Nonlinear Effects in Optical Fibers

Govind P. Agrawal

Institute of Optics
University of Rochester
Rochester, NY 14627

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- Stimulated Brillouin Scattering
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Introduction

Fiber nonlinearities

- Studied during the 1970s.
- Ignored during the 1980s.
- Feared during the 1990s.
- May be conquered in this decade.

Objective:

- Review of *Nonlinear Effects in Optical Fibers*. 
Major Nonlinear Effects

- Stimulated Raman Scattering (SRS)
- Stimulated Brillouin Scattering (SBS)
- Self-Phase Modulation (SPM)
- Cross-Phase Modulation (XPM)
- Four-Wave Mixing (FWM)

Origin of Nonlinear Effects in Optical Fibers

- Ultrafast third-order susceptibility $\chi^{(3)}$.
- Real part leads to SPM, XPM, and FWM.
- Imaginary part leads to SBS and SRS.
Stimulated Raman Scattering

- Scattering of light from vibrating silica molecules.
- Amorphous nature of silica turns vibrational state into a band.
- Raman gain spectrum extends over 40 THz or so.

- Raman gain is maximum near 13 THz.
- Scattered light red-shifted by 100 nm in the 1.5 μm region.
SRS Dynamics

- SRS process is governed by two coupled equations:
  \[ \frac{dI_p}{dz} = -g_R I_p I_s - \alpha_p I_p, \quad \frac{dI_s}{dz} = g_R I_p I_s - \alpha_s I_s. \]

- If we neglect pump depletion \((I_s \ll I_p)\), pump power decays exponentially, and the Stokes beam satisfies
  \[ \frac{dI_s}{dz} = g_R I_0 e^{-\alpha_p z} I_s - \alpha_s I_s. \]

- This equation has the solution
  \[ I_s(L) = I_s(0) \exp(g_R I_0 L_{\text{eff}} - \alpha_s L), \quad L_{\text{eff}} = \frac{1 - \exp(-\alpha_p L)}{\alpha_p}. \]

- SRS acts as an amplifier if pump wavelength is chosen suitably.
Raman Threshold

- Even in the absence of an input, Stokes beam can buildup if pump power is large enough.
- Spontaneous Raman scattering acts as the seed for this buildup.
- Mathematically, the growth process is equivalent to injecting one photon per mode into the fiber:

\[ P_s(L) = \int_{-\infty}^{\infty} \hbar \omega \exp[g_R(\omega_p - \omega)I_0L_{\text{eff}} - \alpha_sL] d\omega. \]

- Approximate solution (using the method of steepest descent):

\[ P_s(L) = P_{s0}^{\text{eff}} \exp[g_R(\Omega_R)I_0L_{\text{eff}} - \alpha_sL]. \]

- Effective input power is given by

\[ P_{s0}^{\text{eff}} = \hbar \omega_s B_{\text{eff}}, \quad B_{\text{eff}} = \left( \frac{2\pi}{I_0L_{\text{eff}}} \right)^{1/2} \left| \frac{\partial^2 g_R}{\partial \omega^2} \right|_{\omega=\omega_s}^{-1/2}. \]
Raman Threshold

- Raman threshold is defined as the input pump power at which Stokes power becomes equal to the pump power at the fiber output:

\[ P_s(L) = P_p(L) \equiv P_0 \exp(-\alpha_p L). \]

- \( P_0 = I_0 A_{\text{eff}} \) is the input pump power.

- For \( \alpha_s \approx \alpha_p \), threshold condition becomes

\[ P_{s0}^{\text{eff}} \exp\left( g_R P_0 L_{\text{eff}} / A_{\text{eff}} \right) = P_0, \]

- Assuming a Lorentzian shape for the Raman-gain spectrum, Raman threshold is reached when (Smith, Appl. Opt. 11, 2489, 1972)

\[ \frac{g_R P_{th} L_{\text{eff}}}{A_{\text{eff}}} \approx 16 \quad \rightarrow \quad P_{th} \approx \frac{16 A_{\text{eff}}}{g_R L_{\text{eff}}}. \]
Estimates of Raman Threshold

Telecommunication Fibers

- For long fibers, \( L_{\text{eff}} = [1 - \exp(-\alpha L)]/\alpha \approx 1/\alpha \approx 20 \text{ km} \) for \( \alpha = 0.2 \text{ dB/km} \) at 1.55 µm.

- For telecom fibers, \( A_{\text{eff}} = 50-75 \mu m^2 \).

- Threshold power \( P_{\text{th}} \sim 1 \text{ W} \) is too large to be of concern.

- Interchannel crosstalk in WDM systems because of Raman gain.

Yb-doped Fiber Lasers and Amplifiers

- For short fibers \( (L < 100 \text{ m}) \), \( L_{\text{eff}} = L \).

- For fibers with a large core, \( A_{\text{eff}} \sim 500 \mu m^2 \).

- \( P_{\text{th}} \) can exceed 100 kW depending on fiber length.

- SRS may limit fiber lasers and amplifiers if \( L \gg 10 \text{ m} \).
SRS: Good or Bad?

- Raman gain introduces interchannel crosstalk in WDM systems.
- Crosstalk can be reduced by lowering channel powers but it limits the number of channels.

On the other hand …

- Raman amplifiers are a boon for WDM systems.
- Can be used in the entire 1300–1650 nm range.
- Erbium-doped fiber amplifiers limited to \( \sim 40 \) nm.
- Distributed nature of amplification lowers noise.
- Likely to open new transmission bands.
Raman Amplifiers

- Pumped in backward direction using diode lasers.
- Multiple pumps used to produce wide bandwidth with a relatively flat gain spectrum.
- Help to realize longer transmission distances compared with erbium-doped fiber amplifiers.
Stimulated Brillouin Scattering

- Scattering of light from acoustic waves.
- Becomes a stimulated process when input power exceeds a threshold level.
- Low threshold power for long fibers ($\sim 5 \text{ mW}$).

- Most of the power reflected backward after SBS threshold is reached.
Brillouin Shift

- Pump produces density variations through electrostriction, resulting in an index grating which generates Stokes wave through Bragg diffraction.

- Energy and momentum conservation require:
  \[
  \Omega_B = \omega_p - \omega_s, \quad \vec{k}_A = \vec{k}_p - \vec{k}_s.
  \]

- Acoustic waves satisfy the dispersion relation:
  \[
  \Omega_B = v_A |\vec{k}_A| \approx 2 v_A |\vec{k}_p| \sin(\theta/2).
  \]

- In a single-mode fiber \( \theta = 180^\circ \), resulting in
  \[
  v_B = \Omega_B / 2\pi = 2 n_p v_A / \lambda_p \approx 11 \text{ GHz},
  \]
  if we use \( v_A = 5.96 \text{ km/s}, \) \( n_p = 1.45 \), and \( \lambda_p = 1.55 \mu m \).
Brillouin Gain Spectrum

- Decay of acoustic waves as $\exp(-\Gamma_B t)$ leads to a Lorentzian gain spectrum of the form

$$g_B(\Omega) = g_p \frac{(\Gamma_B/2)^2}{(\Omega - \Omega_B)^2 + (\Gamma_B/2)^2}.$$ 

- Peak gain depends on the material parameters as

$$g_p \equiv g_B(\Omega_B) = \frac{8\pi^2 \gamma_e^2}{n_p \lambda_p^2 \rho_0 c v_A \Gamma_B}.$$ 

- Electrostrictive constant $\gamma_e = \rho_0 (d\varepsilon/d\rho)_{\rho=\rho_0} \approx 0.902$ for silica.

- Gain bandwidth $\Gamma_B$ scales with $\lambda_p$ as $\lambda_p^{-2}$.

- For silica fibers $g_p \approx 5 \times 10^{-11}$ m/W, $T_B = \Gamma_B^{-1} \approx 5$ ns, and gain bandwidth < 50 MHz.
Brillouin Gain Spectrum

- Measured spectra for (a) silica-core (b) depressed-cladding, and (c) dispersion-shifted fibers.
- Brillouin gain spectrum is quite narrow (∼50 MHz).
- Brillouin shift depends on GeO$_2$ doping within the core.
- Multiple peaks are due to the excitation of different acoustic modes.
- Each acoustic mode propagates at a different velocity $v_A$ and thus leads to a different Brillouin shift ($v_B = 2n_p v_A / \lambda_p$).
Brillouin Threshold

- Pump and Stokes evolve along the fiber as
  \[ -\frac{dI_s}{dz} = g_B I_p I_s - \alpha I_s, \quad \frac{dI_p}{dz} = -g_B I_p I_s - \alpha I_p. \]

- Ignoring pump depletion, \( I_p(z) = I_0 \exp(-\alpha z). \)

- Solution of the Stokes equation:
  \[ I_s(L) = I_s(0) \exp(g_B I_0 L_{\text{eff}} - \alpha L). \]

- Brillouin threshold is obtained from
  \[ \frac{g_B P_{th} L_{\text{eff}}}{A_{\text{eff}}} \approx 21 \quad \rightarrow \quad P_{th} \approx \frac{21 A_{\text{eff}}}{g_B L_{\text{eff}}}. \]

- Brillouin gain \( g_B \approx 5 \times 10^{-11} \) m/W is nearly independent of the pump wavelength.
Estimates of Brillouin Threshold

Telecommunication Fibers

- For long fibers, $L_{\text{eff}} = \frac{1 - \exp(-\alpha L)}{\alpha} \approx \frac{1}{\alpha} \approx 20 \text{ km}$ for $\alpha = 0.2 \text{ dB/km}$ at 1.55 $\mu$m.
- For telecom fibers, $A_{\text{eff}} = 50–75 \mu m^2$.
- Threshold power $P_{th} \sim 1 \text{ mW}$ is relatively small.

Yb-doped Fiber Lasers and Amplifiers

- For short fibers $(L < 100 \text{ m})$, $L_{\text{eff}} = L$.
- $P_{th}$ exceeds 20 W for a 1-m-long fiber.
- Further increase occurs for large-core fibers; $P_{th} \sim 200 \text{ W}$ when $A_{\text{eff}} \sim 500 \mu m^2$.
- SBS is the dominant limiting factor at power levels $P_0 > 1 \text{ kW}$.
Techniques for Controlling SBS

- Pump-Phase modulation: Sinusoidal modulation at several frequencies $>0.1$ GHz or with a pseudorandom bit pattern.
- Cross-phase modulation by launching a pseudorandom pulse train at a different wavelength.
- Temperature gradient along the fiber: Changes in $\nu_B = 2n_pv_A/\lambda_p$ through temperature dependence of $n_p$.
- Built-in strain along the fiber: Changes in $\nu_B$ through $n_p$.
- Nonuniform core radius and dopant density: mode index $n_p$ also depends on fiber design parameters ($a$ and $\Delta$).
- Control of overlap between the optical and acoustic modes.
- Use of Large-core fibers: Wider core reduces SBS threshold by enhancing $A_{\text{eff}}$. 
Fiber Gratings for Controlling SBS

- Fiber Bragg gratings can be employed for SBS suppression [Lee and Agrawal, Opt. Exp. 11, 3467 (2003)].
- One or more fiber grating are placed along the fiber, depending on the fiber length.
- Grating is designed such that it is transparent to the pump beam, but Stokes spectrum falls entirely within its stop band.
- Stokes is reflected by the grating and it begins to propagate in the forward direction with the pump.
- A new Stokes wave can still build up, but its power is reduced because of the exponential nature of the SBS gain.
- Multiple gratings may need to be used for long fibers.
- For short fibers, a long grating can be made all along its length.
Grating-Induced SBS Suppression

[Lee and Agrawal, Opt. Exp. 11, 3467 (2003)]

- (a) 15-ns pulses, 2-kW peak power, 1-m-long grating with $\kappa L = 35$
- (b) Fraction of pulse energy transmitted versus grating strength.
Self-Phase Modulation

- Refractive index depends on optical intensity as
  \[ n(\omega, I) = n_0(\omega) + n_2 I(t). \]

- Leads to nonlinear Phase shift
  \[ \phi_{NL}(t) = \frac{2\pi}{\lambda} n_2 I(t) L. \]

- An optical field modifies its own phase (SPM).
- Phase shift varies with time for pulses.
- Each optical pulse becomes chirped.
- As a pulse propagates along the fiber, its spectrum changes because of SPM.
Nonlinear Phase Shift

- Pulse propagation governed by Nonlinear Schrödinger Equation
  \[ i \frac{\partial A}{\partial z} - \beta_2 \frac{\partial^2 A}{\partial t^2} + \gamma |A|^2 A = 0. \]

- Dispersive effects within the fiber included through $\beta_2$.

- Nonlinear effects included through $\gamma = \frac{2\pi n_2}{(\lambda A_{\text{eff}})}$.

- If we ignore dispersive effects, solution can be written as
  \[ A(L, t) = A(0, t) \exp(i\phi_{\text{NL}}), \text{ where } \phi_{\text{NL}}(t) = \gamma L |A(0, t)|^2. \]

- Nonlinear phase shift depends on the pulse shape through its power profile $P(t) = |A(0, t)|^2$. 
SPM-Induced Chirp

Nonlinear phase shift
Pulse width = 90 ps, Fiber length = 100 m.

Experimental Spectra
SPM: Good or Bad?

- SPM-induced spectral broadening can degrade performance of a lightwave system.
- Modulation instability often enhances system noise.

On the positive side . . .

- Modulation instability can be used to produce ultrashort pulses at high repetition rates.
- SPM can be used for fast optical switching.
- It has been used for passive mode locking.
- Responsible for the formation of optical solitons.
Modulation Instability

Nonlinear Schrödinger Equation

\[ i \frac{\partial A}{\partial z} - \frac{\beta_2}{2} \frac{\partial^2 A}{\partial t^2} + \gamma |A|^2 A = 0. \]

- CW solution unstable for anomalous dispersion \((\beta_2 < 0)\).
- Useful for producing ultrashort pulse trains.
Modulation Instability

- A CW beam can be converted into a pulse train.
- A weak modulation helps to reduce the power level and makes the repetition rate tunable.
- Two CW beams at slightly different wavelengths can initiate modulation instability.
- Repetition rate governed by the wavelength difference.
- Repetition rates $\sim 100$ GHz realized using DFB lasers.

![Graphs showing intensity and SHG intensity vs. wavelength and time delay]
Nonlinear Fiber-Loop Mirror

- An example of the Sagnac interferometer.
- Transmission through the fiber loop:
  \[ T = 1 - 4f(1 - f)\cos^2((f - \frac{1}{2})\gamma P_0 L). \]
- \( f \) = fraction of power in the CCW direction.
- \( T = 0 \) for a 3-dB coupler (loop acts as a perfect mirror).
- Power-dependent transmission for \( f \neq 0.5 \).
Passive Mode Locking

- Figure-8 fiber laser can produce pulses $\sim 100$ fs.
- Amplifier located asymmetrically inside the NFLM.
- SPM-induced phase shift larger in clockwise direction.
- Low-power light reflected by the loop.
- Central part of the pulse transmitted.
- Transmitted pulses become narrower.
Cross-Phase Modulation

- Consider two optical fields propagating simultaneously.
- Nonlinear refractive index seen by one wave depends on the intensity of the other wave as

\[ \Delta n_{NL} = n_2(|A_1|^2 + b|A_2|^2). \]

- Nonlinear phase shift:

\[ \phi_{NL} = \left( \frac{2\pi L}{\lambda} \right)n_2[I_1(t) + bI_2(t)]. \]

- An optical beam modifies not only its own phase but also of other copropagating beams (XPM).
- XPM induces nonlinear coupling among overlapping optical pulses.
XPM-Induced Chirp

- Fiber dispersion affects the XPM considerably.
- Pulses belonging to different WDM channels travel at different speeds.
- XPM occurs only when pulses overlap.
- Asymmetric XPM-induced chirp and spectral broadening.
XPM: Good or Bad?

- XPM leads to interchannel crosstalk in WDM systems.
- It can produce amplitude and timing jitter.

On the other hand …

XPM can be used beneficially for

- Nonlinear Pulse Compression
- Passive mode locking
- Ultrafast optical switching
- Demultiplexing of OTDM channels
- Wavelength conversion of WDM channels
XPM-Induced Mode Locking

- Different nonlinear phase shifts for the two polarization components: nonlinear polarization rotation.

\[ \phi_x - \phi_y = \left( \frac{2\pi L}{\lambda} \right) n_2 \left[ (I_x + bI_y) - (I_y + bI_x) \right] \]

- Pulse center and wings develop different polarizations.
- Polarizing isolator clips the wings and shortens the pulse.
- Can produce \( \sim 100 \) fs pulses.
Synchronous Mode Locking

- Laser cavity contains the XPM fiber (few km long).
- Pump pulses produce XPM-induced chirp periodically.
- Pulse repetition rate set to a multiple of cavity mode spacing.
- Situation equivalent to the FM mode-locking technique.
XPM-Induced Switching

- A Mach–Zehnder or Sagnac interferometer can be used.
- Output switched to a different port using a control signal that shifts the phase through XPM.
- If control signal is in the form of a pulse train, a CW signal can be converted into a pulse train.
- Ultrafast switching time (<1 ps).
Four-Wave Mixing (FWM)

- FWM is a nonlinear process that transfers energy of pumps to signal and idler waves.

- FWM requires conservation of (notation: \( E = \text{Re}[A \exp(i\beta z - i\omega t)] \))
  - Energy: \( \omega_1 + \omega_2 = \omega_3 + \omega_4 \)
  - Momentum: \( \beta_1 + \beta_2 = \beta_3 + \beta_4 \)

- Degenerate FWM: Single pump (\( \omega_1 = \omega_2 \)).
FWM: Good or Bad?

- FWM leads to interchannel crosstalk in WDM systems.
- It generates additional noise and degrades system performance.

On the other hand ... FWM can be used beneficially for

- Parametric amplification
- Optical phase conjugation
- Demultiplexing of OTDM channels
- Wavelength conversion of WDM channels
- Supercontinuum generation
Parametric Amplification

- FWM can be used to amplify a weak signal.
- Pump power is transferred to signal through FWM.
- Peak gain $G_p = \frac{1}{4} \exp(2\gamma P_0 L)$ can exceed 20 dB for $P_0 \sim 0.5$ W and $L \sim 1$ km.
- Parametric amplifiers can provide gain at any wavelength using suitable pumps.
- Two pumps can be used to obtain 30–40 dB gain over a large bandwidth (>40 nm).
- Such amplifiers are also useful for ultrafast signal processing.
Single- and Dual-Pump FOPAs

- Pump close to fiber’s ZDWL
- Wide but nonuniform gain spectrum with a dip

- Pumps at opposite ends
- Much more uniform gain
- Lower pump powers (~0.5 W)
Optical Phase Conjugation

- FWM generates an idler wave during parametric amplification.
- Its phase is complex conjugate of the signal field \((A_4 \propto A_3^*)\) because of spectral inversion.
- Phase conjugation can be used for dispersion compensation by placing a parametric amplifier midway.
- It can also reduce timing jitter in lightwave systems.
Wavelength Conversion

- FWM can transfer data to a different wavelength.
- A CW pump beam is launched into the fiber together with the signal channel.
- Its wavelength is chosen halfway from the desired shift.
- FWM transfers the data from signal to the idler beam at the new wavelength.
Highly Nonlinear Fibers

- Silica nonlinearity is relatively weak ($n_2 = 2.6 \times 10^{-20} \text{ m}^2/\text{W}$).
- Applications of nonlinear effects require high input powers in combination with long fiber lengths (> 1 km).
- Parameter $\gamma = 2\pi n_2 / (\lambda A_{\text{eff}})$ can be increased by reducing $A_{\text{eff}}$.
- Such fibers are called highly nonlinear fibers. Examples include photonic-crystal, tapered, and other microstructure fibers.
Supercontinuum Generation

- FWM in combination with SPM, XPM, and SRS can generate superbroad spectrum extending over $>200$ nm.
- Produced by launching short optical pulses into dispersion- and nonlinearity-controlled fibers.

![Graph showing spectral density and fiber output](image)

Photonic-crystal fiber
Coen et al., JOSA B, Apr. 2002

Tapered fiber
Birk et al., OL, Oct. 2000
Supercontinuum Applications

- Potential applications include optical coherence tomography, carrier-envelope phase locking, telecommunications, etc.

- Spectral slicing can be used to produce 1000 or more channels (Takara et al., EL, 2000).
Concluding Remarks

- Optical fibers exhibit a variety of nonlinear effects.
- Fiber nonlinearities are feared by telecom system designers because they can affect system performance adversely.
- Fiber nonlinearities can be managed thorough proper system design.
- Nonlinear effects are useful for many device and system applications: optical switching, soliton formation, wavelength conversion, broadband amplification, demultiplexing, etc.
- New kinds of fibers have been developed for enhancing nonlinear effects.
- Supercontinuum generation in such fibers is likely to found new applications.