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The Role of Models and Analogies in the Electromagnetic Theory: A Historical Case Study

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Abstract. Despite its great importance, many students and even their teachers still cannot recognize the relevance of models to build up physical knowledge and are unable to develop qualitative explanations for mathematical expressions that exist within physics. Thus, it is not a surprise that analogies play an important role in science education, since students' construction of mental models of abstract phenomena need to be rooted in some existing or previous experience in order to interpret more complex ideas. The present article focuses on some of these issues by analyzing some specific instances of the historical development of the electromagnetic theory. Using the mental models framework, the importance of mechanical analogies to understand some of the electromagnetic concepts are emphasized.

Key words: analogies, electricity, electromagnetic theory, fields, history of science, magnetism, Maxwell, mental models, models

1. Introduction

In the last years, researchers in the sciences teaching area have been increasingly interested in the role of models in science teaching. Nowadays one can find a wide range of research publications on this matter, from various viewpoints (Duit 1991; Nersessian 1995; Clement 2000; Gilbert & Boulter 2000; Greca & Moreira 2000; Greca & Moreira 2002; Heywood 2002; Coll et al. 2005). Despite its great relevance, however, many students and teachers still cannot recognize the relevance of models in the construction of scientific knowledge (Van Driel & Verloop 1999; Islas & Pesa 2003). Furthermore they are not able to develop qualitative explanations for mathematical expressions that exist within physical models (Lozano & Cardenas 2002; Silva & Pietrocola 2003).

It is positively important for both teachers and students to comprehend the historical and epistemological features related to the role of models and analogies in the construction of physical knowledge. Coll et al. (2005), for example, state that the use of models and analogies within science teaching may help students to gain some understanding concerning the nature of science, to develop a metacognitive awareness as well as to provide them the tools with which to reflect their own scientific understanding. From a pedagogical point of view, there is a prevailing assumption among science educators that students should learn how to modelling scientific models (Greca & Moreira 2000, p. 2).

Instead of considering such issues in an abstract manner, the present article focuses on some of those features by analyzing some specific instances from the history of the electromagnetic theory in the 19th century. The electromagnetic theory was chosen due to the fact that it is quite an abstract theory and students usually encounter many difficulties to build mental models as close as possible to physical models even regarding some of its basics concepts, as for example the concepts of electric and magnetic fields (Rainson et al. 1994; Borges 1998; Greca & Moreira 1998). In this specific case, knowledge of historical roots concerning these concepts and how the founders of the electromagnetic theory built them may help students and teachers to gain some understanding regarding physical concepts and also to learn about scientific endeavour.

2. Brief Considerations on Models and Analogies

As there are many published works that deal with cognitive, epistemological and pedagogical (Achinstein 1964; Hesse 1972; DeKleer & Brown 1983; Gentner & Stevens 1983; Johnson-Laird 1983; Clement 1989; Giere 1992; Thagard 1992; Greca & Moreira 2000; Adúriz-Bravo & Morales 2002; Justi & Gilbert 2002; Kipnis 2005) features in this matter, it is necessary to specify the conception of models and analogies adopted in this paper. This section presents a brief characterization of the conception of models and analogies considered in this paper.

Models play a core role in science because they are representations of ideas, objects, phenomena or systems. They are also important in science teaching because it is usually assumed that modeling, that is, to establish relations between theory and objects or phenomena should be learned by students. Greca & Moreira (2000) point out the difference between the notion of mental models and scientific models. The notion of mental model in the instructional approach is

> ... a type of knowledge representation which is implicit, incomplete, imprecise, incoherent with normative knowledge in various domains, but it is a useful one, since it results in a powerful explicative and predictive tool for the interaction of subjects with the world, and a dependable source of knowledge, for it comes from the subjects' own perceptive and manipulative experience with this world. (Barquero 1995, *apud* Greca & Moreira 2000, p. 3)

The mental models built by the students have a very strong mechanistic character and enables them to explain and make predictions about the physical system represented. These mental models are considered as intermediate levels of analysis between the modelled phenomenon and the resulting final scientific model. A scientific model is created by researchers, scientists, teachers, etc., in order to facilitate the comprehension or teaching of natural phenomena or systems. The scientific models can materialize as mathematical formulations, analogies or material artifacts (Greca & Moreira 2000, p. 5). In the specific case of a physical model, it is expected that it be materialized as a mathematical formulation.

According to the philosopher Mary Hesse, the relation between model and modelled phenomenon is generally analogical. She differentiates two kinds of analogies: the formal analogy and the material analogy. In the first case, the same axiomatic and deductive relations associate both subjects and objects of similar systems. In this case, these relations are described through similar equations. For instance, a pendulum and an oscillating electric circuit are formally analogous since both systems can be described through the same differential equation. The existence of a material similarity between both systems is not necessary. When material analogies are taken into account, there is a physical similarity between the systems; as an example, one can take the kinetic theory of gases that considers a gas as being a set of tiny spheres (Hesse 1972). Gas molecules may be conceptualized as a collection of billiard balls randomly moving and hitting one another, even though gas molecules and billiard balls are not quite identical – but there are either identical or similar properties for both systems. In the billiard ball model case, both billiard balls and gas molecules can be seen as spherical and both obey Newtonian mechanics.

The analogical relation (either formal or material) usually considers differences and similarities between the analyzed objects. These should be emphasized under pedagogical circumstances because we often use analogies and hardly discuss such differences and similarities.

The historical development of the electromagnetic theory was strongly based on these two kinds of analogies (Nersessian 2002). In spite of this fact, nowadays the electromagnetic theory is taught without any discussion on the models upon which it was built, and students may think that it was just built from empirical knowledge.

3. Analogies and Models in the 19th-Century Electromagnetism

Throughout the second half of the 19th century, mechanics and its applications was a well-developed research field. The British natural philosophy was a mechanical philosophy that investigated explanations for the physical phenomena in terms of matter, movement and forces based on Newtonian laws by making use of concepts such as force, velocity, acceleration, energy and many other mechanical concepts. Within such context, ether was considered as the base for many physical phenomena. It was considered as the medium through which light was propagated, and many functions were attributed to it, such as that of explaining the electric and magnetic phenomena as "fields" that existed within it. Thus, the idea of electromagnetic ether gradually became more common throughout the 19th century and led to the concept of electric and magnetic fields as the physical structures of ether.

19th century physicists paid much attention to the relation between models and physical reality. At that time, a new way of interpreting the relations between mathematical and physical models gained space and brought an interesting view to the development of explanations to physical phenomena, such as the electric and magnetic ones.

One of the methods to develop electromagnetic models made use of analogies with well-known and studied physical systems, such as, heat propagation, motion of a fluid, study of elastic solid mediums etc. (Nersessian 1992). These analogies bore a strong mathematical feature but, on the other hand, were also concerned with the construction of a mental image of the electromagnetic phenomena. This way, such analogies were formal and material, following Hesse's distinction.

There was a strong tendency for the development of realistic models to understand electromagnetic phenomena among the British physicists community of the 19th century. Most of the Victorian physicists believed that matter and ether could bear some mechanical nature; hence, they believed it was possible to build models that could truly explain the physical reality or, at least, could be similar to it.

William Thomson (1824–1907), for instance, considered the mechanical models essential to understand electromagnetic phenomena:

> I never satisfy myself until I can make a mechanical model of a thing. If I can make a mechanical model I understand I can understand it. [...] I firmly believe in an electromagnetic theory of light, and that when we understand electricity and magnetism and light we shall see them all together as parts of a whole. But I want to understand light as well as I can, without introducing things we understand even less of. That is why I take plain dynamics; I can take a model in plain dynamics. I cannot in electromagnetics. (Thomson 1884, *apud* Darrigol 2000, p. 178)

Thomson's mechanical models also bore mathematical features enclosed within the physical ones. He dedicated himself to developing models whose formal aspects (i.e., equations) had a material analogy with mechanical systems as well.

William Thomson, James C. Maxwell and others developed models and analogies to explain both electric and magnetic phenomena based on the existence of ether. The equations and physical concepts that were drawn through the analogical method are still used – and taught – until today, despite the fact that we do not believe that a material medium such as ether pervades space.

3.1. THOMSON'S ANALOGY BETWEEN ELECTRICITY AND HEAT FLOW

William Thomson was one of the first researchers who in 1847 introduced the concept of field with mathematical formalities; he also sought for explanations based on a dynamic theory of ether, as shown below.

Thomson was a mathematician with solid knowledge of analytical mechanics and was perfectly able to appreciate and develop the mathematical works carried out in France. At the age of 16 he studied Fourier's *Théorie analytique de la chaleur* in two weeks (Darrigol 2000, p. 114). Fourier's works on heat called the attention of the British natural philosophers because they had a strong geometric aspect and did not speculate on the nature of heat. Their basic equations bore a direct and empirical meaning and they attributed a central role to the concept of heat flow as well.

In 1842, Thomson started to explore the new formal processes in physics by making use of the analogical method. He started to develop formal analogies between electrostatics and heat flow without trying to understand physically what took place in the space left between conductors. At this point, Thomson just meant to find relations between the equations that could describe both phenomena (Silva & Martins 2003). According to Thomson, such mathematical analogies could be start-points towards more realistic mechanical analogies that could bring a physical theory for the propagation of both electric and magnetic fields (Thomson 1872).

By considering these ideas, Thomson studied physical phenomena that were completely unlike, such as heat flow problems, electrostatic attraction and gravitational attraction. He realized that such phenomena could be described through equations of the same kind by simply attributing the right meanings to each symbol, in each equation. As an example, let us focus on Gauss's law applied to temperature distribution. Thomson over positioned punctual heat sources with density σ on a surface $\mathrm{d}S$ and found the following expression (for temperature θ at a distance r between sources) (Thomson 1872, pp. 3–5):

$$\theta = \iint \frac{\sigma \mathrm{d}S}{r}.$$

This expression is identical to the one used for the electric potential of a σ-charge density positioned at a distance r. Table I shows the elements used by Thomson in his analogy:

Thomson thought it was possible to develop new concepts for electrostatics by studying equations that described the conduction of heat (and vice-versa) because the sets of equations are identical. In his logical

Table I. Thomson's analogy between heat flow and electrostatic

Symbol	Heat flow	Electrostatic attraction
	Heat	Electricity
θ	Temperature	Electric potential
σ	Heat source	Electric charge

thought, the starting point of a theory (Coulomb's law) became the result of the other one (punctual source temperature distribution). An obvious consequence of a theory (local heat transference in Fourier's theory) became an essential principle of the other (field flow through a surface) (Silva & Martins 2003).

This was not a new result. Thomson's contribution was to build a model for the electrostatic based on a better-known phenomenon – heat flow. This is quite an important result currently taught, however, it is usually taught in a very abstract manner, making it difficult for students to build mental models about it.

3.2. THE ANALOGY WITH AN ELASTIC MEDIUM

As was shown in the previous section, the analogy between electrostatics and heat propagation is a formal analogy – it does not focus on the construction of a model-object to explain the electromagnetic phenomena, but rather describes them on a mathematical basis through a mathematical structure that had been "borrowed" from another theory. In order to set a mental image of both electric and magnetic phenomena, Thomson considered that the electric and magnetic fields were propagated quite in the same way as the displacements in an elastic medium. By coincidence, at that time George Gabriel Stokes had just finished a brilliant study on the elasticity of solids and continuous media, presenting a new approach to the study of dynamics of continuous media. Stokes studied mathematically the most general movement of an element of fluid, interpreting this movement as a superposition of a rotation and three dilatations (or contractions) around the three orthogonal axes x, y and z (Darrigol 2000, p. 126).

In his paper *On a mechanical representation of electric, magnetic, and galvanic forces* published in 1847, Thomson observed the equations that described some electromagnetic phenomena and compared them with the equations for the general motion (dilatations, contractions and rotations) of an element of an elastic solid. Thomson interpreted the solutions of Stokes' equations as formally analogous to the electromagnetic equations, where α,β,γ are the projections over the Cartesian axes of a small displacement of the incompressible medium in a point determined by x, y, z:[1]

(a) $\alpha = \frac{x}{r^3}, \beta = \frac{y}{r^3}, \gamma = \frac{z}{r^3}$.

Interpreted as the electrical field due to a unitary electrical charge. Thomson (1882, pp. 78–79) considered the electrical field as materially analogous to a linear displacement in the elastic medium.

(b) $X = \frac{d\beta}{dz} - \frac{d\gamma}{dy}, Y = \frac{d\gamma}{dx} - \frac{d\alpha}{dz}, Z = \frac{d\alpha}{dy} - \frac{d\beta}{dx}$.

Interpreted as the magnetic field between a magnetic dipole and a unit of magnetism.

Thomson regarded each of these components of the magnetic field as analogous to rotations of an element of elastic medium around the axes x, y, and z respectively (Thomson 1882, p. 79). These equations are the components of the curl of a vector withα,β,γ, in modern terminology.

(c) $\frac{d\beta}{dz} - \frac{d\gamma}{dy} = \frac{mz-ny}{r^3}, \frac{d\gamma}{dx} - \frac{d\alpha}{dz} = \frac{nx-lz}{r^3}, \frac{d\alpha}{dy} - \frac{d\beta}{dx} = \frac{ly-mx}{r^3}$.

Interpreted as the force between a unitary current element in the direction l, m, n and a unit of magnetism in the point x, y, z. For Thomson (1882, p. 80) the rotation of any element of elastic medium, expressed by this solution, represented the electromagnetic field due to a current element in direction and intensity.

Thomson applied Stokes' ideas and interpreted the above three simplest kinds of solutions of the equations for an incompressible solid as being similar to the fields of a punctual charge of a magnetic dipole and a current element. By comparing equations that described these phenomena with those that described the general movement (translation and rotation) of an elastic solid element, Thomson came to the conclusion that such phenomena were not only formally analogous, but also materially analogous. The analogy was between the electric field and the elastic displacement; and between the electromagnetic and magnetic fields and rotations in ether with the same properties as the elastic solid described by Stokes. Thomson's analogies can be summarized in Table II.

These analogies are important because they suggest explicitly the propagation of electric and magnetic forces occurring through mechanical processes in the ether (displacements, rotations, forces and other mechanical quantities). Thomson's analogical approach to electromagnetic problems strongly influenced the young Maxwell.

Table II. Thomson's analogy between movements in an elastic solid medium and fields

Elastic solid problem	Electromagnetic problem
Displacement	Electric field
Rotation	Magnetic field
Rotation	Electromagnetic field

3.3. ANALOGIES AND MECHANICAL MODELS IN MAXWELL'S WORKS

Following the tradition of the 19th century mathematical physics in Great Britain, James Clerk Maxwell extensively employed mechanical models in the study and representation of electromagnetic phenomena. In his early works, Maxwell considered the development of a connection between the electromagnetic theory and a theory about the ether as quite significant. Maxwell did not consider the field equations as the only important features of his electromagnetic theory; actually he thought that the field itself bore mechanical properties and physical reality. Within this context, the existence of mechanical models to explain physical phenomena was essential. This section discusses some of Maxwell's early models, focusing on the use of both formal and material analogies.

For instance, in the article *On Faraday's lines of force* published in 1856, Maxwell developed a mathematical model to describe the lines of force (Maxwell 1965, pp. 155–229). He intended to

> [...] discover some method of investigation which allows the mind at every step to lay hold of a clear physical conception, without being committed to any theory founded on the physical science from which that conception is borrowed [...]. In order to obtain physical ideas without adopting a physical theory we must make ourselves familiar with the existence of physical analogies. (Maxwell 1965, p. 156)

In this article, Maxwell developed a formal analogy between the movement of an incompressible fluid and electrostatics to reach a material analogy between both physical phenomena. Initially, he found the expressions below for velocity v and pressure p of the fluid at a distance r from the source (Maxwell 1965, pp. 160–167)

$$v = \frac{1}{4\pi r^2} \quad \text{and} \quad p = \frac{1}{4\pi r}.$$

These equations are quite similar to the ones that express the electrostatic field and potential. Because of such similarity, Maxwell thought that the lines of force could be interpreted as analogous to the lines of fluid motion and the concept of lines of fluid motion "may be modified so as to be applicable to the sciences of statical electricity, permanent magnetism, magnetism of induction, and uniform galvanic currents; reserving the laws of electro-magnetism for special consideration" (Maxwell 1965, p. 175–177).

Despite Maxwell applying the idea of lines of force in several different cases (electrostatic, magnetism, electrodynamics) in the paper *On Faraday's line of force*, Maxwell did not bring forward an unified model that could lead to a clear idea of the mechanisms for explaining the electric and magnetic phenomena, because each phenomenon was explained in a different and independent way.

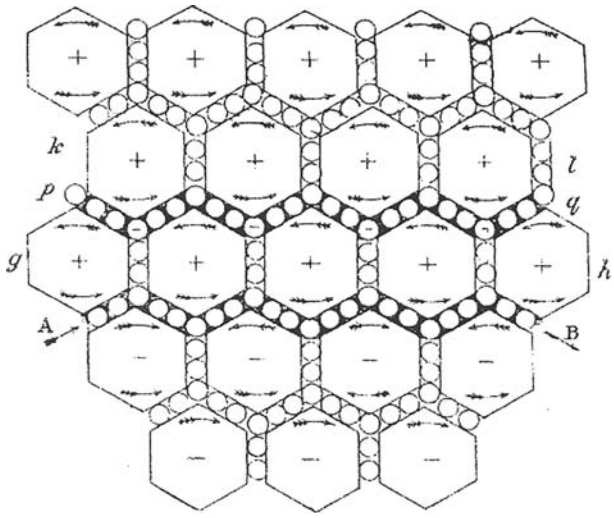

Figure 1. Maxwell's vortices and idle wheels model.

Maxwell's following paper on this matter was *On physical lines of force*, published in two parts, in 1861 and 1862. In this paper, Maxwell developed an analogy between the electromagnetic phenomena and the movement of molecular vortices in a medium. Besides the fluid vortices, Maxwell introduced between them a layer of small idle wheels, as shown in Figure 1 (Maxwell 1861, p. 283). He interpreted the tangential action of the vortices on the particles as the electromotive force, the angular momentum of the vortices as proportional to the magnetic force and the displacement of the idle wheels as proportional to the current. In this model, Maxwell demonstrated, by use of formal and material analogies, the possibility of reducing the electromagnetic phenomena to mechanical phenomena taking place in a physical medium.

Maxwell strongly believed that it was possible to explain the electromagnetic phenomena using mechanical concepts; therefore he explored many different mechanical models to describe ether and its properties. In 1873 Maxwell published the book *Treatise on electricity and magnetism*, where he exposed his ideas in a more mature form. In spite of believing in the existence of mechanical models for ether, in this book Maxwell did not restrict himself to any particular mechanical model since the details of the various possible models were quite controversial at that time. He still adopted a mechanical approach, however, a more abstract one: he employed a dynamical method by the use of the Lagrange equations method, which is more general and formal than the use of specific mechanical models.

A question may be pointed out here: how far would such images go as a literal representation of reality? Maxwell discussed physical questions and commented on philosophical bases regarding the use of analogies in the article *On Faraday's lines of force*. Maxwell believed physical analogies could offer an investigation method that would allow each used step to be visualized until a clear physical conception was obtained. For instance, the analogy with the movement of a fluid could be applied to the separate parts within electromagnetism (electrostatics, magnetostatics, electrodynamics) without considering mechanical forces between charged bodies or magnets. This analogy considers that the action between bodies propagates through a "field of forces" similarly to what happens with the movement of a fluid. Maxwell considered the dynamical methods he used in the book *Treatise* as only a provisional step and lamented his incapacity to "take the next step, namely, to account by mechanical considerations for these stresses in the dielectrics" (Maxwell 1954, p. 166).

4. Implications for Teaching

This paper discusses the role of analogies in the development of electromagnetic theory in the 19th century by transferring knowledge from some branches of physics (such as mechanics and heat flow) into another (such as electrostatic and electrodynamics). However the central role of analogies in the understanding of phenomena may be found in many historical cases, including the works of Darwin, Kekule and Einstein. Analogies, models and modelling are key features of science and consequently of science education. This crucial role justifies the inclusion of this kind of study in science teaching. Thus, it is important for teachers to develop pedagogical practices that provide opportunities for students to get into the type of work done by the scientists and to realize the role and value of analogies and models (Coll et al. 2005).

The historical case study presented in this paper shows that models and analogies play an essential role in the practice of science. Furthermore, the students can learn that new physical theories and their equations do not emerge completely ready from brilliant minds. The electromagnetic field equations are not derived by induction from the experimental data and electrical considerations alone. On the contrary, they were constructed by the use of a wide range of heuristic procedures, among whose are the analogies (Nersessian 1992, p. 12).

In the case of the electromagnetic theory, this statement becomes very clear because the founders of the electromagnetic theory made extensive use of such analogies. Using the mental models framework, it can be said that some of the Victorian physicists, such as Thomson and Maxwell, were looking for physical models to describe the electromagnetic phenomena that enable to form mental models the nearest possible to the physical models (Nersessian 2002). This goal could be reached by the hypothesis of existence of a physical medium with mechanical properties pervading all space – the ether. The requisite of having mechanical properties was quite fundamental because people understand and reason about specific domains of knowledge by constructing mental models with mechanical features (Nersessian 1992).

Currently, we do not consider ether as a real physical entity, however it still has an important role on the modelling process of electromagnetic concepts. Is it possible to explain (and to understand) the magnetic field produced around a wire carrying current, or the flow of energy in an electromagnetic wave without appealing to some mechanical rotation or flux of some material entity such as ether? (Nersessian 2002, p. 144) points out that the mechanical reasoning process has the potential to lead to new representational structures. To appeal to a material entity (such as the ether) to understanding the field concept is a necessary intermediate step towards the construction of abstract concepts as the field concept currently accepted. Teachers very often use some medium like ether to explain the immaterial processes that take place in the "empty space", however it occurs in a subliminal and unconscious way.

It is not sufficient that students are able to deal with the mathematical aspects of a theory. In order to be able to build mental models of the electromagnetic phenomena, understanding the physical concepts involved in the mathematical formulae is a necessary condition (Silva & Pietrocola 2003). Knowing the historical roots of physical ideas can facilitate learning because students can have a context in which to place them. In this perspective the historical analogies can potentially help students in constructing mental models of difficult scientific concepts such as the field concept and developing qualitative understandings of mathematical expressions as well.

To use analogies previously used by scientists in the past while they were developing their theories has advantages over other kinds of analogies. It is due to the fact that their strong points and limitations are well known (Kipnis 2005). Historical analogies can be helpful to teachers who aim at helping students in constructing their own mental models on physical phenomena, which will be as close as possible to currently accepted scientific models.

The use of historical analogies provides a link between the scientific and the educational domains and this connection contributes to a better understanding of the scientific nature (Gilbert et al. 2000). Moreover, if teachers knew the heuristic role of analogies, teaching activities that enable better understanding concerning the importance of other factors, besides experimental factors, could be more easily carried out in order to develop science teaching and this would lead to a better understanding of the scientific dynamics as well.

Despite their important role in the development of the electromagnetic theory, analogies are rarely discussed when electromagnetic theory is taught. This paper suggests that teacher awareness concerning the role played by the models and analogies in the historical development of the electromagnetic theory may help them to become acquainted with students' understandings and the difficulties they face in constructing mental models on electromagnetic theory. Therefore, it is no surprise that analogies play an important role in science education, since explanation of the abstract phenomena needs to be rooted in some existing or previous experience in order to interpret more complex ideas.

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Note

[1] The vector calculus had not yet been developed in the 19th century. As was common at this time, Thomson used the components formalism in order to represent the vector quantities. In this formalism a vector quantity is represented by its three components and three different letters are used to represent these components. That is, symbols such as E_x, E_y, E_z were not used, but *F*, *G*, *H* were used (Crowe 1967; Silva 2002).

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