}} \right).$$

Ground State Energy

Excitations

Phase Transition in a simpler problem



Energy cost of a loop $\approx 4J\ell$

Length of the loop

$$\sum_{S_\pm^\gamma} z^\gamma = e^{\beta E_F}\left(1 + \sum_{\text{loops}} e^{-2\beta \sum_{\mathbf{a}\in\text{loop},\mathbf{c}\notin\text{loop}} J_{\mathbf{ac}}}\right).$$

Order-Disorder Transition

Number of Loops with Length $\ell \approx \mu^{\ell}$

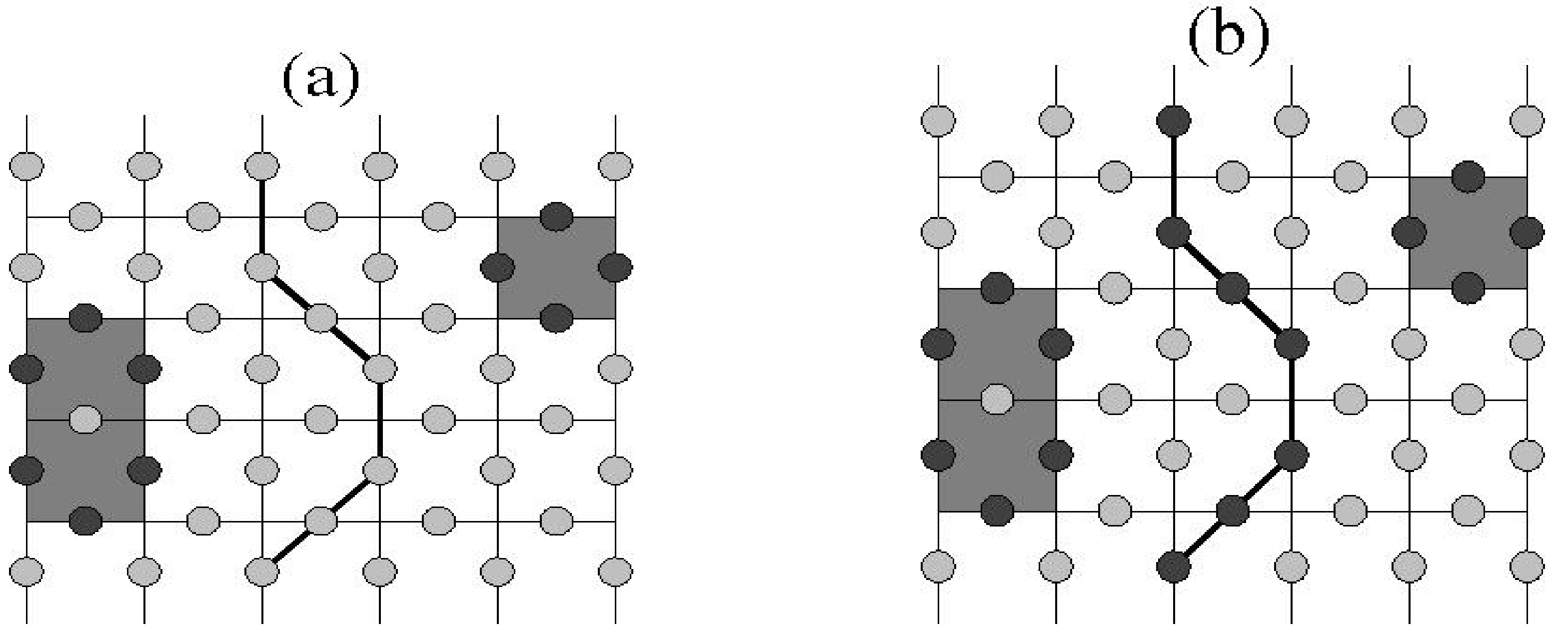
Connectivity constant $\mu \approx 2.638$

Contribution from loops of length ℓ to the "partition function"

$$\mu^{\ell}\, e^{-8\beta J\ell}$$

Order-Disorder transition at $\beta_c \approx \dfrac{\ln \mu}{8J}.$

Order-Disorder Transition



$$\beta_c \approx \frac{\ln \mu}{8J}.$$

Above the critical temperature we have a paramagnetic phase and the boundary field is irrelevant. The Fidelity is unity.

Below the critical temperature we have an ordered phase and the boundary magnetic field splits the topological degenerescence. The Fidelity is smaller than one.

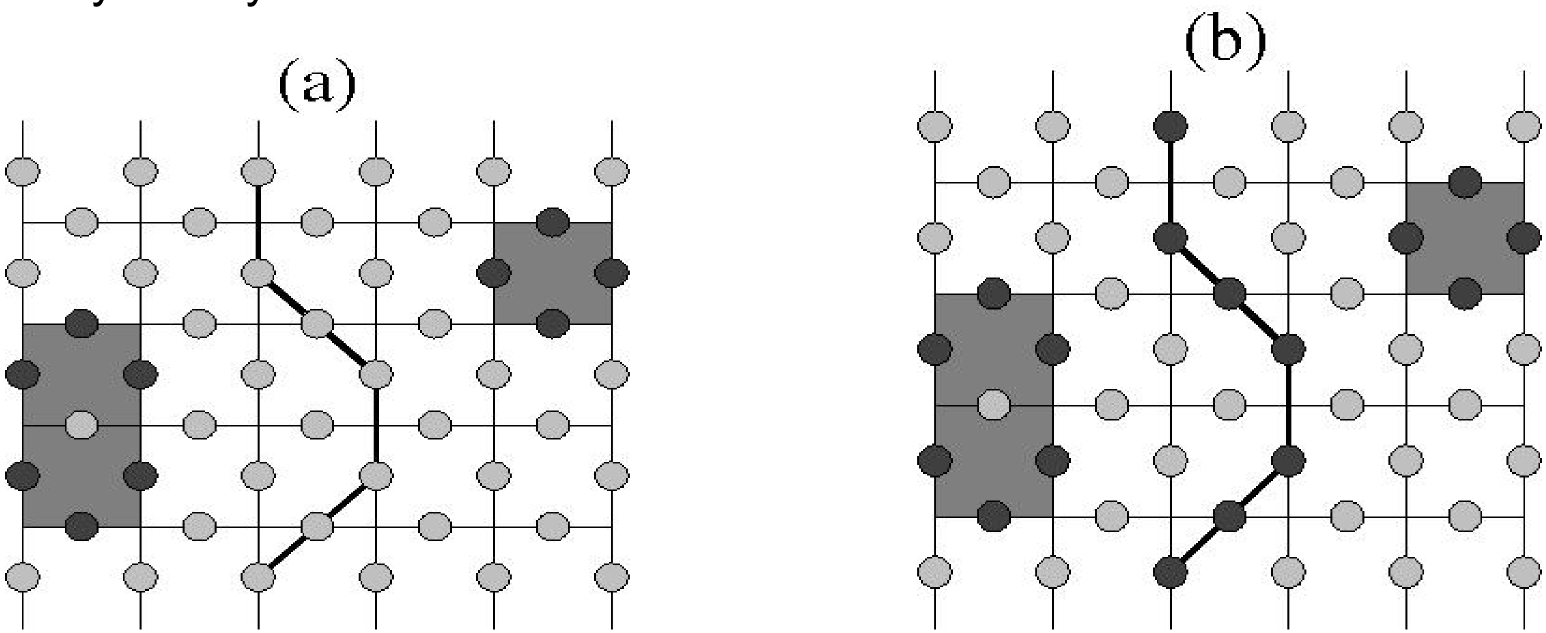
What about more complicated problems?

Very hard to tell:

- Imaginary parts of the correlators

- Frustration

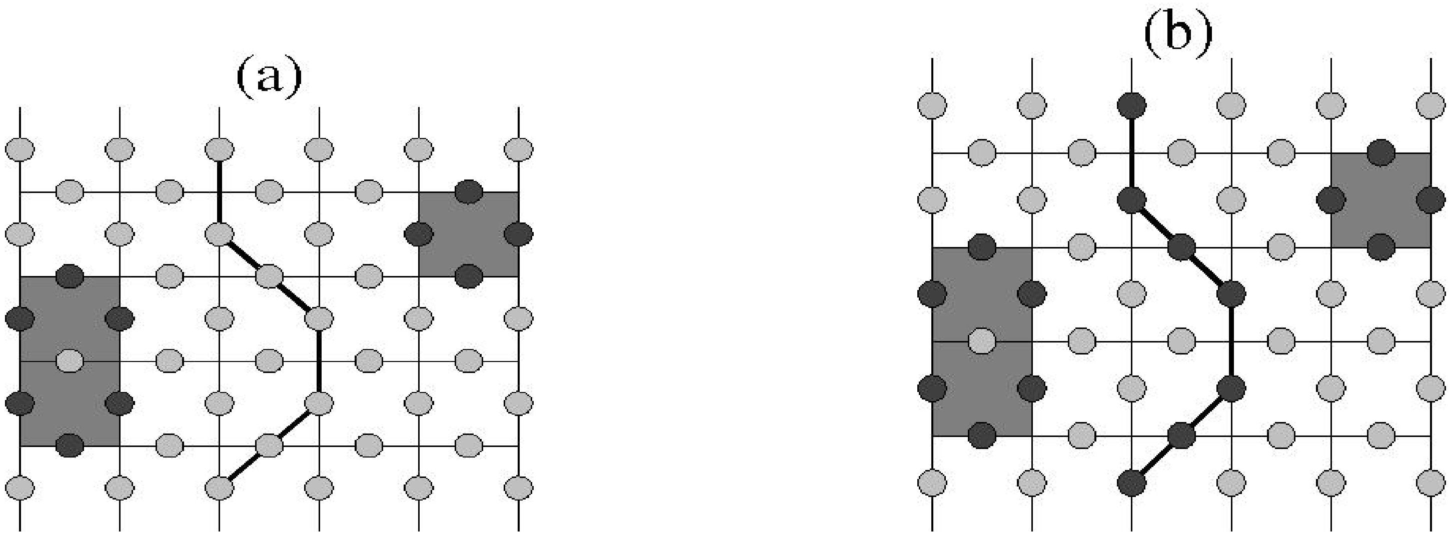
Maybe only with numerics....



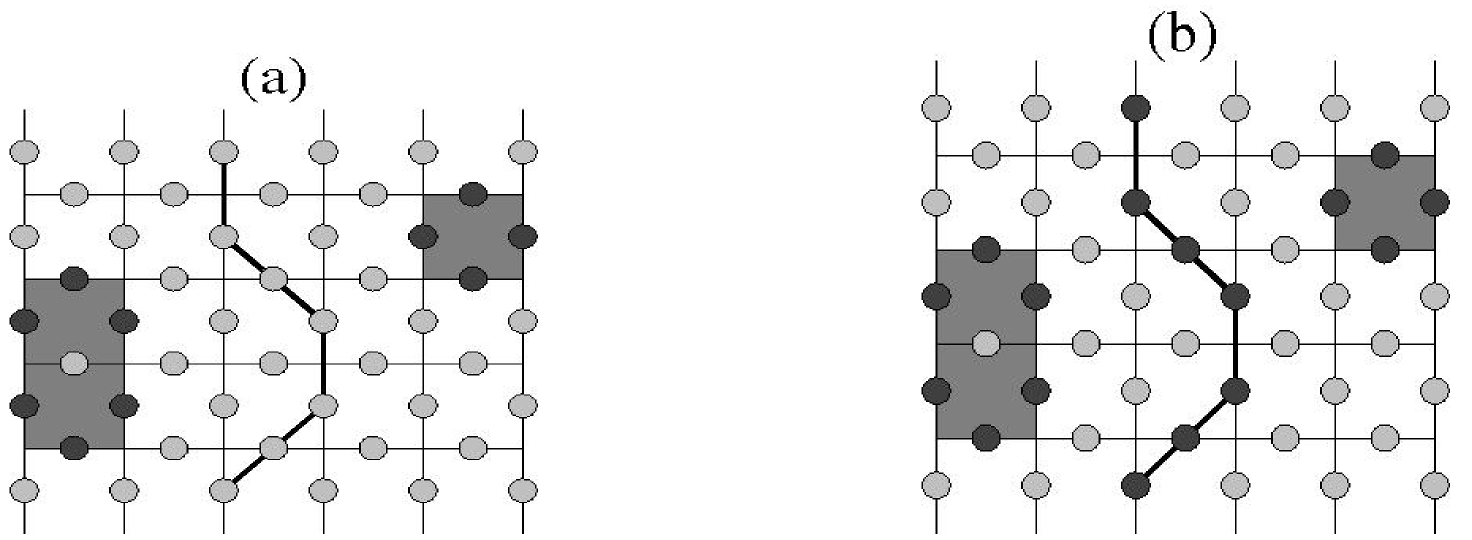
Maybe something not so hard

Consider only the real part of the ohmic correlator and only between different sublattices:

$$\Phi_{\mathbf{rs}}(\Delta) = \left(\frac{\omega_0}{v}\right)^2 \begin{cases} \operatorname{arcsh}\left(\frac{v\Delta}{|\mathbf{r}-\mathbf{s}|}\right) + \frac{i\pi}{2}, & 0 < |\mathbf{r}-\mathbf{s}| < v\Delta, \\ i\arcsin\left(\frac{v\Delta}{|\mathbf{r}-\mathbf{s}|}\right), & 0 < v\Delta < |\mathbf{r}-\mathbf{s}|. \end{cases}$$



The order-disorder transition

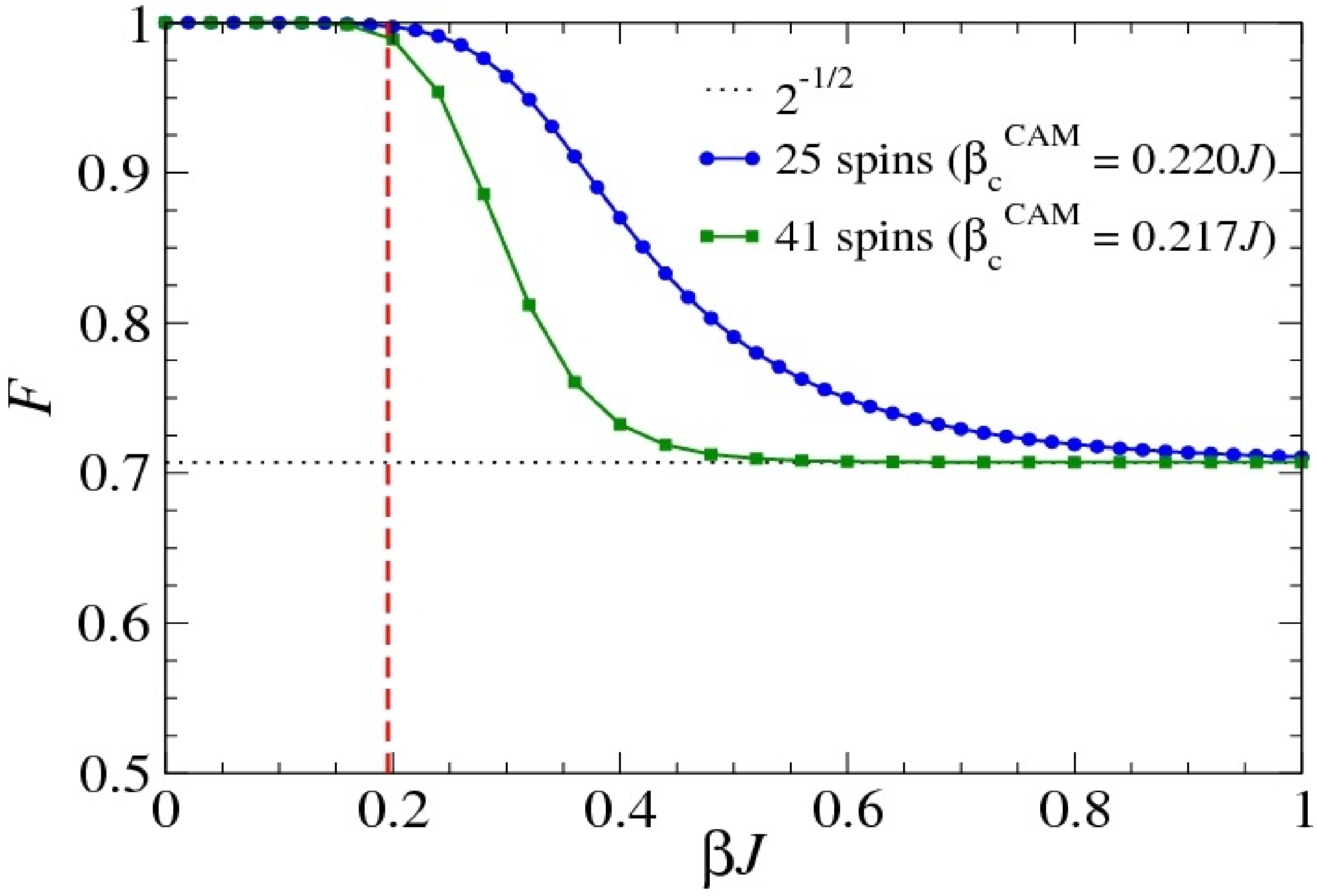


$$\beta_c \approx \frac{\ln \mu}{2(n-2)J}.$$

Number of qubits connected by the interaction

Numerical Evaluations

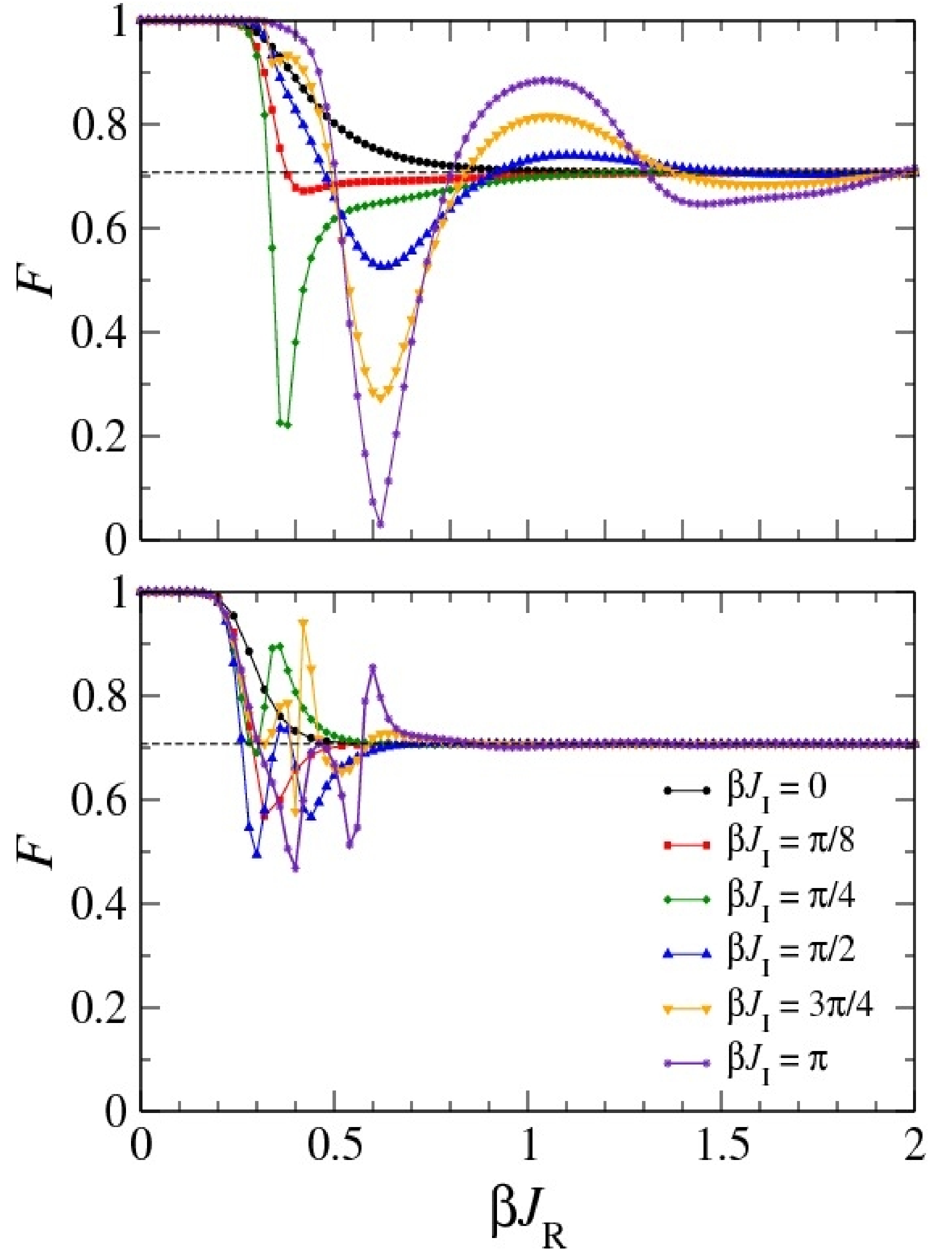
1- Exact diagonalization for small lattices – nearest neighbor and real interaction



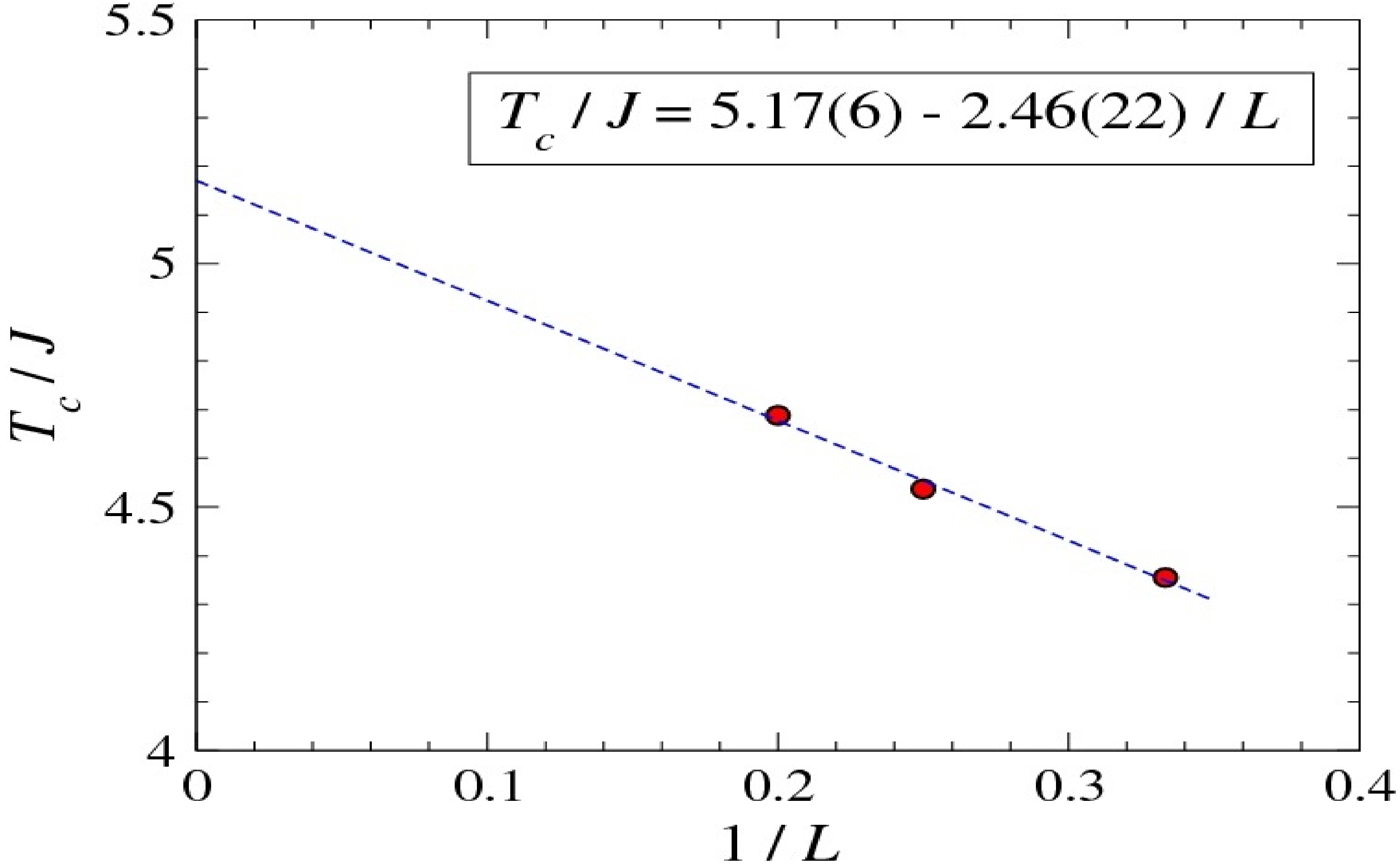
Numerical Evaluations

2-Mean-field solution:

coherent anomaly method
With nearest neighbor interaction



Numerical Evaluations



Finite-size scaling of the critical fictitious temperature Tc obtained from cluster mean-field calculations for lattice of sizes 13, 25, and 41. A real Ising interaction of strength J involving only nearest neighbors was used. The circles are the numerical data and the dashed line is a linear fit.

How does correlations change the usual threshold?

Probability of an error in a physical qubit

$$p = \langle 0| \otimes \langle \uparrow_j \, | U_j^\dagger(\Delta) | \downarrow_j \rangle \langle \downarrow_j \, | U_j(\Delta) | \uparrow_j \rangle \otimes |0\rangle,$$

$$p = \frac{1}{2} \left\{ 1 - \exp \left[-\frac{\lambda^2}{4} \, \mathcal{G}^{(R)}_{\mathbf{r}_j \mathbf{r}_j}(\Delta) \right] \right\}.$$

$$\ln(1-2p) = -\frac{\pi \beta (v\Delta)^{D+2s-2}}{L^D} \sum_{\mathbf{k} \neq 0} |\mathbf{k}|^{2s-2} [1 - \cos(|\mathbf{k}| v \Delta)].$$

How does correlations change the usual threshold? (ohmic example)

Let us take the example of the ohmic case

$$p = \tfrac{1}{2}\left[1 - (2v\Delta\Lambda)^{-\beta/2}\right]$$

$$J_{ij} \approx \frac{1}{2}\ln\left(\frac{v\Delta}{|\mathbf{r}_i - \mathbf{r}_j|}\right)$$

How does correlations change the usual threshold? (ohmic example)

Using that the logs produce numbers of the same order, we find the relations

$$\beta \approx \frac{2p}{J}, \qquad \beta_c \approx \frac{\ln \mu}{2nJ},$$

To be resilient, we must have $\beta < \beta_c$

$$p < \frac{\ln \mu}{4n}. \xrightarrow[\text{Nearest Neighbors}]{} p < \frac{\ln \mu}{16} \sim 6\%.$$

Conclusion

We map the Fidelity Calculation into a "Stat Mec" problem.

We showed that the "threshold" for quantum computing exist and evaluate the critical coupling.

Thank you !!!