Loop Quantum Gravity in 3D. May a Quantum Observable Vanish in the Classical Limit?

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- Dirac canonical formalism:
  - Assume space-time manifold splitted as: $\mathbb{R} \times \Sigma =$ times $\times$ space. No metric now!
  - Hamiltonian $H[\mathcal{A}, \mathcal{P}] = \sum_{i=1}^{3} \lambda^\alpha C_\alpha[\mathcal{A}, \mathcal{P}]$
    Metric is function of some components of $\mathcal{A}$ and/or $\mathcal{P}$.
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- Quantization – formal: Wave functional $\Psi[\mathcal{A}]$
  Operators $\hat{\mathcal{A}}$, $\hat{\mathcal{P}}$: $\hat{\mathcal{A}}^i(\mathbf{x})\Psi = \mathcal{A}^i(\mathbf{x})\Psi$, $\hat{\mathcal{P}}_j(\mathbf{x})\Psi = -i\hbar\dfrac{\delta}{\delta\mathcal{A}^j(\mathbf{x})}\Psi$.

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- Apply the Constraints: $\hat{C}_\alpha\Psi[\mathcal{A}] = 0$.

Note:

Constraints $\leftrightarrow$ Gauge invariance:

- Yang-Mills G-invariance $\sim$ local SU(2)($\subset$ Lorentz) invariance.
- Space diffeomorphisms
- Time diffeomorphisms ("coordinate time" evolution is a gauge transformation!)

"Loop" Quantization (Ashtekar, Rovelli, Smolin, etc.)

$$\Psi[\mathcal{A}] \to \psi_\Gamma(h_{\gamma_1}, \cdots, h_{\gamma_N}) \quad (\text{“holonomy functions”}),$$

where $\Gamma = \{\gamma_1, \cdots, \gamma_N\}$ ("graph"), γ_n = curve in space Σ, and

$$h_{\gamma_n} = P\, e^{\int_{\gamma_n} \mathcal{A}} \quad (\text{“holonomy”}); \ P \text{ means path ordering}.$$

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Gauge transformation:

$$g(x) \in G \ : \quad h'_{\gamma_n} = g(x_{\mathrm{f}}) h_{\gamma_n} g^{-1}(x_{\mathrm{i}}),$$

where x_{i}, x_{f} : initial and final points of γ_n.

Scalar product:

$$\langle \psi_\Gamma | \psi'_{\Gamma'} \rangle = \delta_{\Gamma\Gamma'} \int d\mu(g_1) \cdots \int d\mu(g_N) \overline{\psi\,(g_1, \cdots g_N)} \psi'\,(g_1, \cdots g_N) \ ,$$

where $g_n \in G$, $d\mu(g_n)$ is the Haar integration measure on the group G.

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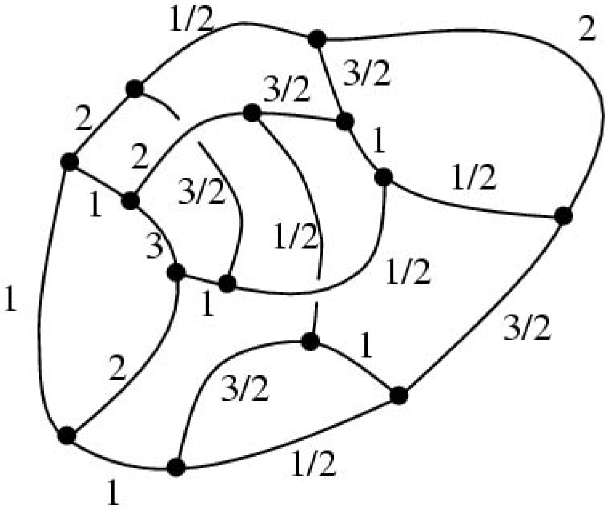
$\rightarrow$ Kinematical Hilbert Space $\mathcal{H}_{\mathrm{kin}}$ (non-separable!)

Orthonormal basis:

$\psi_{\Gamma, j_1, \cdots j_N}$ = gauge invariant combinations of products $R^{j_1}_{\alpha_1\beta_1} \cdots R^{j_N}_{\alpha_N\beta_N}$, where $R^{j_n}_{\alpha_n\beta_n}$ = matrix elements of the spin j_n representation of $h_{\gamma_n} \in G$.

$\longrightarrow$ Spin networks, represented by graphs:



Consider all possible graphs!

Finally, apply the constraints (the difficult part!)
$\rightarrow$ (separable) Physical Hilbert space.

Note: Due to the diffeomorphism invariance constraint, states are now labeled by diffeomorphism equivalence classes of graphs. The set of such classes is countable $\rightarrow$ separability.

1. $D = (2+1)$ – Gravity as a Chern-Simons theory with a Barbero-Immirzi parameter

(Valentin Bonzom and Etera R. Livine, CQG 25 (2008) 195024)

Independent fields ("first order formalism"):

$$e^I = e^I_\mu dx^\mu \text{ (Triad)}, \quad \omega_I = \tfrac{1}{2}\varepsilon_{IJK}\,\omega^{JK} = \omega_{I\mu}dx^\mu \text{ (Spin connection)},$$

$$\left(\text{metric } g_{\mu\nu}(x) = \eta_{IJ}e^I_\mu(x)e^J_\nu(x)\right),$$

$$\eta_{IJ} = \mathrm{diag}(-1,1,1) \text{ (Minkowski metric)},$$

$$I, J, \dots = 0,1,2, \quad \mu, \nu, \dots = t, x, y.$$

Cosmological constant $\boldsymbol{\Lambda > 0}$ $\rightarrow$ de Sitter gauge group **SO(3,1)**
Generators: J^I (Lorentz), P_I ("translations")

$$[J^I, J^J] = \varepsilon^{IJ}{}_K J^K, \quad [J^I, P_J] = \varepsilon^I{}_J{}^K P_K, \quad [P_I, P_J] = \Lambda\varepsilon_{IJK}J^K.$$

SO(3,1) – connection: $A = \omega_I J^I + e^I P_I$.

$D = 2 + 1$ de Sitter gravity described by a sum of two SO(3,1) Chern-Simons action:

$$S = S_{(1)} - \frac{1}{\gamma} S_{(2)} \,.$$

$$S_{(i)}(A) = -\frac{\kappa}{2} \int_{\mathcal{M}=\mathbb{R}\times\Sigma} \left\langle A,\, dA + \frac{2}{3} A \wedge A \right\rangle_{(i)} , \quad i = 1, 2$$

(For SO(1,3) there exist two invariant quadratic forms $\langle \ \rangle_{(i)}$).

$\kappa^{-1} \approx G_{\text{“Newton”}}$, γ is a Barbero-Immirzi-like parameter.

N.B.: *Classical field equations are independent of* γ:

$$F(A) = dA + A^2 = 0 \,, \quad \text{or:} \quad \mathbf{R} - \frac{\Lambda}{2} \mathbf{e} \wedge \mathbf{e} = 0 \,, \quad \mathbf{T} \equiv \mathbf{de} + \boldsymbol{\omega} \wedge \mathbf{e} = 0 \,.$$

Why two quadratic invariant forms?

Achucarro and Townsend, Witten: For SO(3,1) there exists two quadratic Casimir operators:

$$C_{(1)} = P_I J^I \,, \quad C_{(2)} = \eta_{IJ} \left(\frac{1}{\Lambda} P^I P^J - J^I J^J \right) \,,$$

corresponding to the two quadratic invariant forms

$$\langle P_I, J_J \rangle_1 = \eta_{IJ}, \; \langle P_I, P_J \rangle_1 = 0, \; \langle J_I, J_J \rangle_1 = 0,$$

$$\langle P_I, J_J \rangle_2 = 0, \; \langle P_I, P_J \rangle_2 = \sigma \Lambda \eta_{IJ}, \; \langle J_I, J_J \rangle_2 = \eta_{IJ}.$$

The inner product $< \, , \, >_1$ is non-degenerate for all Λ, whereas $< \, , \, >_2$ only for $\Lambda \neq 0$.

Partial gauge fixing...

(R.M.S. Barbosa, C.P. Constantinidis, O. Piguet and Zui Oporto, Proceedings of Loops11-Madrid (2011), and Class.Quant.Grav. 29 (2012) 155011)

... such that the residual gauge invariance be SU(2) – which is compact!

Idea: Separate the generators of SO(3,1) between

- The ones which generate the compact subgroup SO(3): L_i, $i = 1, 2, 3$;

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- Expand the SO(3,1) connection in the new basis:
  $A = A^i L_i + B^i K_i$
  A^i: SO(3) (or SU(2)) – connection.

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- Expand the SO(3,1) connection in the new basis:
  $A = A^i L_i + B^i K_i$
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- Impose the gauge condition $B^i_y = 0$

  i.e., $\mathbf{e^0_y = 0}\,, \quad \omega^1_{\mathbf{y}} = \omega^1_{\mathbf{y}} = \mathbf{0}$.

→ Chern Simon theory of an SU(2) connection $\mathcal{A}$:

$$\mathcal{A}_x = (\sqrt{\Lambda}e_x^2 - \gamma\omega_x^2\,,\ -\sqrt{\Lambda}e_x^1 + \gamma\omega_x^1\,,\ -\omega_x^0 - \gamma\sqrt{\Lambda}e_x^0)\,,$$

$$\mathcal{A}_y = \left(\sqrt{\Lambda}e_y^2 \qquad\qquad ,\ -\sqrt{\Lambda}e_y^1 \qquad\qquad ,\ -\omega_y^0 \qquad\qquad\qquad \right).$$

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Hamiltonian:

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Constraint: $F^i(\mathcal{A}) \equiv \partial_x \mathcal{A}^i_y - \partial_y \mathcal{A}^i_x + (\mathcal{A}_x \times \mathcal{A}_y)^i \approx 0.$

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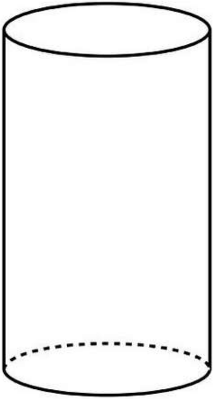
Brackets:

$$\{\mathcal{A}_x^i(\mathbf{x}),\, \mathcal{A}_y^j(\mathbf{x}')\}_{\mathrm{D}} = \frac{\gamma}{\kappa}\delta^{ij}\delta^2(\mathbf{x}-\mathbf{x}')\,,$$

Special case: space Σ has topology of a cylinder:



Coordenadas

$x\,,\;\; y\,, \quad 0 \leq x < 2\pi\,,\; -\infty < y < +\infty$

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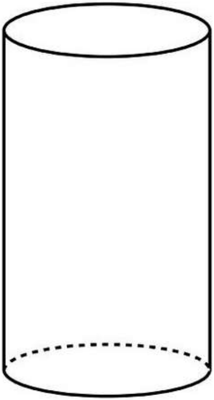

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$\frac{\gamma}{\kappa}\mathcal{A}_y^i =$ conjugate monentum of $\mathcal{A}_x^i$

Fully constrained Hamiltonian.

Constraint is:

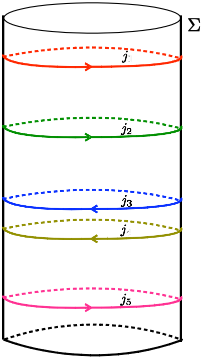
$\mathcal{F}^i(\mathcal{A}) \approx 0$ ("YM-Curvature constraint").

(analogous to the Gauss constraint of QED)

Loop quantization

(C.P. Constantinidis, G. Luchini and O. Piguet, CQG27 (2010) 065009

Graph Γ:



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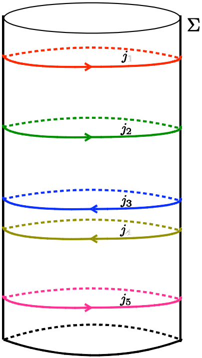

Holonomies $h_n[\mathcal{A}_x]$
on closed $y =$ const. pathes (Loops)

Wave functionals

$$\Psi_{\Gamma,f}[\mathcal{A}_x] = e^{2i\pi\alpha_0[\mathcal{A}_x]} f\left(h_1[\mathcal{A}_x], \cdots h_N[\mathcal{A}_x]\right)$$

$$\hat{\mathcal{A}}^i_x \Psi_{\Gamma,f} = \mathcal{A}^i_x \Psi_{\Gamma,f}\ , \quad \hat{\mathcal{A}}^i_y \Psi_{\Gamma,f} = -i\frac{\hbar\gamma}{\kappa}\frac{\delta}{\delta\mathcal{A}^i_x}\Psi_{\Gamma,f}$$

Diffeomorphism invariance
$\rightarrow$ State vectors independent
of “heights” y of the curves

First result:

Separable Hilbert space $\mathcal{H}_{\mathrm{phys}}$, with orthonormal basis

$$\left\{ |0\rangle \right\} \oplus \left\{ |j_1, j_2, \cdots j_N\rangle\,; \quad N = 0, 1, 2, \cdots\,; \quad j_n = \frac{1}{2}, 1, \frac{3}{2}, \cdots \right\}.$$

with:

$$\Psi_0[\mathcal{A}_x] = e^{2i\pi\alpha_0}$$

$$\Psi_{j_1, j_2, \cdots j_N} = e^{2i\pi\alpha_0} \mathrm{Tr}\, R^{(j_1)} \mathrm{Tr}\, R^{(j_2)} \ldots \mathrm{Tr}\, R^{(j_N)} \Big|_{\mathrm{inv/diff}}$$

Observable L

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1. The classical gauge invariant observable $L_{\rm class}$

$$L_{\rm class} = \int_{-\infty}^{+\infty} dy \sqrt{\left(\mathcal{A}_y - \mathcal{G}^{-1}\partial_y\mathcal{G}\right)^i \left(\mathcal{A}_y - \mathcal{G}^{-1}\partial_y\mathcal{G}\right)_i}\,,$$

$\mathcal{G} = \mathcal{G}[\mathcal{A}_x]$ defined as solution of: $\mathcal{G}^{-1}\partial_x\mathcal{G} = \mathcal{A}_x$

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Consequence of the constraints $\mathcal{F}^i = 0$:

$$\mathbf{L}_{\rm class} = \mathbf{0}\ !$$

Observable L

2. The quantum gauge invariant observable $\hat{L}$

$$\hat{L} = \int_{-\infty}^{+\infty} dy \sqrt{\left(\hat{\mathcal{A}}_y - \mathcal{G}^{-1}\partial_y\mathcal{G}\right)^i \left(\hat{\mathcal{A}}_y - \mathcal{G}^{-1}\partial_y\mathcal{G}\right)_i}$$

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Remember: $\Psi_{\Gamma,f}[\mathcal{A}_x] = e^{2i\pi\alpha_0[\mathcal{A}_x]} f\left(h_1[\mathcal{A}_x], \cdots h_N[\mathcal{A}_x]\right)$
We have:

$$\left(\hat{\mathcal{A}}_y - \mathcal{G}^{-1}\partial_y \mathcal{G}\right)^i \Psi_{\Gamma,f} = e^{2i\pi\alpha_0} \hat{\mathcal{A}}_y^i\, f\left(h_1, \cdots h_N\right)$$

Special case: $f = R^{(j)}(h_\gamma)$ (spin j representation matrix of the holonomy $h_\gamma[\mathcal{A}_x]$):

$$\mathcal{A}^i_y f \propto i\frac{\hbar\gamma}{\kappa} R^{(j)}(h_{\gamma_2}) L^i R^{(j)}(h_{\gamma_1}) \, ,$$

with curve γ splitted in $\gamma = \gamma_2 \circ \gamma_1$.

Hence, since $L^i L^i R^{(j)} = -j(j+1) R^{(j)}$,

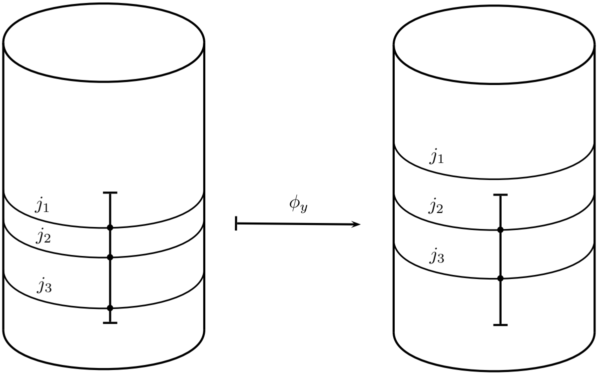
$$\int_{-\infty}^{+\infty} \sqrt{\mathcal{A}^i_y \mathcal{A}^i_y} f = \frac{\hbar\gamma}{\kappa} \sqrt{j(j+1)} f \ .,$$

$\Rightarrow$ **Second result:**

$$\hat{L}\,|0\rangle = 0\,,$$

$$\hat{L}\,|j_1, j_2, \cdots j_N\rangle = \frac{\hbar\gamma}{\kappa}\left(\sum_{n=1}^{N}\sqrt{j_n(j_n+1)}\right)|j_1, j_2, \cdots j_N\rangle$$

(Needs a regularization; but the result is finite– and non-zero! Some analogy with the Area operator in D=4 LQG.)



j_1
j_2
j_3
ϕ_y
j_1
j_2
j_3

Quantization among the fundamental constants

(With $\hbar = 1$)

In the SU(2) CS theory: $S_{\rm CS}(\mathcal{A}) = -\frac{1}{2}\frac{\kappa}{\gamma}\int_{\mathcal{M}} \left\langle \mathcal{A},\, d\mathcal{A} + \frac{2}{3}\mathcal{A}\wedge\mathcal{A} \right\rangle$,

the coupling $\frac{\kappa}{\gamma}$ is quantized (Witten, etc.):

$$\frac{\kappa}{\gamma} \equiv \frac{c^3}{16\pi\gamma\sqrt{\Lambda}G_{(\rm Newton)}} = \frac{1}{4\pi}\nu\,, \quad \nu \in \mathbb{Z}\,.$$

Third result:

$$\Rightarrow \quad \hat{\mathbf{L}}\,|\mathbf{j_1}, \mathbf{j_2}, \cdots \mathbf{j_N}\rangle = \frac{\mathbf{4}\pi}{\nu}\left(\sum_{\mathbf{n}=\mathbf{1}}^{\mathbf{N}} \sqrt{\mathbf{j_n}(\mathbf{j_n}+\mathbf{1})}\right)|\mathbf{j_1}, \mathbf{j_2}, \cdots \mathbf{j_N}\rangle\,.$$

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ν is a quantized fundamental constant. Compare with J.Zanelli, "Quantization of the gravitational constant in odd-dimensional gravity", *Phys. Rev.* D51 (1995) 490, [arXiv:hep-th/9406202].

Conclusion

- We have succeeded to give a complete quantum description of a pure gravitational $D = 3$ model with positive cosmological constant.
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Muito Obrigado!

References

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