Quantum phase transitions and disorder: Griffiths singularities, infinite randomness, and smearing

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- Phase transitions and quantum phase transitions
- Effects of impurities and defects: the common lore
- Rare regions, Griffiths singularities and smearing
- Experiments in $Ni_{1-x}V_x$ and $Sr_{1-x}Ca_xRuO_3$
- Classification of weakly disordered phase transitions

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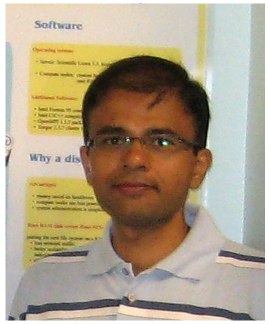

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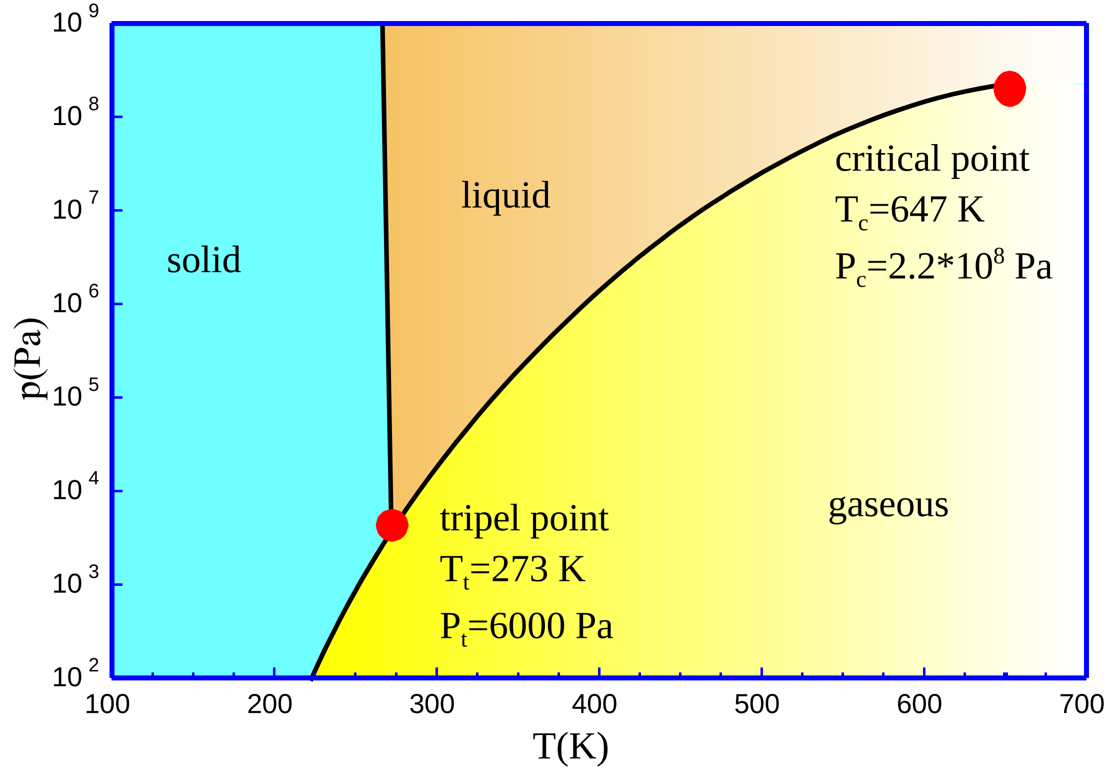
Phase transitions: the basics

Phase transition:
singularity in thermodynamic quantities

occurs in macroscopic (infinite) systems



1st order transition:
phase coexistence, latent heat, finite correlations

Continuous transition:
no phase coexistence, no latent heat, critical behavior:

diverging **correlation length** $\xi \sim |T-T_c|^{-\nu}$ **and time** $\xi_\tau \sim \xi^z \sim |T-T_c|^{-\nu z}$

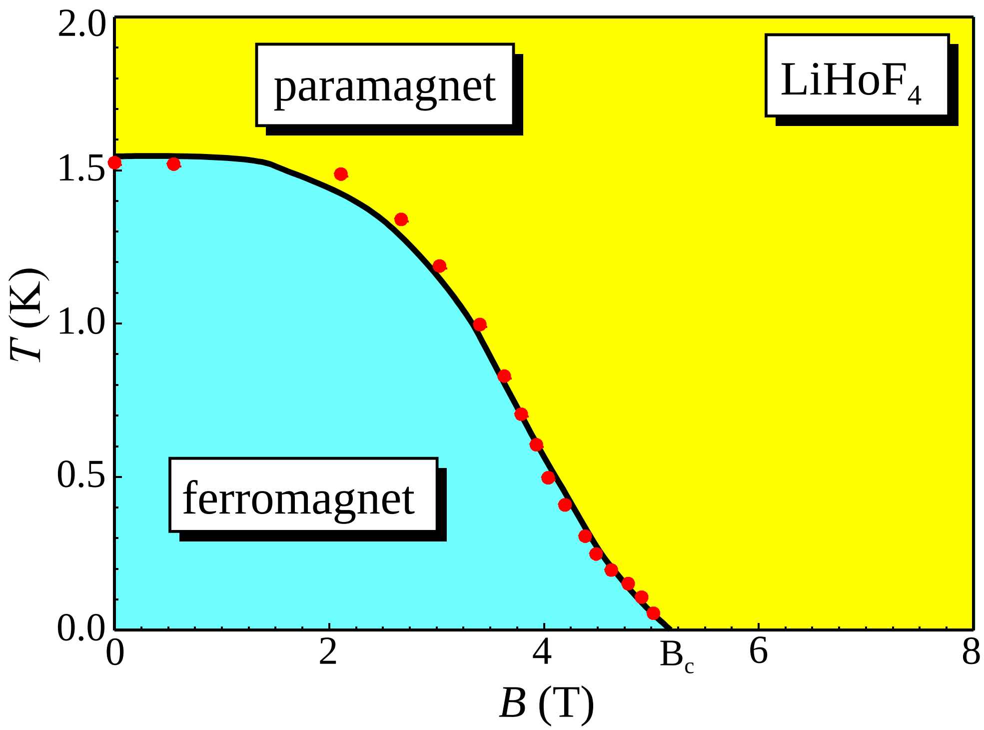
power-laws in thermodynamic observables: $\Delta\rho \sim |T-T_c|^{\beta},\ \kappa \sim |T-T_c|^{-\gamma}$

critical exponents are universal = independent of microscopic details

Quantum phase transitions

occur at **zero temperature** as function of pressure, magnetic field, chemical composition, ...

driven by **quantum** rather than thermal fluctuations



phase diagram of $LiHoF_4$ (Bitko et al. 96)

Transverse-field Ising model

$$H = -J \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z - h \sum_i \sigma_i^x$$

transverse magnetic field induces spin flips via $\sigma^x = \sigma^+ + \sigma^-$

transverse field suppresses magnetic order

Imaginary time and quantum to classical mapping

Classical partition function: statics and dynamics decouple

$$Z = \int dpdq \; e^{-\beta H(p,q)} = \int dp \; e^{-\beta T(p)} \int dq \; e^{-\beta U(q)} \sim \int dq \; e^{-\beta U(q)}$$

Quantum partition function: statics and dynamics coupled

$$Z = \mathrm{Tr} e^{-\beta \hat{H}} = \lim_{N\to\infty} (e^{-\beta \hat{T}/N} e^{-\beta \hat{U}/N})^N = \int D[q(\tau)] \; e^{S[q(\tau)]}$$

imaginary time τ acts as additional dimension
at $T=0$, the extension in this direction becomes infinite

Caveats:

- mapping holds for thermodynamics only
- resulting classical system can be unusual and anisotropic ($z \neq 1$)
- if quantum action is not real, extra complications may arise, e.g., Berry phases

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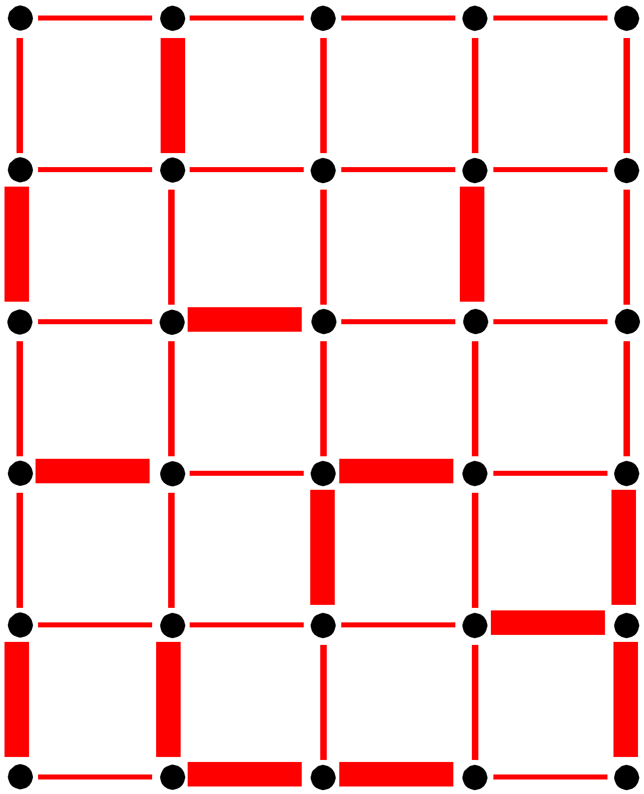
Phase transitions and (weak) disorder



Real systems always contain
impurities and other **imperfections**

Weak (random-T_c) disorder:

spatial variation of coupling strength but
no change in character of the ordered phase

Will the phase transition remain sharp or become smeared?

Will the critical behavior change?

How important are rare strong disorder fluctuations?

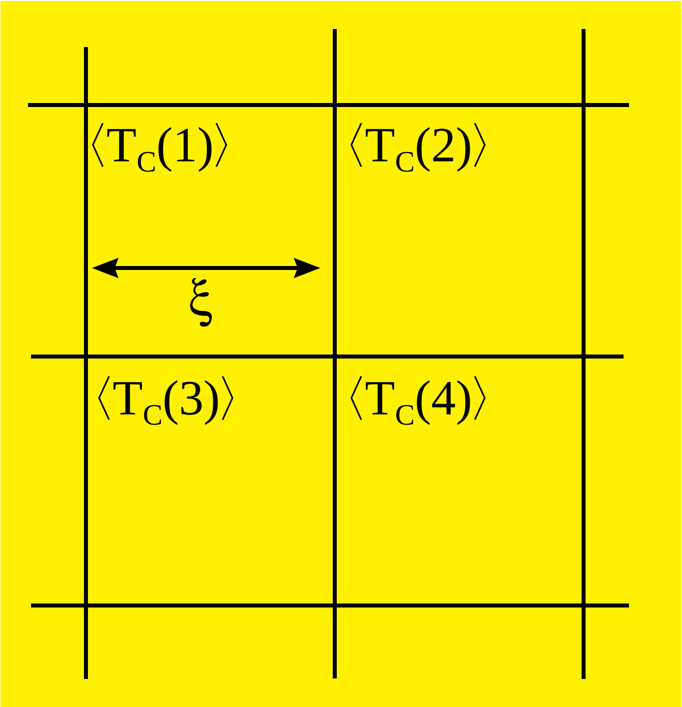
Harris criterion

Harris criterion:
variation of average local $T_c(i)$ between correlation volumes must be smaller than distance from global T_c

variation of average $T_c(i)$ in volume ξ^d
$\Delta T_c(i) \sim \xi^{-d/2}$

distance from global critical point
$T - T_c \sim \xi^{-1/\nu}$

$\Delta T_c(i) < T - T_c \qquad \Rightarrow \qquad \boxed{d\nu > 2}$



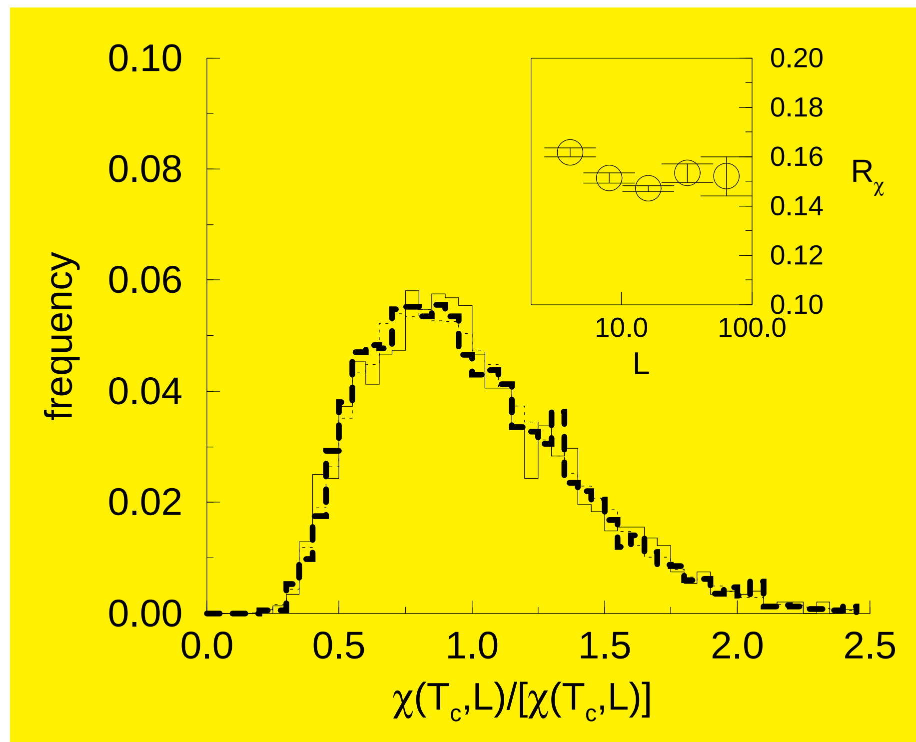
- if clean critical point fulfills Harris criterion $\Rightarrow$ stable against disorder
- inhomogeneities vanish at large length scales
- macroscopic observables are self-averaging
- example: 3D classical Heisenberg magnet: $\nu = 0.711$

Finite-disorder critical points

if critical point violates Harris criterion $\Rightarrow$ unstable against disorder

Common lore:

- new, different critical point which fulfills $d\nu > 2$
- inhomogeneities finite at all length scales ("finite disorder")
- macroscopic observables not self-averaging
- example: 3D classical Ising magnet: clean $\nu = 0.627 \quad \Rightarrow \quad$ dirty $\nu = 0.684$



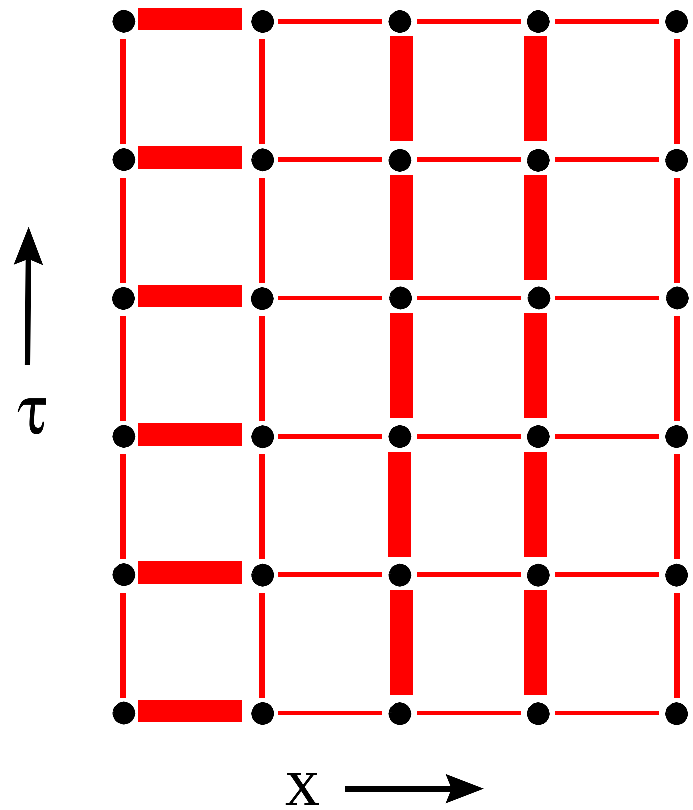
Distribution of critical susceptibilities of 3D dilute Ising model
(Wiseman + Domany 98)

Disorder and quantum phase transitions

Disorder is quenched:

- impurities are time-independent
- disorder is perfectly correlated in imaginary time direction

$\Rightarrow$ correlations increase the effects of disorder ("it is harder to average out fluctuations")



Disorder generically has stronger effects on quantum phase transitions than on classical transitions

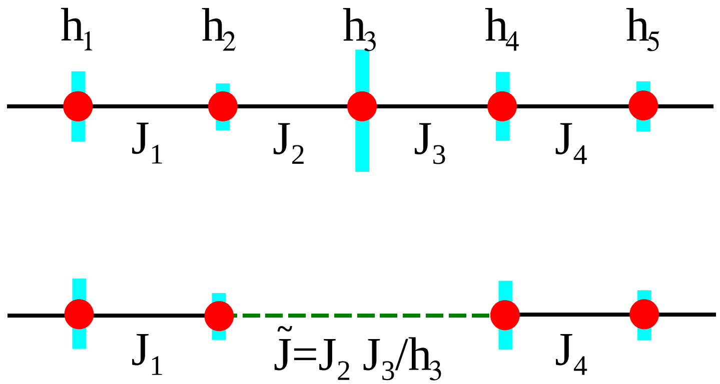
Random transverse-field Ising model

$$H = -\sum_{\langle i,j\rangle} J_{ij}\sigma_i^z\sigma_j^z - \sum_i h_i\sigma_i^x$$

nearest neighbor interactions J_{ij} and transverse fields h_i both random

Strong-disorder renormalization group: exact solution in 1+1 dimensions:

- Ma, Dasgupta, Hu (1979), Fisher (1992, 1995)
- in each step, integrate out largest of all J_{ij}, h_i
- cluster aggregation/annihilation process
- exact in the limit of large disorder

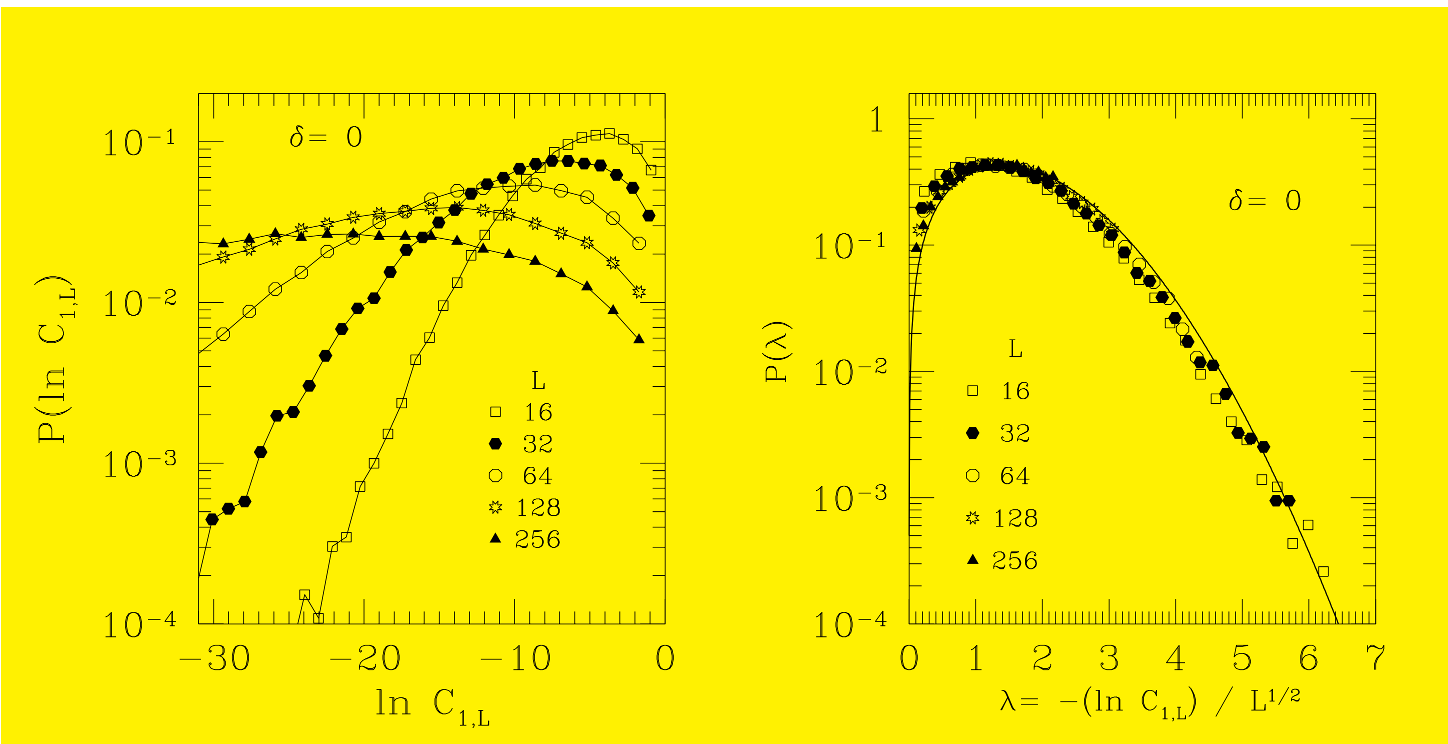


Infinite-disorder critical point:

- under renormalization the disorder increases without limit
- relative width of the distributions of J_{ij}, h_i diverges

Infinite-disorder critical point

- extremely slow dynamics $\log \xi_\tau \sim \xi^\mu$ (activated scaling)
- distributions of macroscopic observables become infinitely broad
- average and typical values drastically different
  correlations: $G_{av} \sim r^{-\eta}$, $-\log G_{typ} \sim r^\psi$
- averages dominated by rare events



Probability distribution of end-to-end correlations in a random quantum Ising chain
(Fisher + Young 98)

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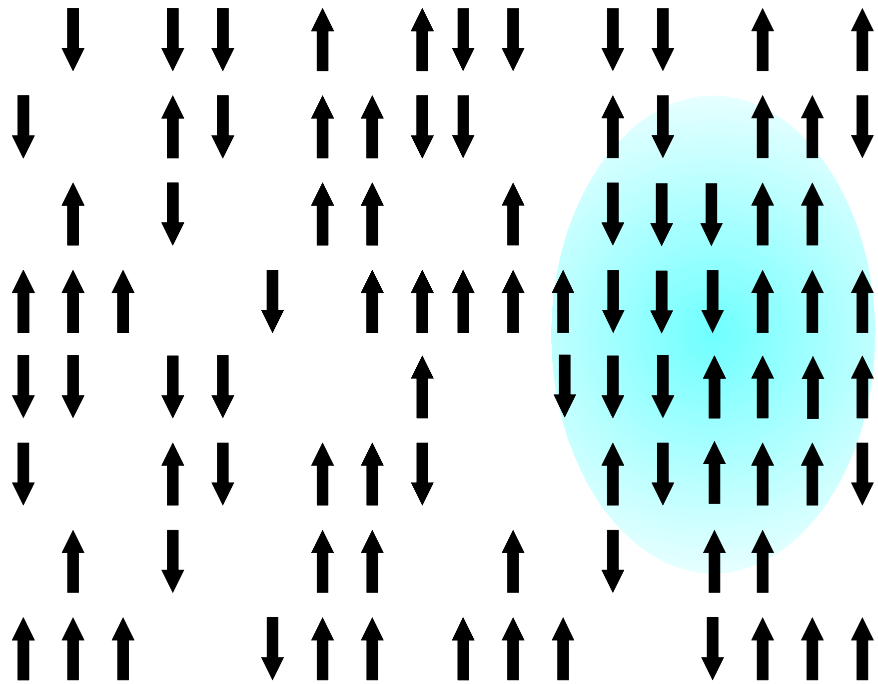
Rare regions in a classical dilute ferromagnet

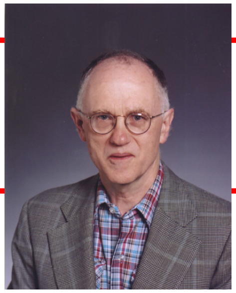
- critical temperature T_c is reduced compared to clean value T_{c0}
- for $T_c < T < T_{c0}$: no global order but **local order** on **rare regions** devoid of impurities
- probability: $w(L) \sim e^{-cL^d}$:

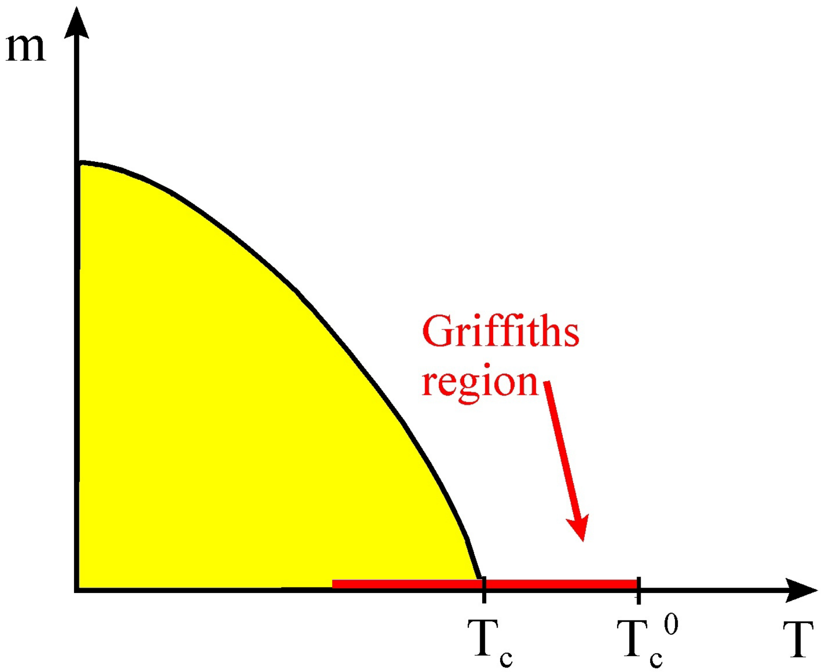


rare regions cannot order statically but act as large **superspins**

$\Rightarrow$ **very slow** dynamics, **large** contribution to thermodynamics

Griffiths region or Griffiths "phase"





Griffiths:

rare regions lead to singular free energy everywhere in the interval $T_c < T < T_{c0}$

Rare region susceptibility:

- susceptibility of single RR: $\chi \lesssim L^{2d}/T$
- sum over all RRs:

$$\chi_{RR} \sim \int dL \; e^{-cL^d} L^{2d}$$

- essential singularity
- large regions make negligible contribution

In generic classical systems:

Thermodynamic Griffiths effects are weak and essentially unobservable

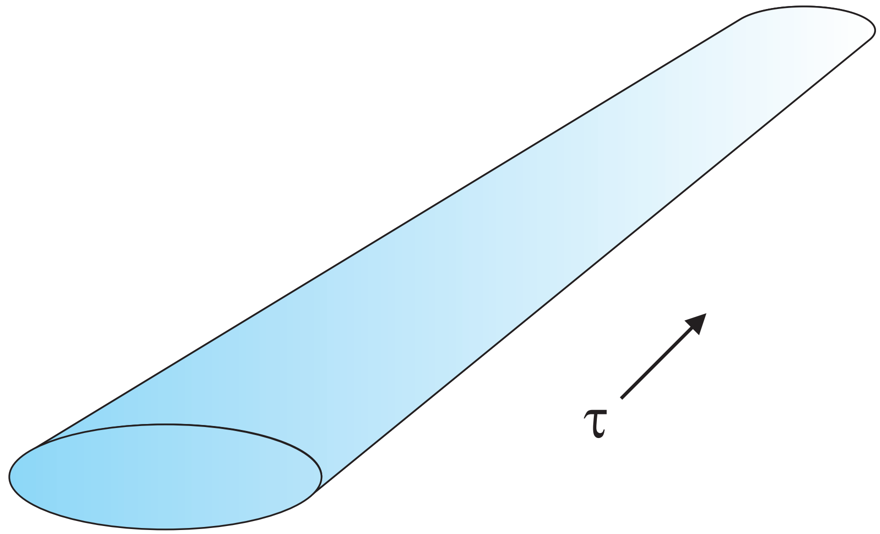
Long-time dynamics can be dominated by rare regions

Quantum Griffiths effects

Quantum phase transitions:

- rare regions are finite in space but **infinite in imaginary time**
- fluctuations **even slower** than in classical case

Griffiths singularities enhanced

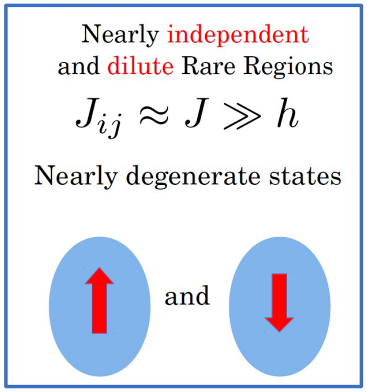


rare region at a quantum phase transition

Random quantum Ising systems:

- susceptibility of rare region: $\chi_{loc} \sim \Delta^{-1} \sim e^{aL^d}$

  $\chi_{RR} \sim \int dL\; e^{-cL^d} e^{aL^d}$ can diverge inside Griffiths region



- **power-law quantum Griffiths singularities**

  susceptibility: $\chi_{RR} \sim T^{d/z'-1}$

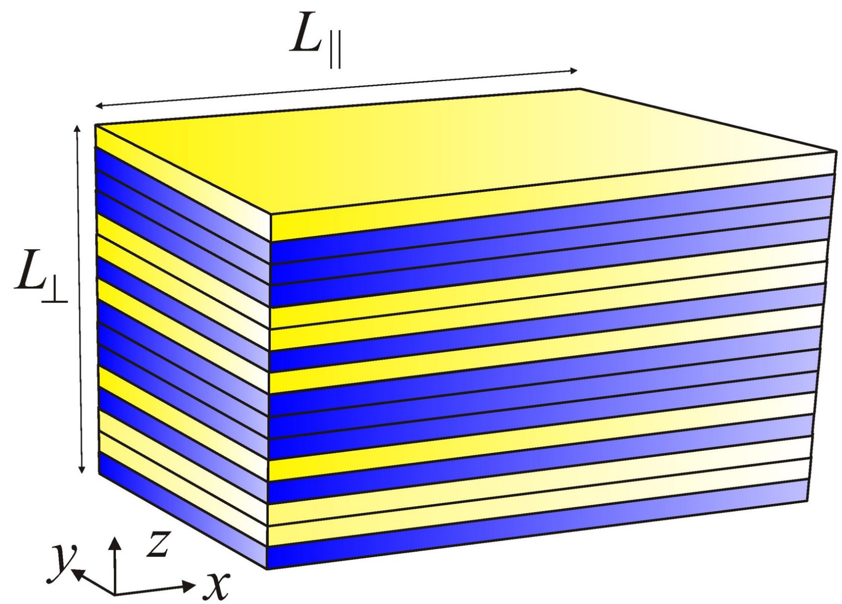
  specific heat: $C_{RR} \sim T^{d/z'}$

z' is continuously varying Griffiths dynamical exponent, diverges at criticality

Smeared phase transitions

Randomly layered classical magnet:

- layers of two different ferromagnetic materials grown in random order
- rare regions are **thick slabs** of the material with higher T_c
- rare regions are **two-dimensional**
- two-dimensional (Ising) magnets have **true phase transition**



$\Rightarrow$ global magnetization develops gradually as rare regions order **independently**

$\Rightarrow$ no Griffiths region

global phase transition is smeared by disorder

T.V., Phys. Rev. Lett **90**, 107202 (2003); J. Phys. A **36**, 10921 (2003)

Isolated islands – Optimal fluctuation arguments

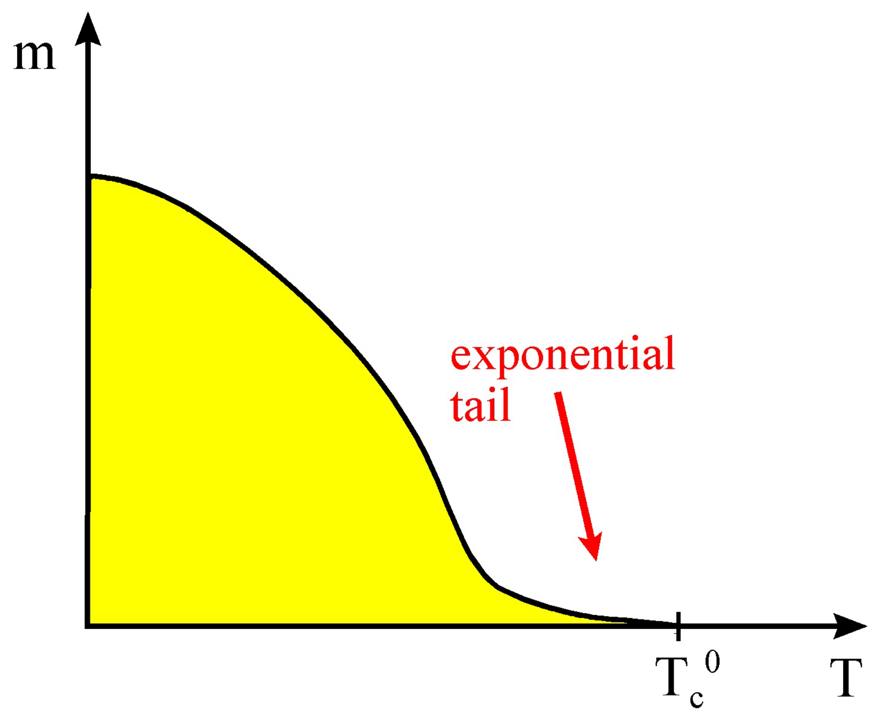
- probability for finding region of size L devoid of weak planes: $w \sim e^{-cL_\perp^d}$
- region has transition at temperature $T_c(L) < T_c^0$ $\quad$ (T_c^0 = higher of the two bulk T_c)
- finite size scaling: $|T_c(L) - T_c^0| \sim L^{-\phi}$ $\quad$ (ϕ = clean shift exponent)

probability for finding a region which becomes critical at T_c:

$$w(t_c) \sim \exp(-B\ |T_c - T_c^0|^{-d_\perp/\phi})$$

total magnetization at temperature T:
sum over all rare regions with $T_c > T$:

$m(t) \sim \exp(-B\ |T - T_c^0|^{-d_\perp/\phi}) \quad (T \to T_c^0-)$



Dissipative random transverse-field Ising model

$$H = -\sum_i J_i \sigma_i^z \sigma_{i+1}^z - \sum_i h_i \sigma_i^x + \sum_{i,n} \sigma_i^z \lambda_{i,n} (a_{i,n}^\dagger + a_{i,n}) + \sum_{i,n} \nu_{i,n} a_{i,n}^\dagger a_{i,n}$$

- nearest neighbor interactions J_i and transverse fields h_i both random
- bath oscillators $a_{i,n}^\dagger, a_{i,n}$ have Ohmic spectral density

$$\mathcal{E}(\omega) = \pi \sum_n \lambda_{i,n}^2 \delta(\omega - \nu_{i,n}) = 2\pi\alpha\omega e^{-\omega/\omega_c}$$

- damping due to baths leads to long-range interaction in time: $\sim 1/(\tau - \tau')^2$

1D Ising model with $1/r^2$ interaction is known to have an ordered phase

$\Rightarrow$ isolated rare region can develop a static magnetization, i.e.,
large islands do not tunnel

$\Rightarrow$ quantum Griffiths behavior does **not** exist
magnetization develops **gradually** on **independent** rare regions

quantum phase transition is smeared by disorder

J.A. Hoyos and T.V., PRL **100**, 240601 (2008)

Universality of the smearing scenario

Condition for disorder-induced smearing:

isolated rare region can develop a static order parameter
$\Rightarrow$ rare region has to be above lower critical dimension

Examples:

- quantum phase transitions in dissipative quantum magnets
  (disorder correlations in imaginary time + long-range interaction $1/\tau^2$)
- classical Ising magnets with planar defects
  (disorder correlations in 2 dimensions)
- classical non-equilibrium phase transitions in the directed percolation universality class with extended defects
  (disorder correlations in at least one dimension)

Disorder-induced smearing of a phase transition is a ubiquitous phenomenon

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Rare regions in metallic quantum magnets

Gaussian propagator:

$$G_0^{-1}(\mathbf{q},\omega_n) = \begin{cases} t + \mathbf{q}^{d-1} + |\omega_n|/|\mathbf{q}| & \text{ferromagnet} \\ t + \mathbf{q}^2 + |\omega_n| & \text{antiferromagnet} \end{cases}$$

magnetic fluctuations are **damped** due to coupling to electrons

in imaginary time: long-range power-law interaction $\sim 1/(\tau - \tau')^2$

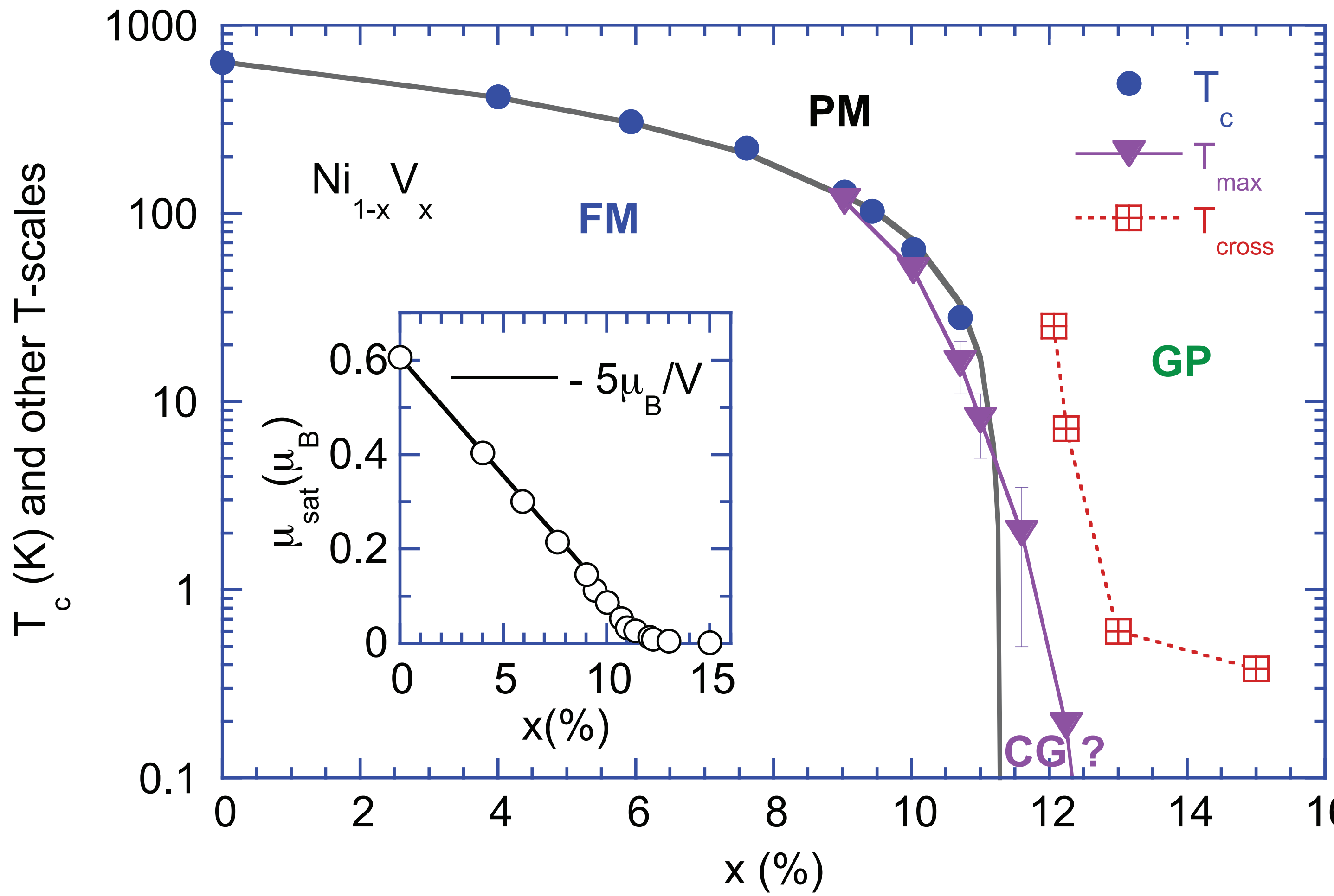
Consider single rare region:

Itinerant Ising magnets: rare region can order by itself
(1D Ising model with $1/r^2$ interaction has an ordered phase)
$\Rightarrow$ **global phase transition is smeared**

Itinerant Heisenberg magnets: rare region is at the lower critical dimension
(1D Heisenberg model with $1/r^2$ interaction does NOT have an ordered phase)
$\Rightarrow$ **strong power law quantum Griffiths effects**

T.V., Phys. Rev. Lett **90**, 107202 (2003); T.V. + J. Schmalian, Phys. Rev. B **72**, 045438 (2005)

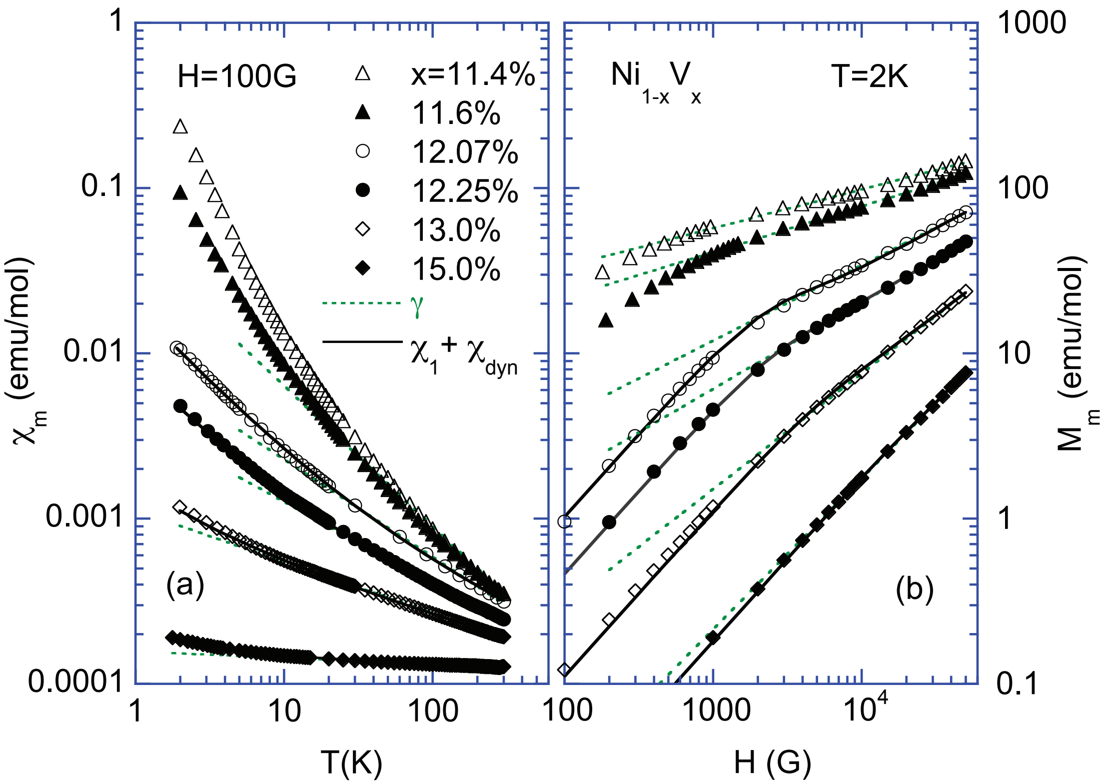
Phase diagram of $Ni_{1-x}V_x$

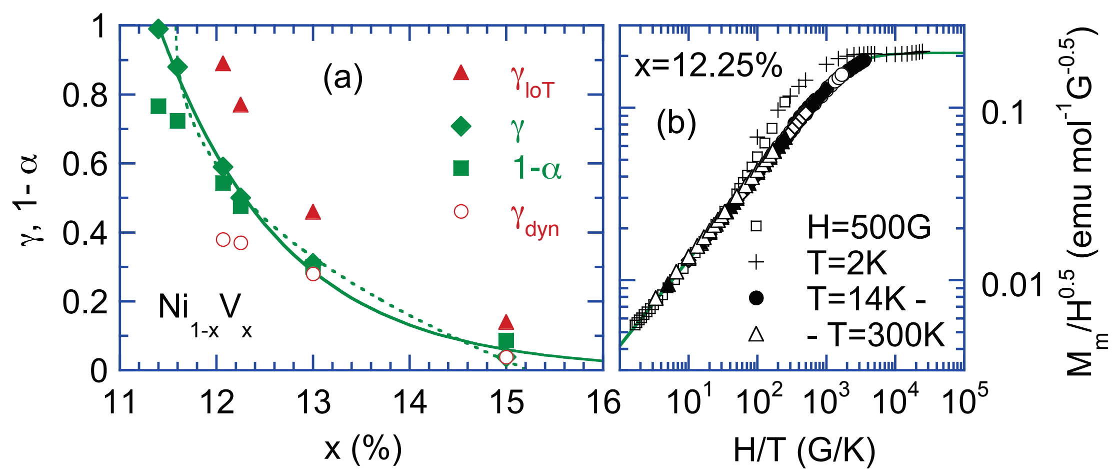


S. Ubaid-Kassis, T. V. and **A. Schroeder**, Phys. Rev. Lett. **104**, 066402 (2010)

A. Schroeder, S. Ubaid-Kassis and T. V., J. Phys. Condens. Matter **23**, 094205 (2011)

Quantum Griffiths singularities in $\mathbf{Ni}_{1-x}\mathbf{V}_x$





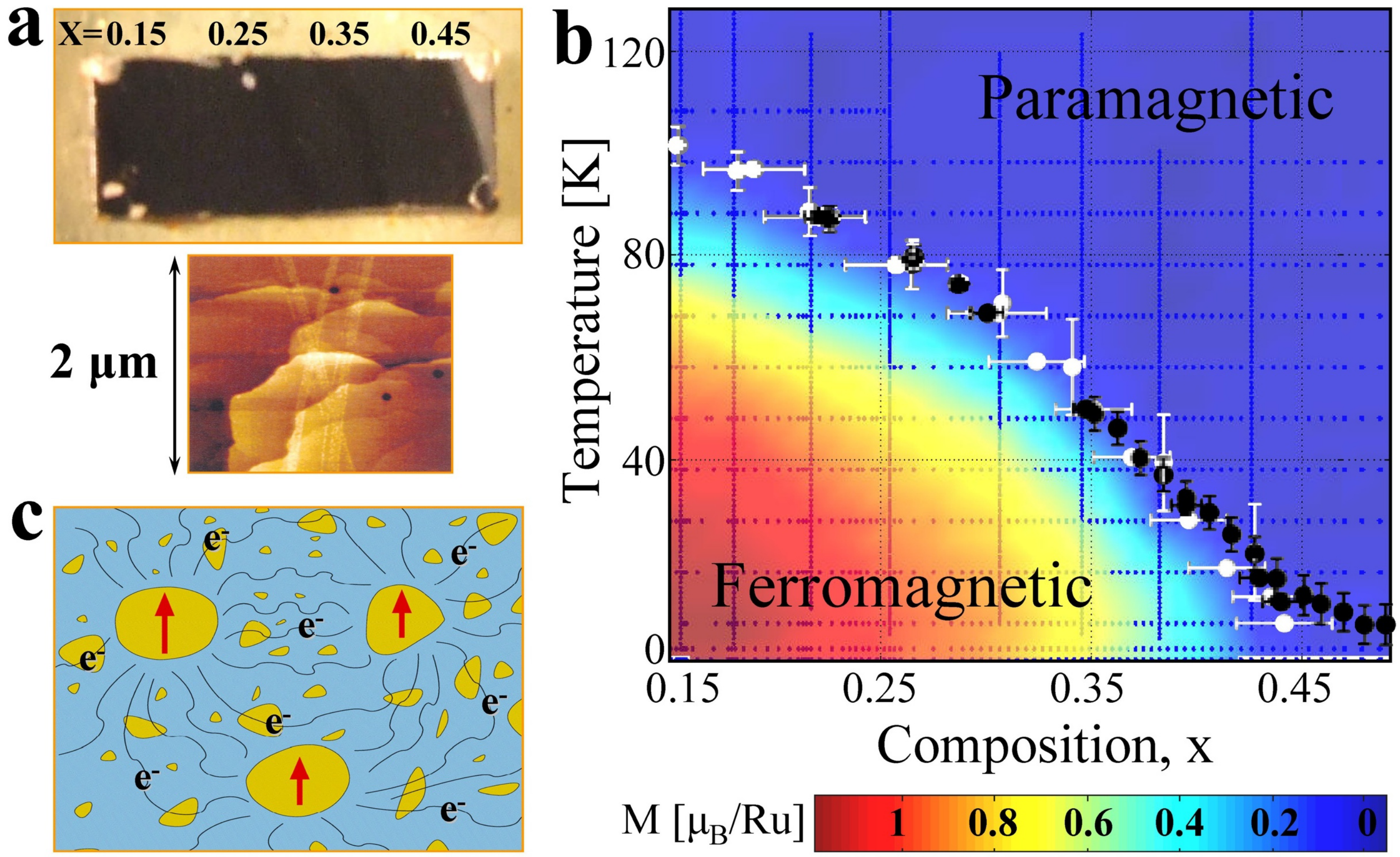
- $\chi(T)$ and $m(H)$ show nonuniversal power laws above x_c
- Griffiths exponent $\lambda = d/z'$ varies systematically
- $\lambda = 1 - \gamma$ vanishes at criticality

improved theory:

- includes Landau damping and order parameter conservation
- $\chi \sim \frac{1}{T} \exp\left[-\frac{d}{z'} |\ln T|^{3/5}\right]$

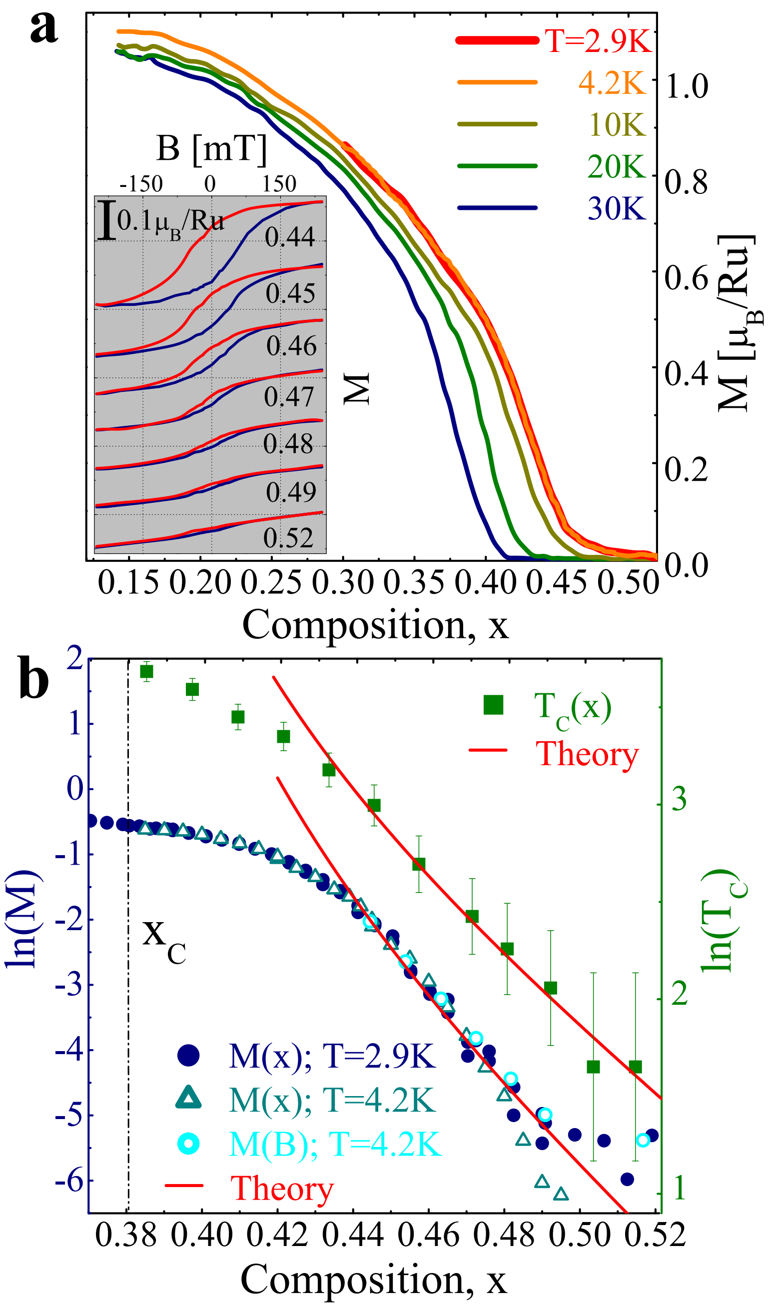
D. Nozadze + T. V., Phys. Rev. B **85**, 174202 (2012)

Phase diagram of $Sr_{1-x}Ca_xRuO_3$



L. Demkó, S. Bordács, T. Vojta, D. Nozadze, F. Hrahsheh, C. Svoboda, B. Dóra, H. Yamada, M. Kawasaki, Y. Tokura and **I. Kézsmárki**, Phys. Rev. Lett. **108**, 185701 (2012)

Composition-tuned smeared phase transitions



Magnetization and T_c in tail:

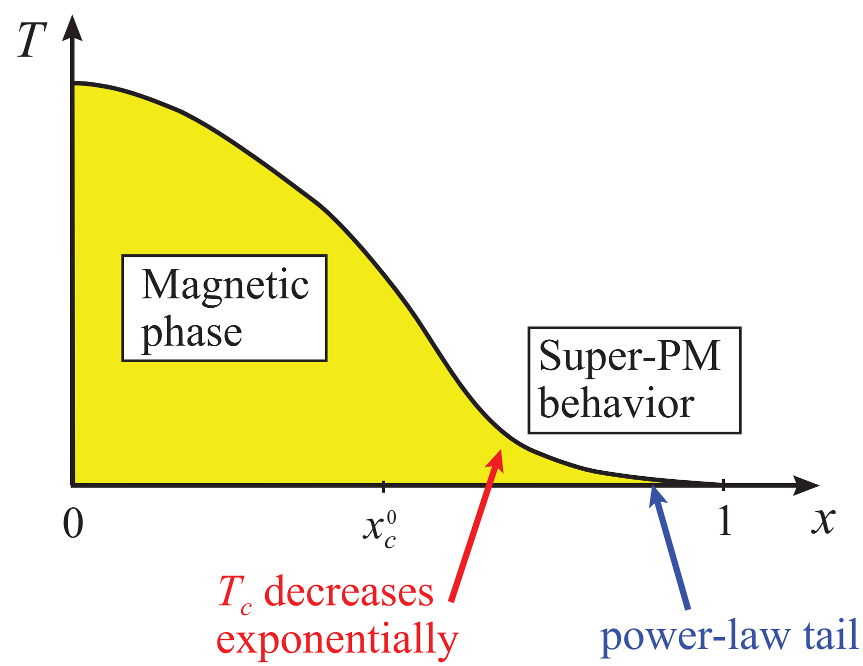
$$M, T_c \sim \exp\left[-C\frac{(x-x_c^0)^{2-d/\phi}}{x(1-x)}\right]$$

for $x \to 1$:

$$M, T_c \sim (1-x)^{L^d_{\min}}$$

F. Hrahsheh et al., PRB **83**, 224402 (2011)

C. Svoboda et al., EPL **97**, 20007 (2012)



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Disorder at phase transitions: two frameworks

- fate of average disorder strength under coarse graining
- importance of rare regions and strength of Griffiths singularities

Recently:

- general relation between **Harris criterion and rare region physics**
  T.V. + J.A. Hoyos, Phys. Rev. Lett. **112**, 075702 (2014), Phys. Rev. E **90**, 012139 (2014)
- **below** d_c^+, same inequality, $d\nu > 2$, governs relevance or irrelevance of disorder and fate of the Griffiths singularities

Class	RR dimension	Subclass	Harris criterion	Griffiths effects	Critical behavior of disordered system
A	$d_{\mathrm{RR}} < d_c^-$	A1	$d\nu > 2$	weak exponential	clean
		A2	$d\nu < 2$	weak exponential	conventional finite disorder
B	$d_{\mathrm{RR}} = d_c^-$	B1	$d\nu > 2$	power law, z' remains finite	clean
		B2	$d\nu < 2$	power law, z' diverges	strong or infinite randomness
C	$d_{\mathrm{RR}} > d_c^-$			rare regions freeze	smeared transition

- **above** d_c^+, behavior is even richer
- relevance of rare regions depends on inequality $d_c^+\nu > 2$

Conclusions

- even weak disorder can have surprisingly strong effects on a phase transition
- **rare regions** play a much bigger role at quantum phase transitions than at classical transitions
- **effective dimensionality** of rare regions determines **character** of Griffiths singularities
- in recent years, experimental evidence for **quantum Griffiths singularities** and **smeared phase transitions** has been found at quantum phase transitions in dirty metals

Quenched disorder at quantum phase transitions leads to a rich variety of new effects and exotic phenomena

Reviews: T.V., J. Phys. A **39**, R143 (2006); J. Low Temp. Phys. **161**, 299 (2010)