

Universidade de São Paulo
Instituto de Física de São Carlos - IFSC

FCM 0410 Física para Engenharia Ambiental

Rotação, torque e momento angular

Prof. Dr. José Pedro Donoso

Agradescimentos

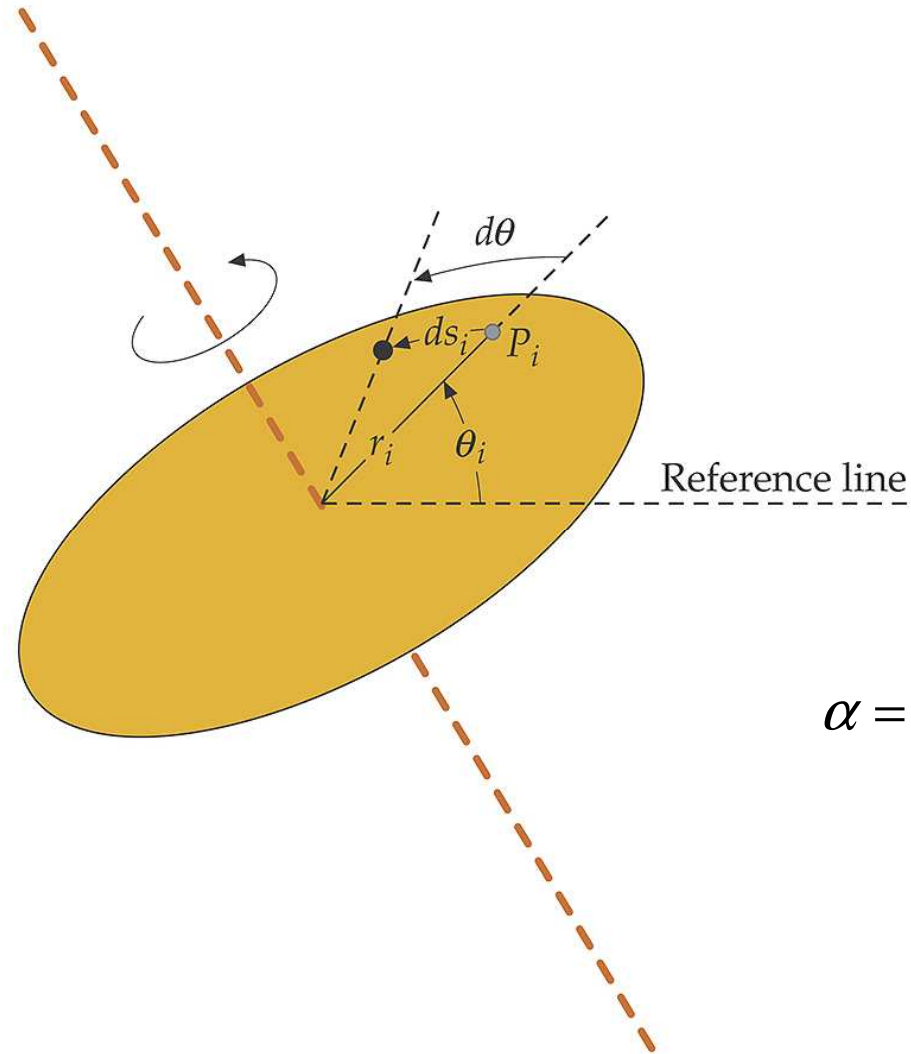
O docente da disciplina, Jose Pedro Donoso, gostaria de expressar o seu agradecimento a Flávia O. S. de Sá Lisboa pelo auxílio na montagem da página /web/ da disciplina.

Parte das figuras utilizadas nos slides foram obtidas do texto "*Física*" de P.A. Tipler e G. Mosca, através do acesso às páginas para os professores das editora LTC (Livros Técnicos e Científicos).



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Disco girando em torno de um eixo: velocidade e aceleração angular



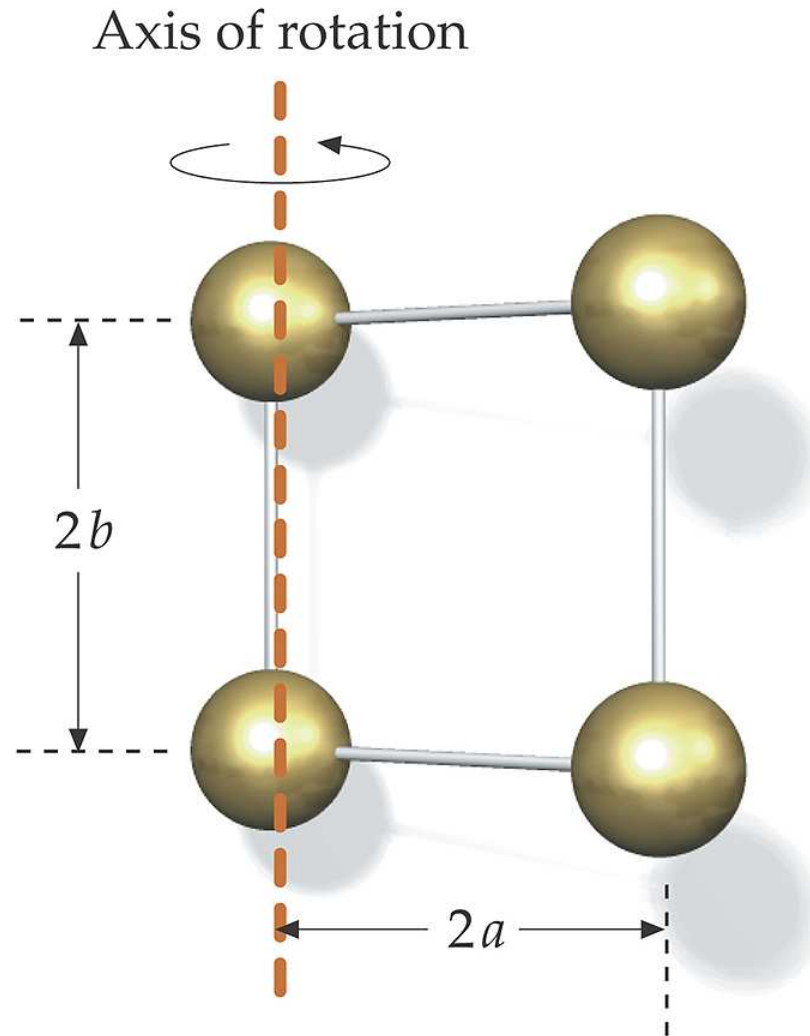
$$ds = r d\theta$$

$$\omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

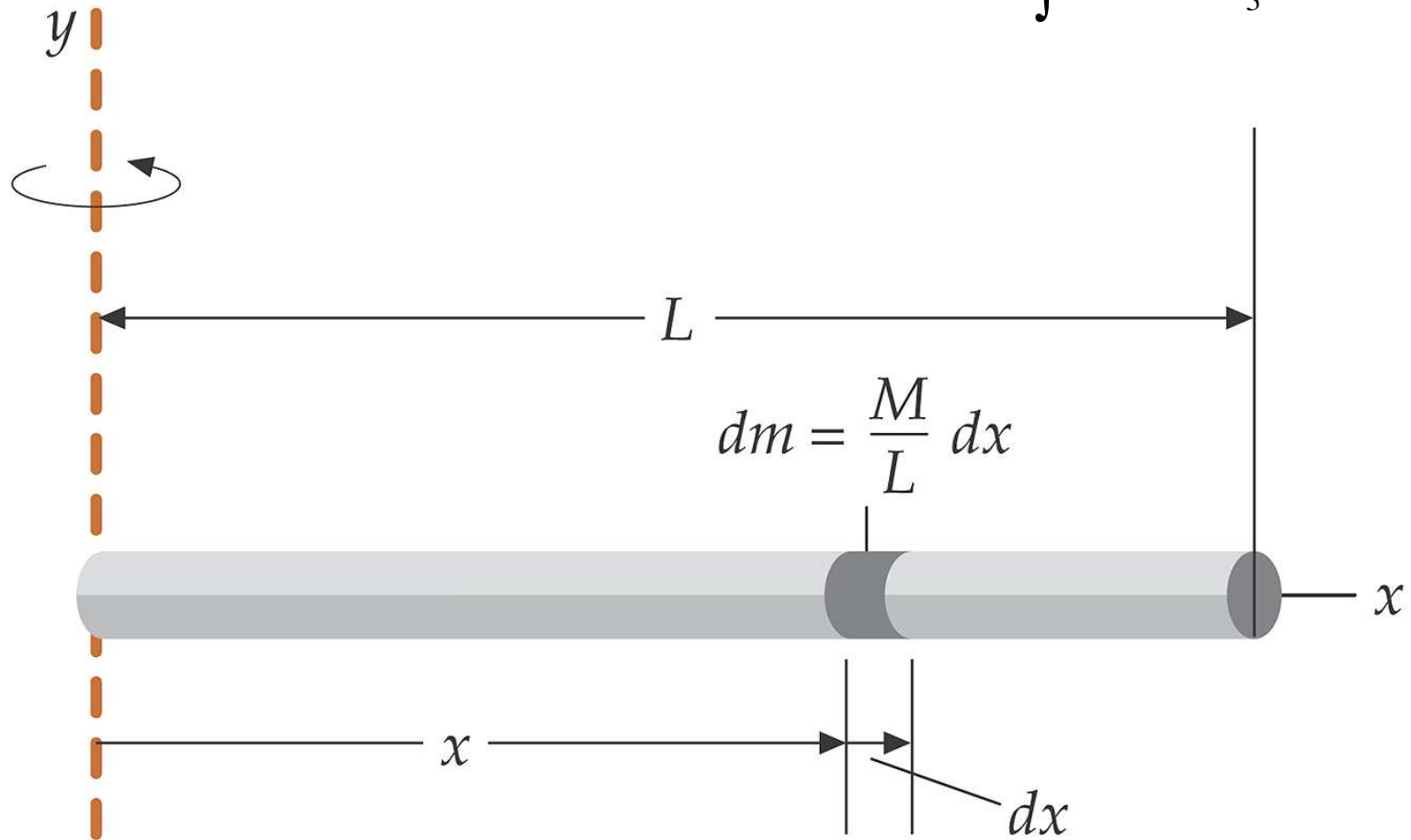
Sistema de partículas girando

$$I = \sum m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2$$

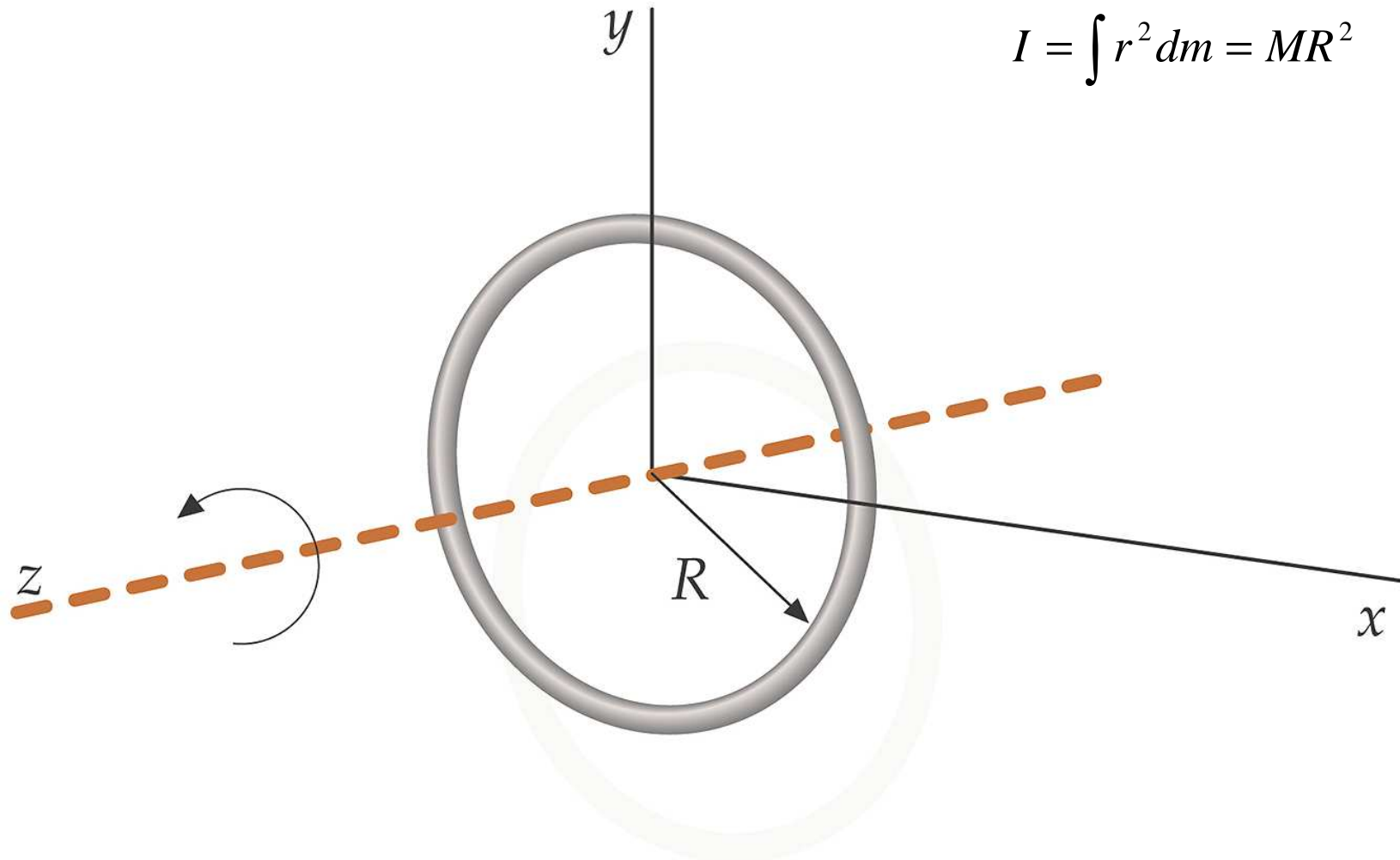


Momento de inercia de uma barra (eixo na extremidade)

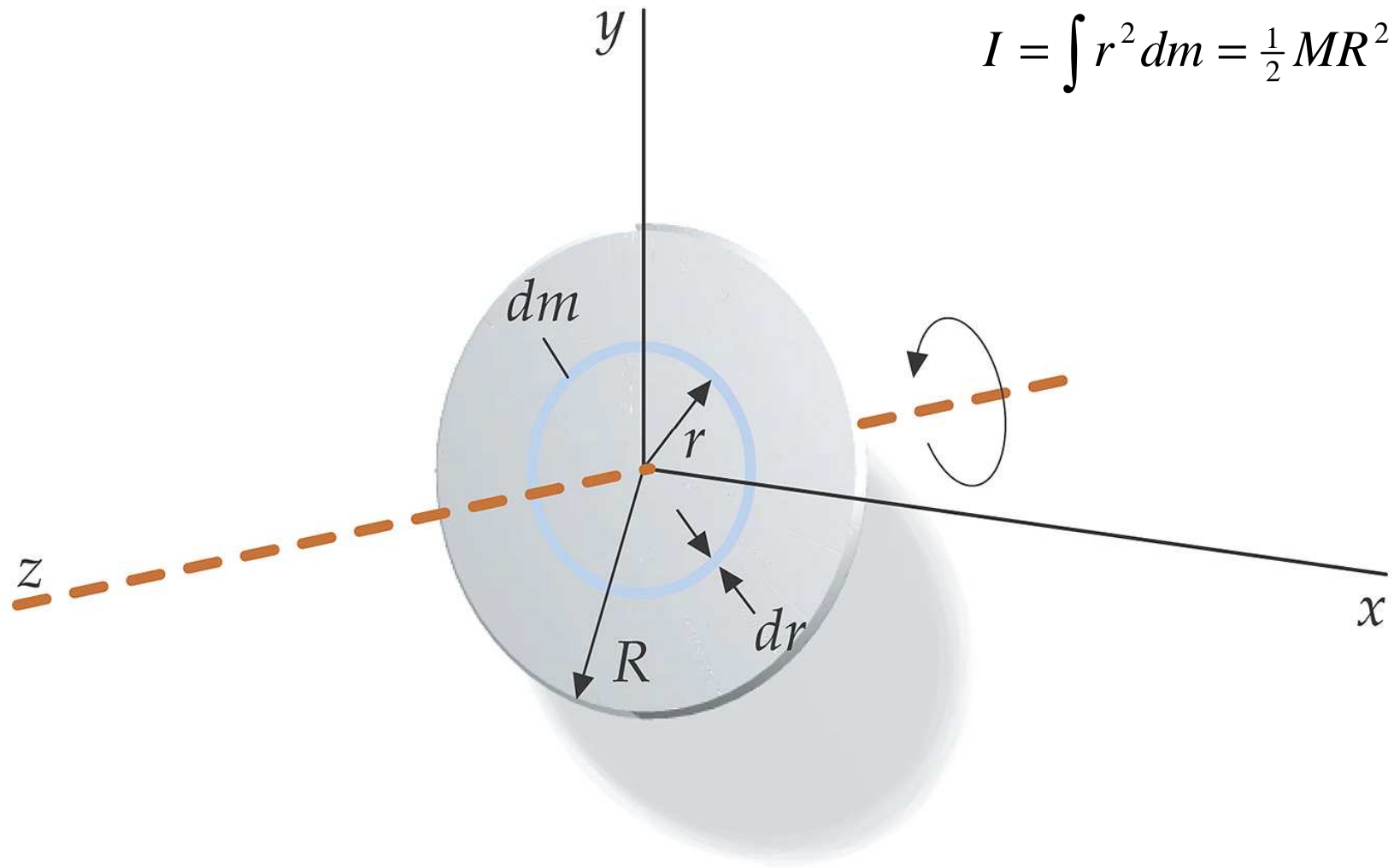
$$I = \int x^2 dm = \frac{1}{3} ML^2$$



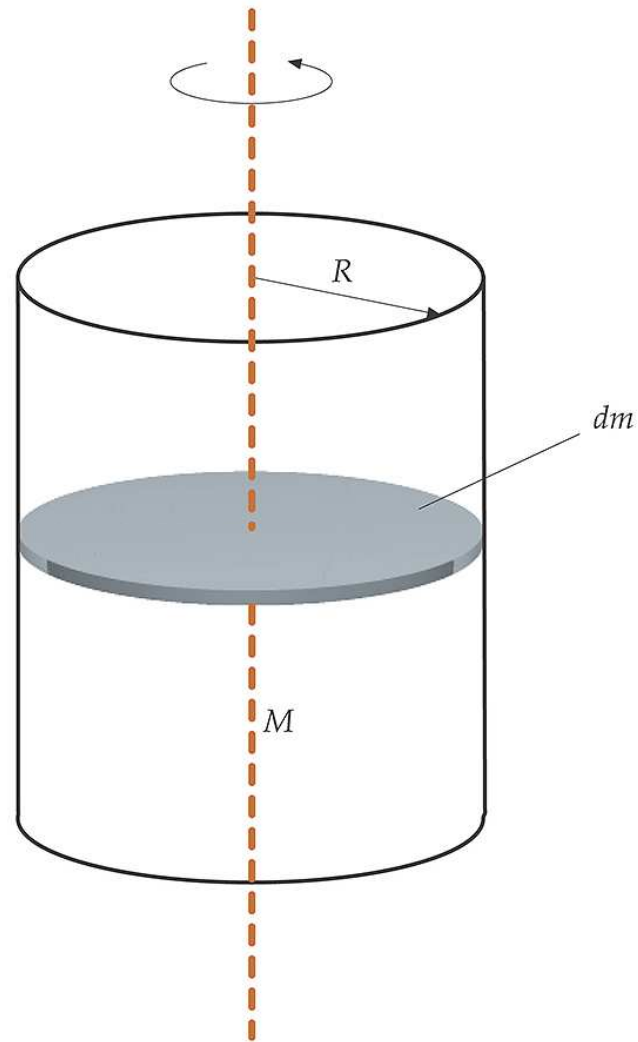
Momento de inercia de um aro (eixo pelo centro)



Momento de inercia de um disco (eixo pelo centro)



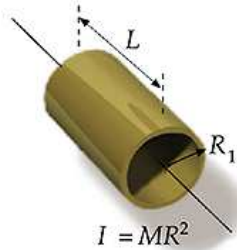
Momento de inercia de um cilindro (eixo pelo centro)



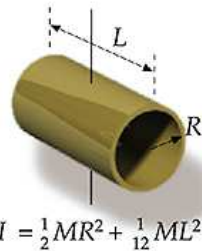
$$I = \int r^2 dm = \frac{1}{2} MR^2$$

Table 9-1 Moments of Inertia of Uniform Bodies of Various Shapes

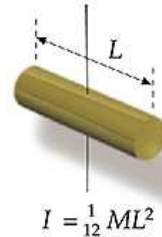
Thin cylindrical shell about axis



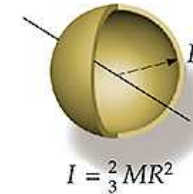
Thin cylindrical shell about diameter through center



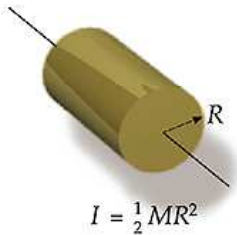
Thin rod about perpendicular line through center



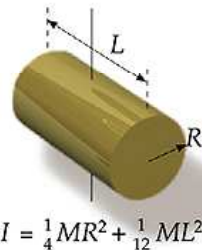
Thin spherical shell about diameter



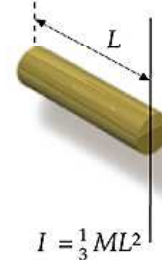
Solid cylinder about axis



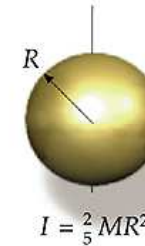
Solid cylinder about diameter through center



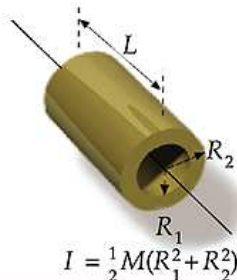
Thin rod about perpendicular line through one end



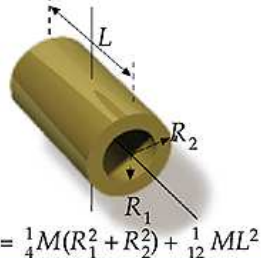
Solid sphere about diameter



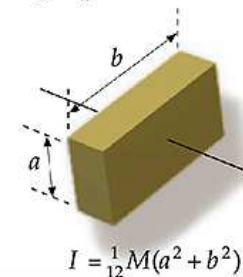
Hollow cylinder about axis



Hollow cylinder about diameter through center

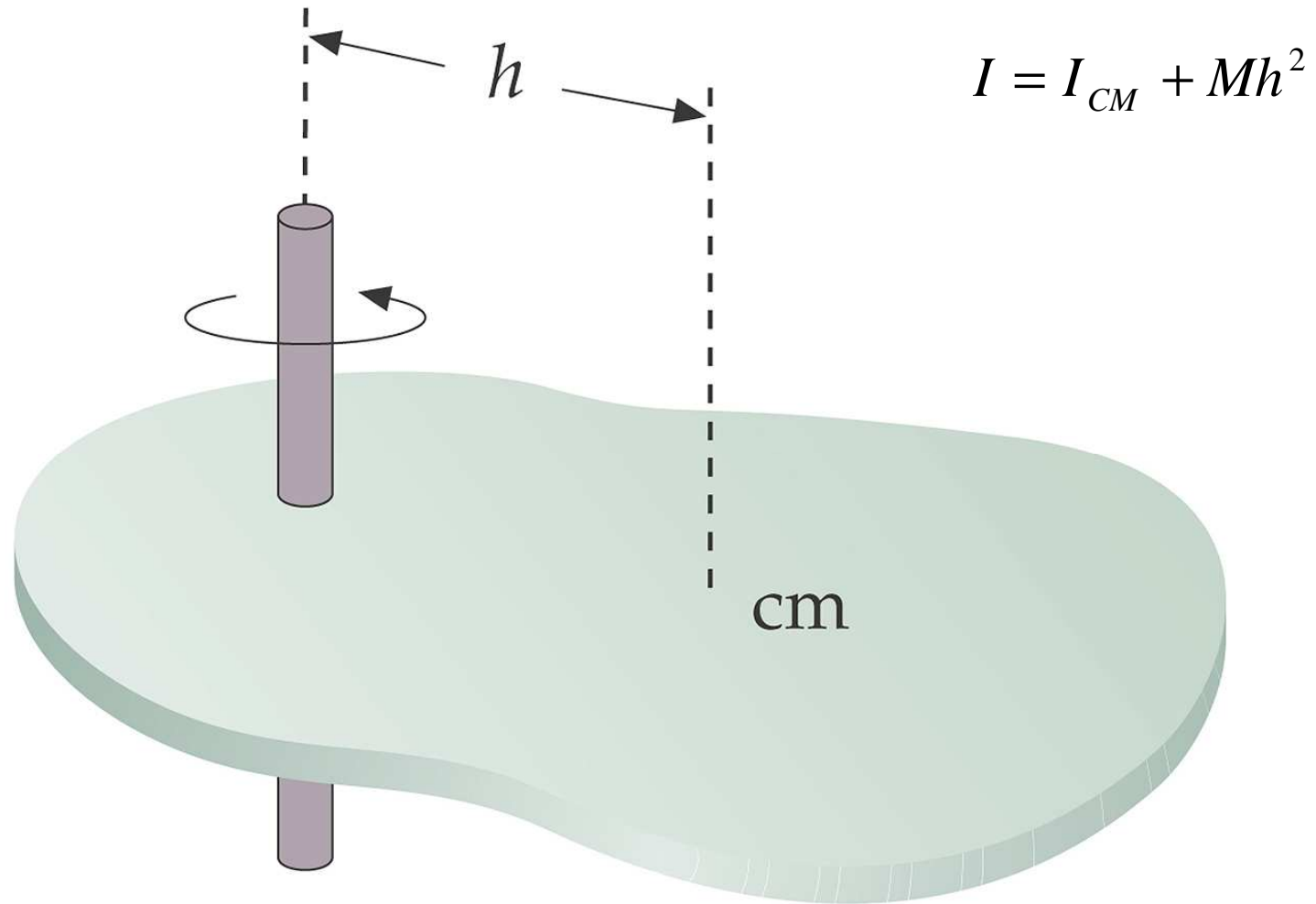


Solid rectangular parallelepiped about axis through center perpendicular to face



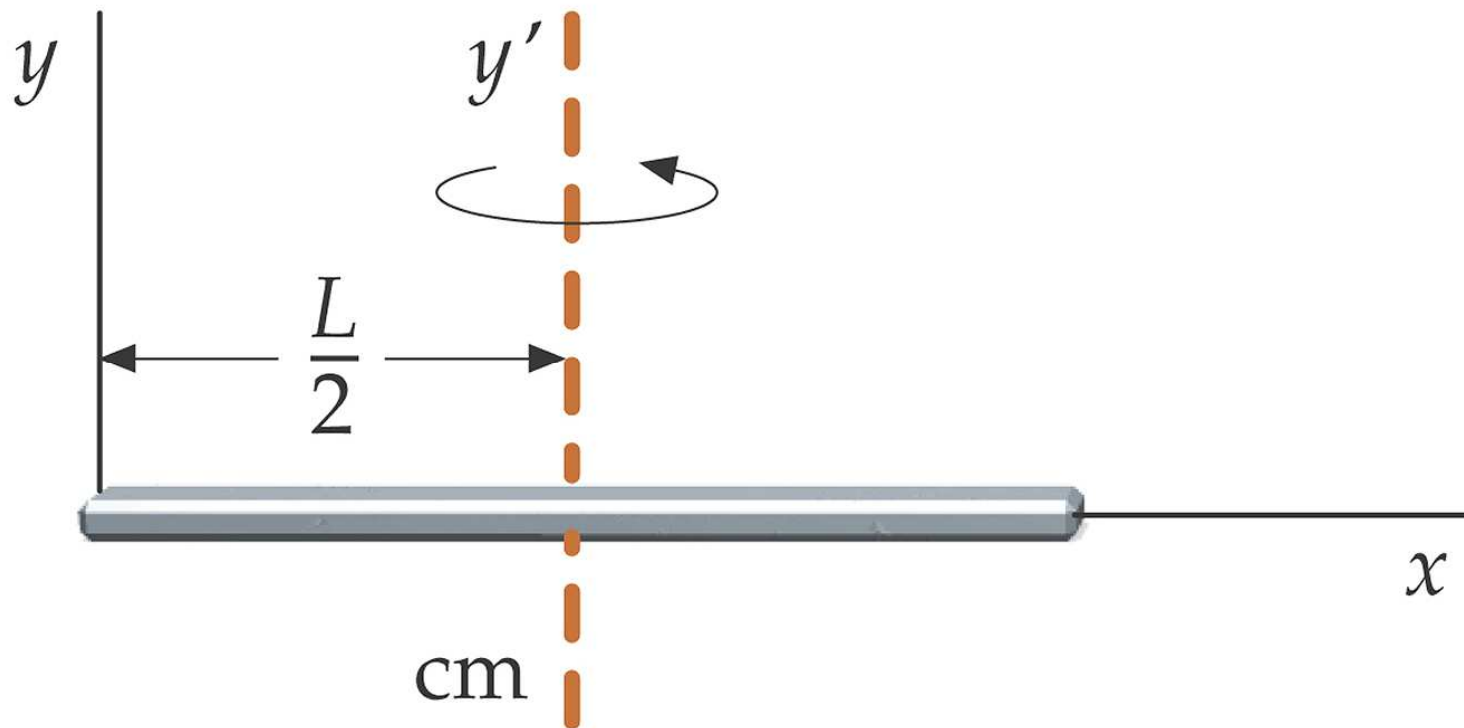
*A disk is a cylinder whose length L is negligible. By setting $L = 0$, the above formulas for cylinders hold for disks.

Momento de inercia: teorema dos eixos paralelos

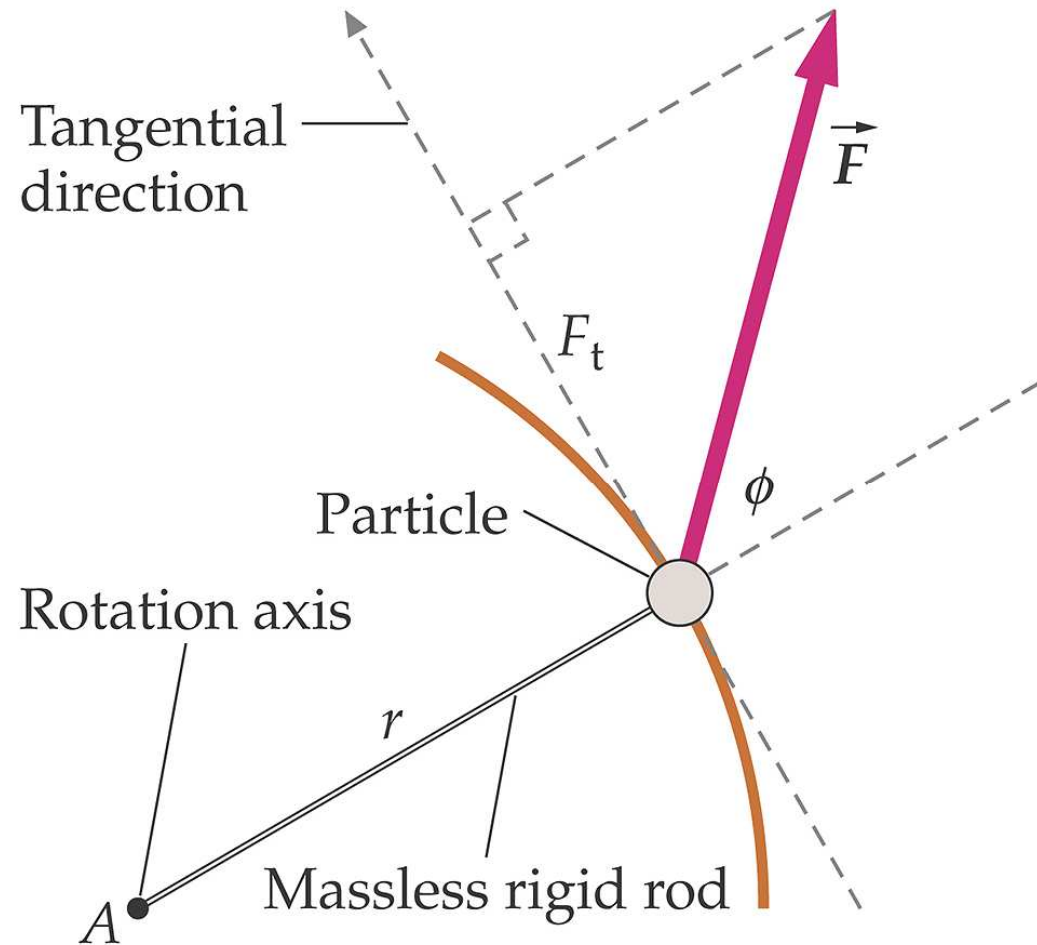


Aplicando o teorema dos eixos paralelos: momento de inercia de uma barra

$$I_{CM} = I_y - Mh^2 = \frac{1}{3}ML^2 - M\left(\frac{1}{2}L\right)^2 = \frac{1}{12}ML^2$$



Partícula pressa a uma barra que pode girar livremente: torque

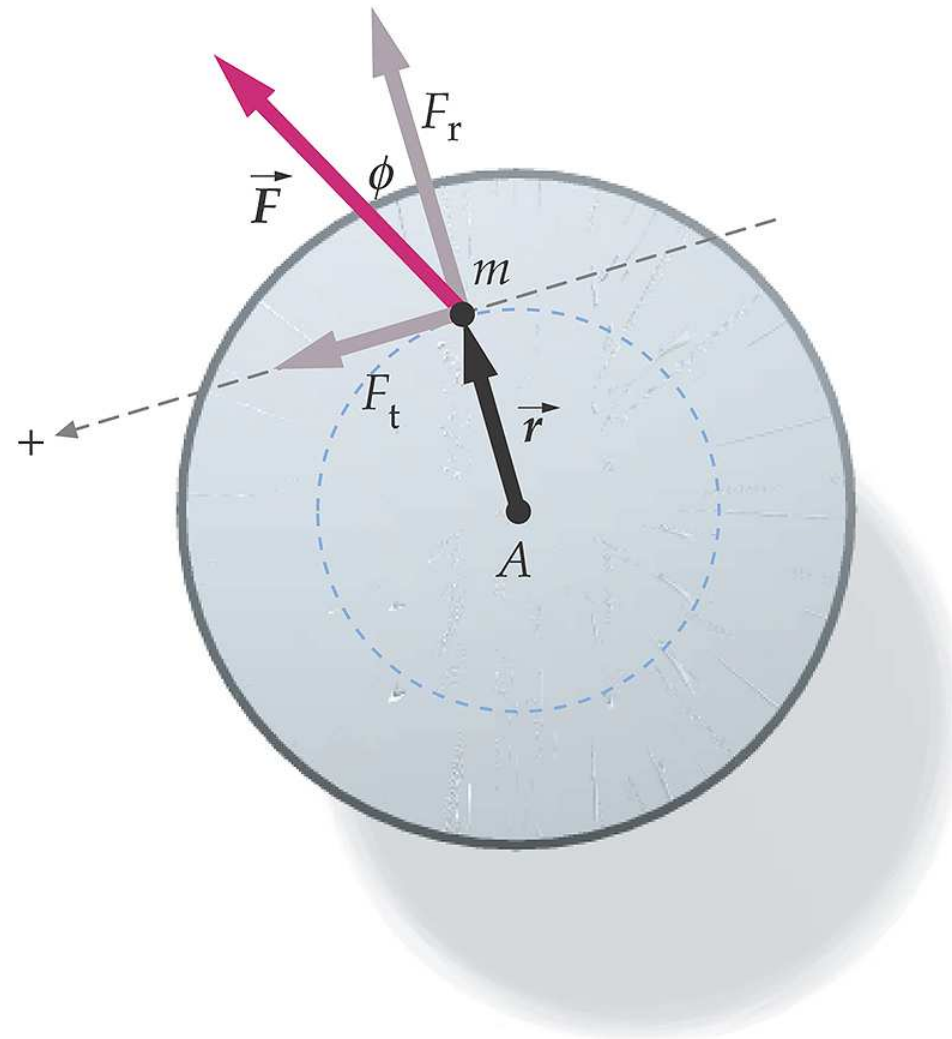


$$\tau = F_t r$$



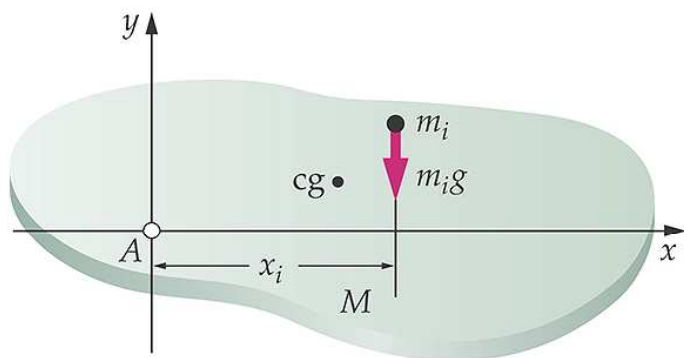
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A força F produz um torque $F_t r$ em relação ao centro

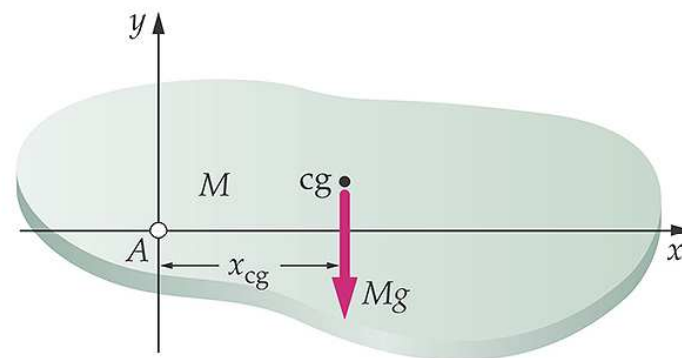


Torque devido à gravidade: ele é calculado como se toda a força gravitacional fosse aplicada no centro de massa

$$\tau = Mg x_{CM}$$



(a)



(b)

Barra pivotada: determine a aceleração angular

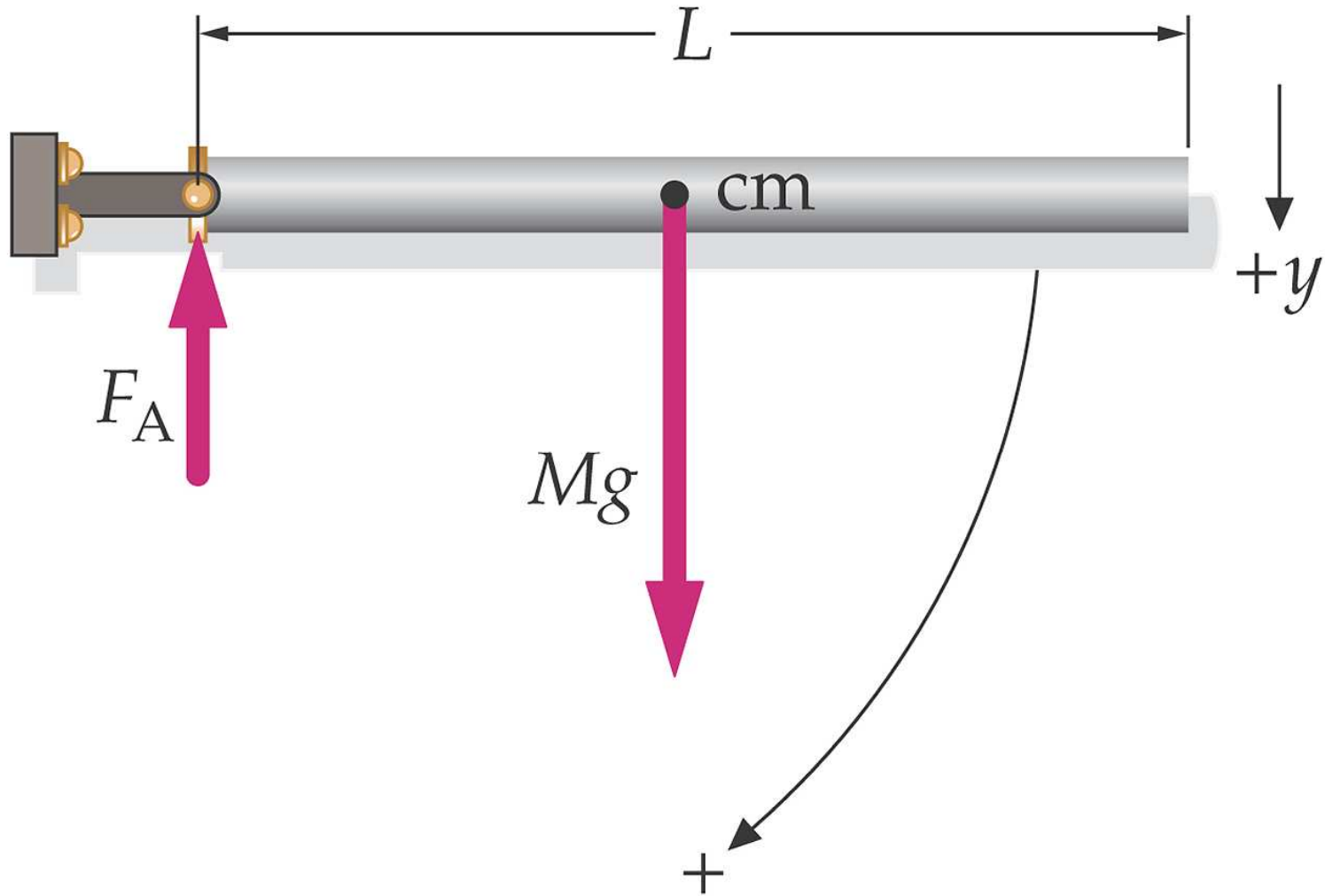


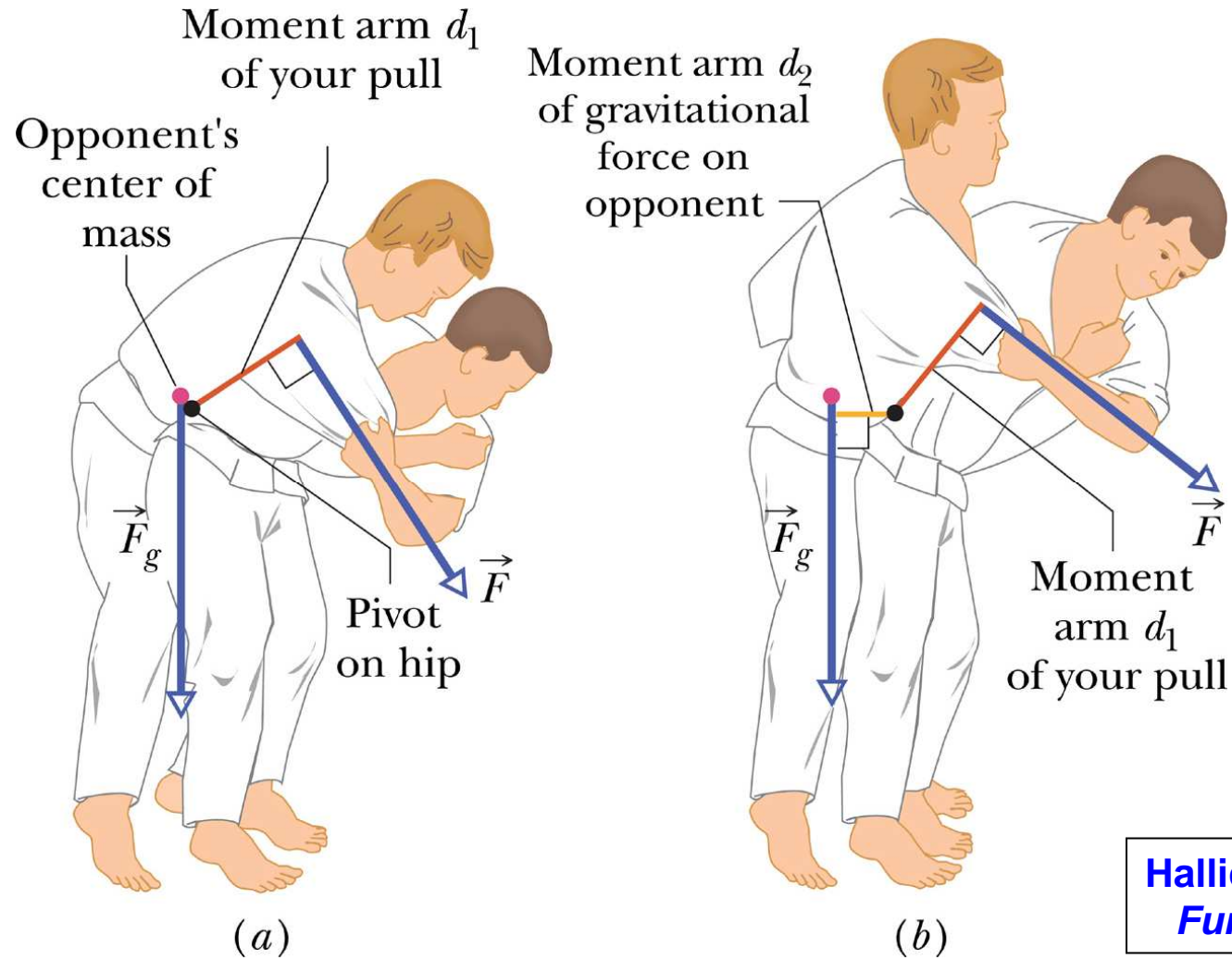
Table 9-2 Analogs in Fixed-Axis Rotational and One-Dimensional Linear Motion

Rotational Motion		Linear Motion	
Angular displacement	$\Delta\theta$	Displacement	Δx
Angular velocity	$\omega = \frac{d\theta}{dt}$	Velocity	$v_x = \frac{dx}{dt}$
Angular acceleration	$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$	Acceleration	$a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$
Constant-angular-acceleration equations	$\omega = \omega_0 + \alpha t$ $\Delta\theta = \omega_{av} \Delta t$ $\omega_{av} = \frac{1}{2}(\omega_0 + \omega)$ $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$ $\omega^2 = \omega_0^2 + 2\alpha \Delta\theta$	Constant-acceleration equations	$v_x = v_{0x} + a_x t$ $\Delta x = v_{avx} \Delta t$ $v_{avx} = \frac{1}{2}(v_{0x} + v_x)$ $x = x_0 + v_{0x} t + \frac{1}{2}a_x t^2$ $v_x^2 = v_{0x}^2 + 2a_x \Delta x$
Torque	τ	Force	F_x
Moment of inertia	I	Mass	m
Work	$dW = \tau d\theta$	Work	$dW = F_x dx$
Kinetic energy	$K = \frac{1}{2}I\omega^2$	Kinetic energy	$K = \frac{1}{2}mv^2$
Power	$P = \tau\omega$	Power	$P = F_x v_x$
Angular momentum*	$L = I\omega$	Momentum	$p_x = mv_x$
Newton's second law	$\tau_{net} = I\alpha = \frac{dL}{dt}$	Newton's second law	$F_{netx} = ma_x = \frac{dp_x}{dt}$

*Angular momentum is introduced in Chapter 10.

Judô: qual valor da força F para derrubar o adversário?

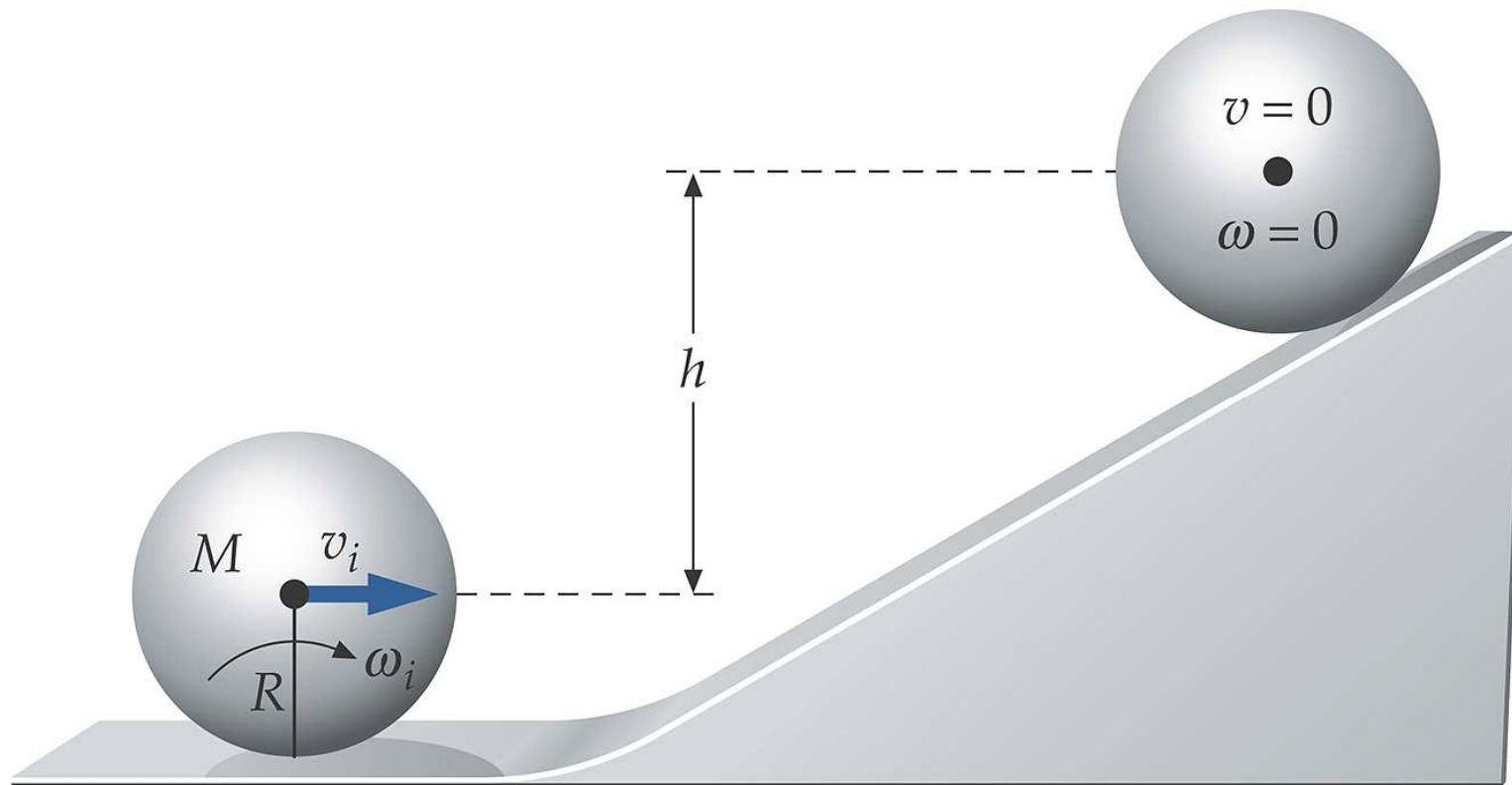
$$\text{Torque} : -d\vec{F} = I\alpha$$



Halliday, Resnick, Walker,
Fundamentos da Física

Conservação da energia

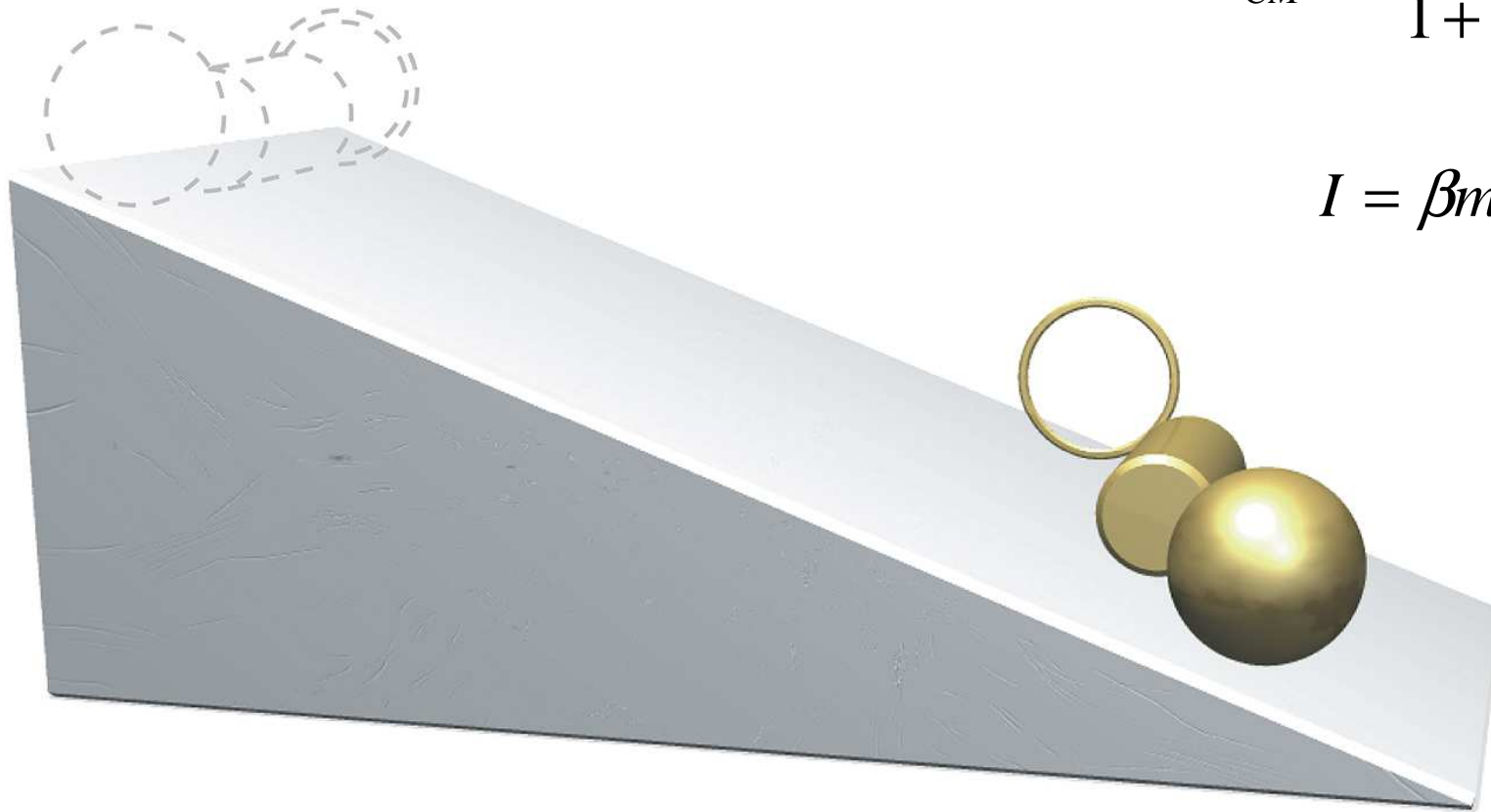
$$Mgh = \frac{1}{2} Mv_{CM}^2 + \frac{1}{2} I_{CM} \omega^2$$



Uma esfera ($\beta = 2/5$), um cilindro ($\beta = 1/2$) e um aro ($\beta = 1$) num plano inclinado

$$a_{CM} = \frac{g \sin \phi}{1 + \beta}$$

$$I = \beta m R^2$$



Centrífugas de laboratório



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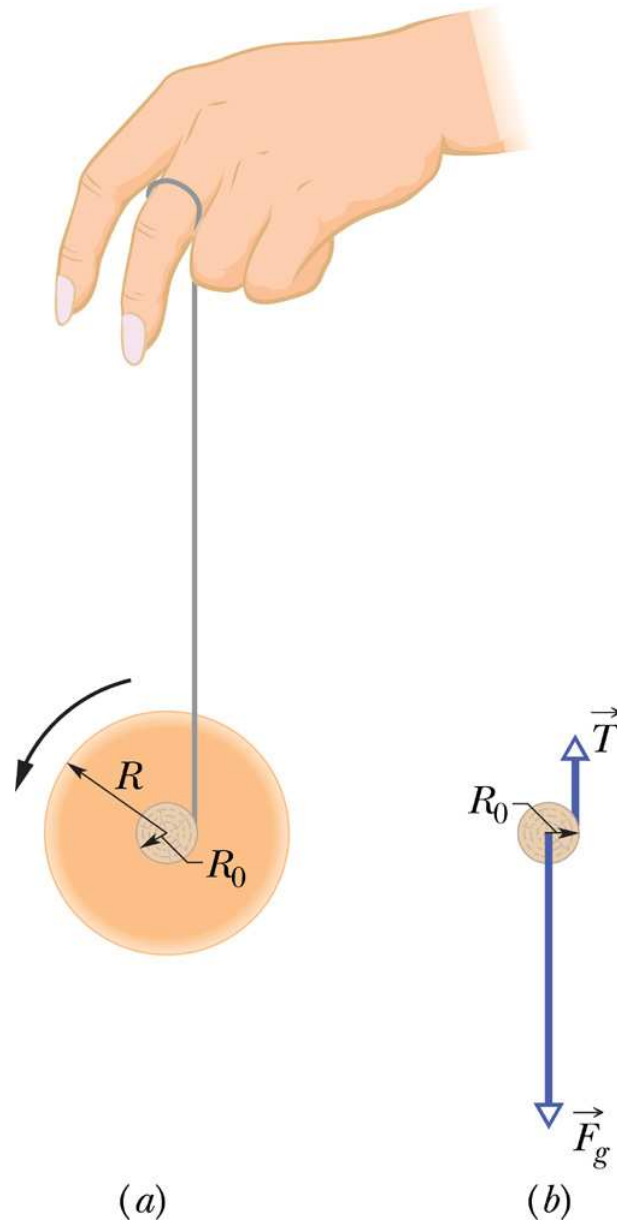
Explosão de um rotor de aço maciço em forma de disco

Empresa *Test Devices Inc.* (1985)



Halliday, Resnick, Walker, *Fundamentos da Física*

O loiô



Halliday, Resnick, Walker
Fundamentos da Física

Correspondências entre movimentos de traslação e rotação

Pure Translation (Fixed Direction)		Pure Rotation (Fixed Axis)	
Position	x	Angular position	θ
Velocity	$v = dx/dt$	Angular velocity	$\omega = d\theta/dt$
Acceleration	$a = dv/dt$	Angular acceleration	$\alpha = d\omega/dt$
Mass	m	Rotational inertia	I
Newton's second law	$F_{\text{net}} = ma$	Newton's second law	$\tau_{\text{net}} = I\alpha$
Work	$W = \int F dx$	Work	$W = \int \tau d\theta$
Kinetic energy	$K = \frac{1}{2}mv^2$	Kinetic energy	$K = \frac{1}{2}I\omega^2$
Power (constant force)	$P = Fv$	Power (constant torque)	$P = \tau\omega$
Work–kinetic energy theorem	$W = \Delta K$	Work–kinetic energy theorem	$W = \Delta K$

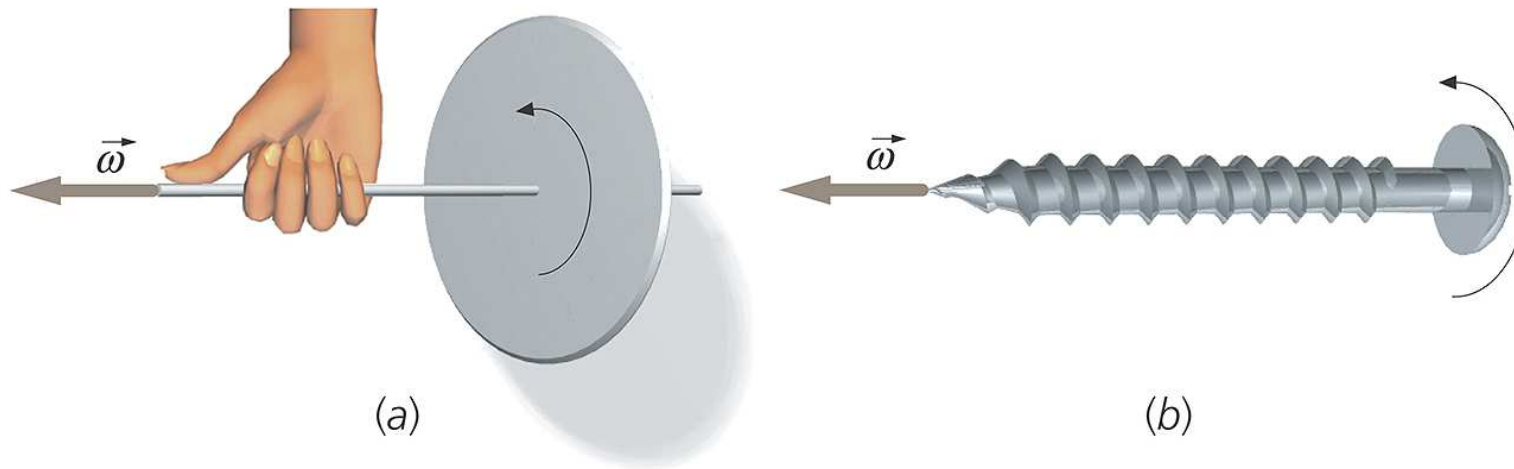
Halliday, Resnick, Walker, *Fundamentos da Física*

Momento angular

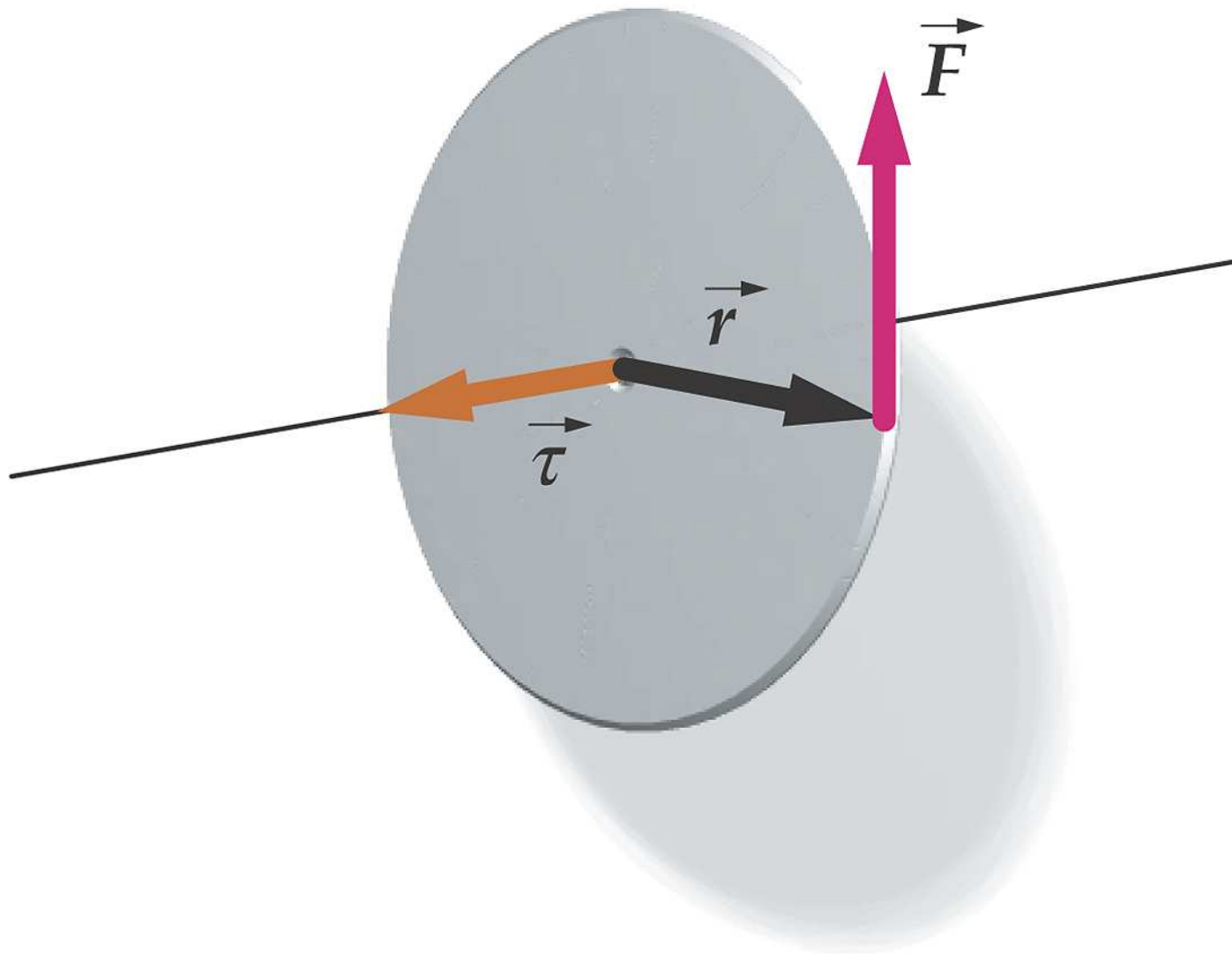


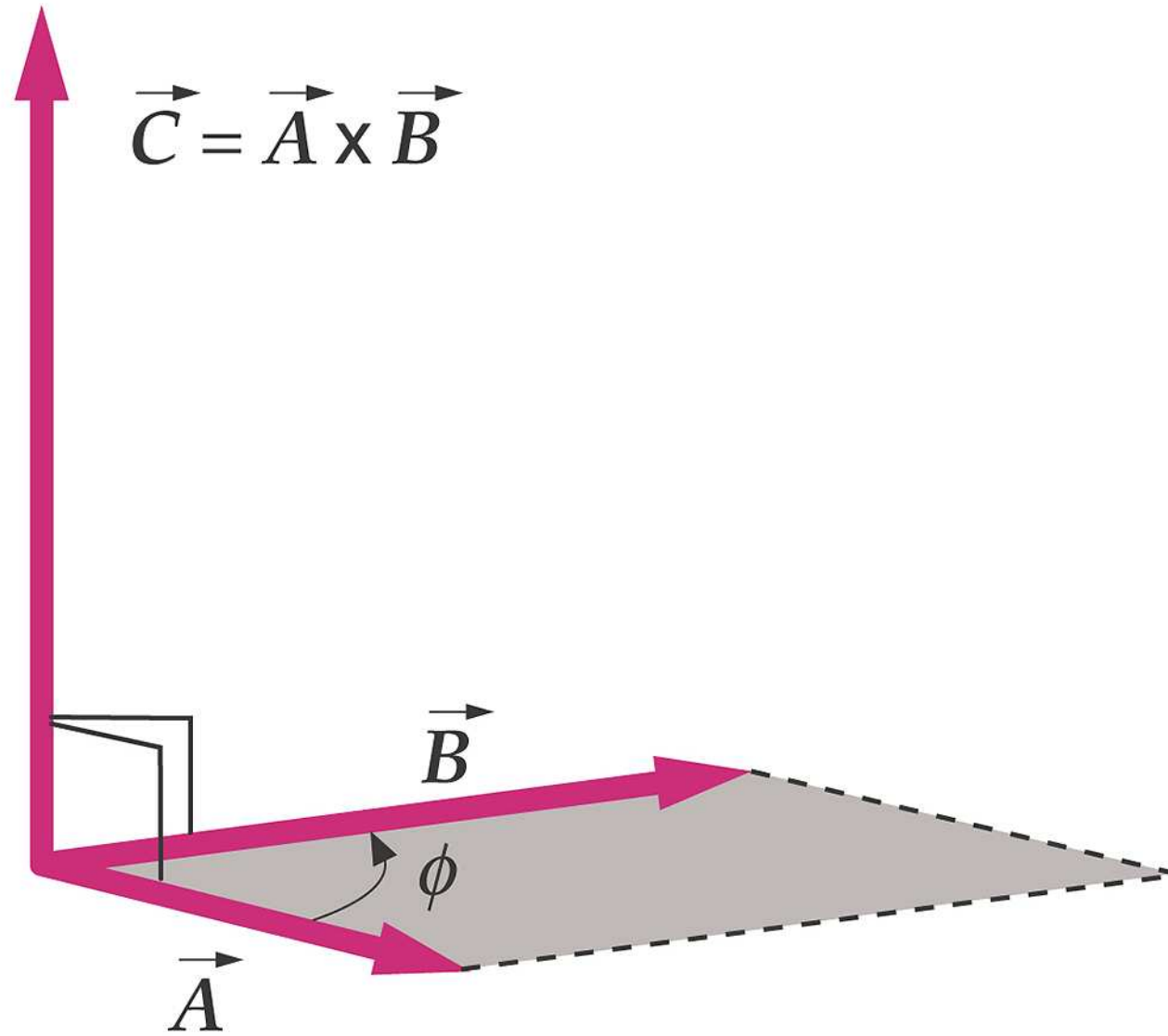
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Natureza vetorial da rotação

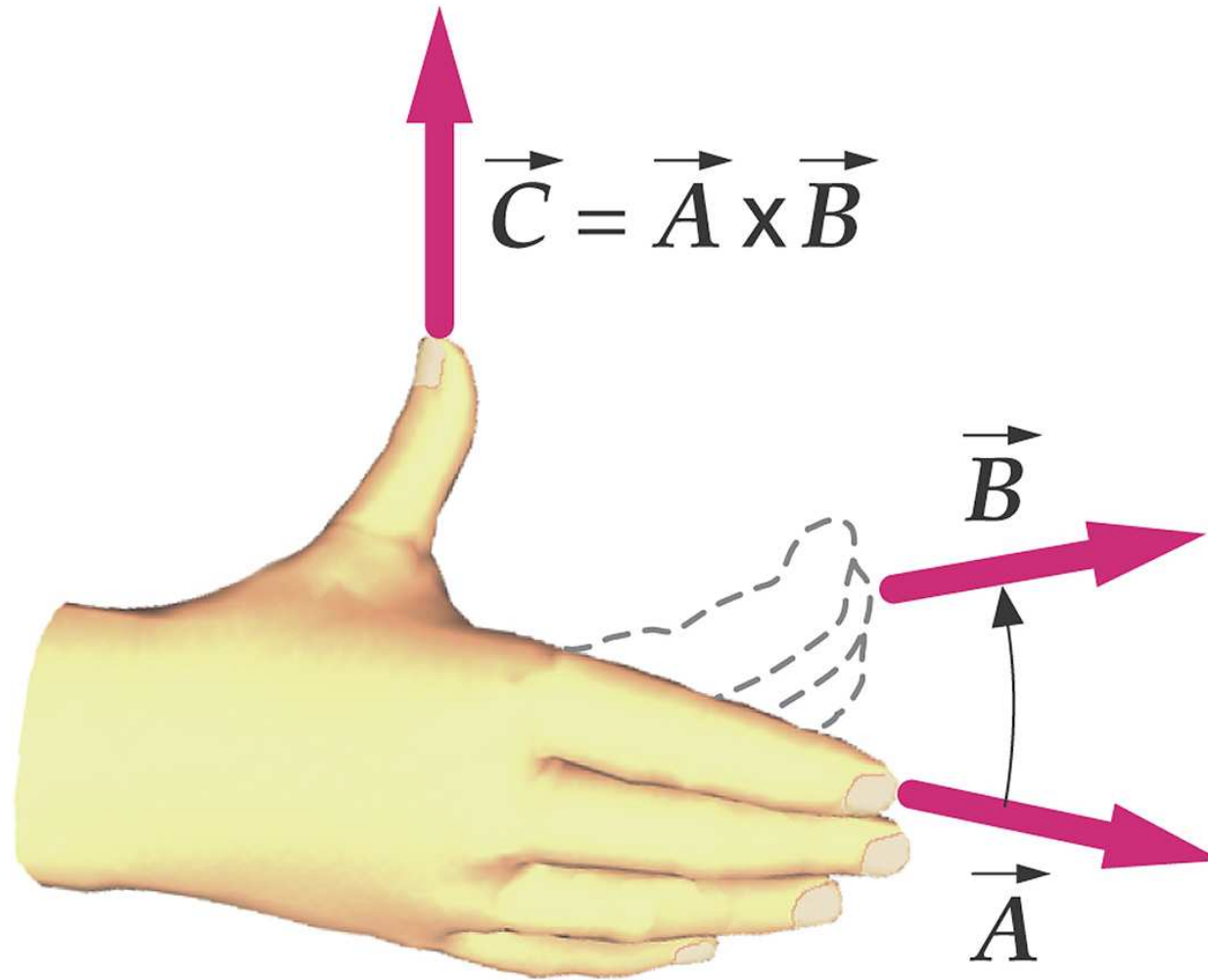


Torque: $\tau = r \times F$

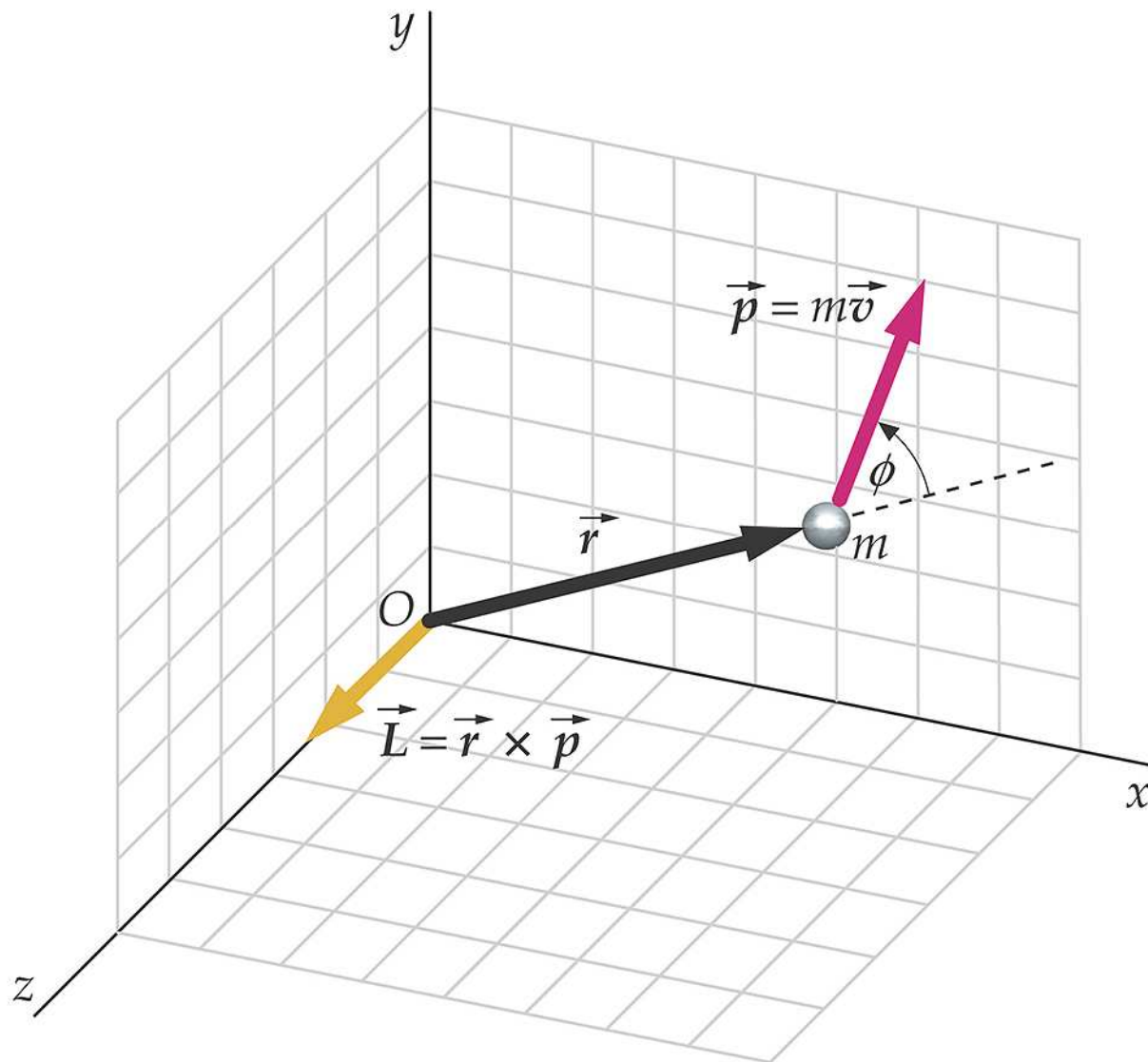




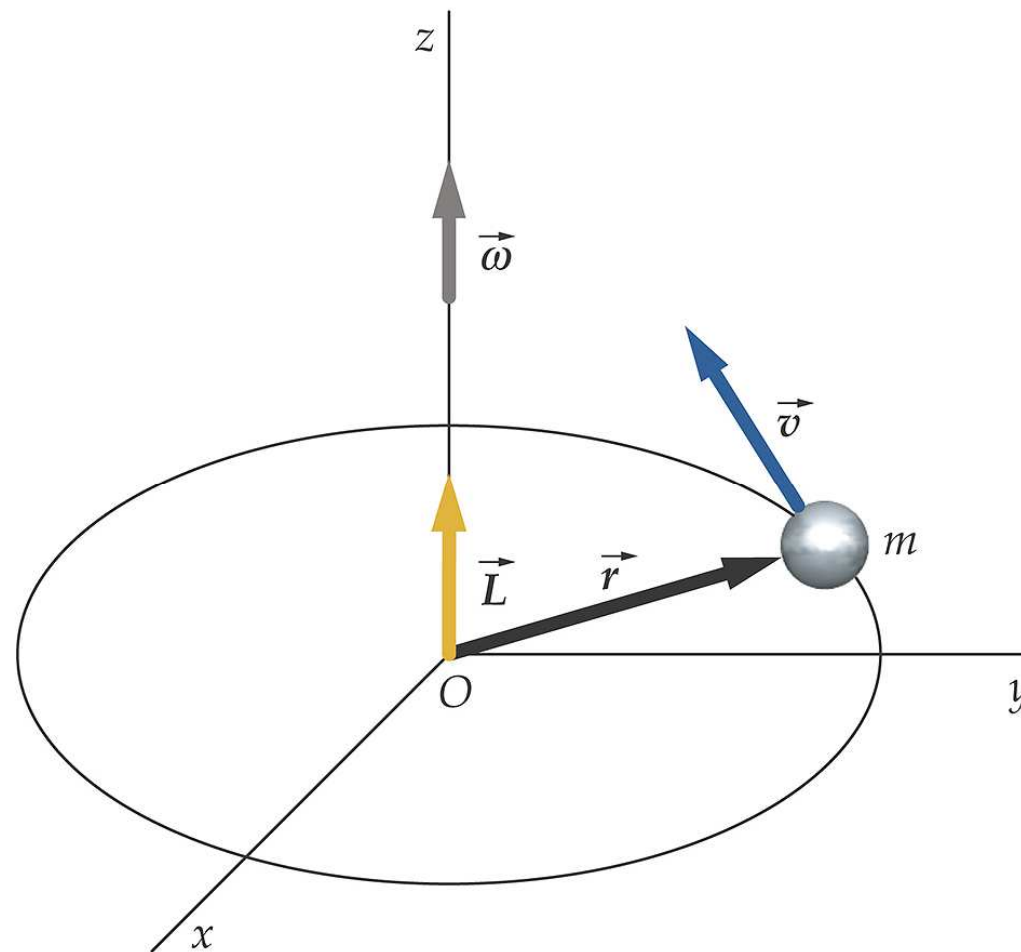
Regra da mão direita



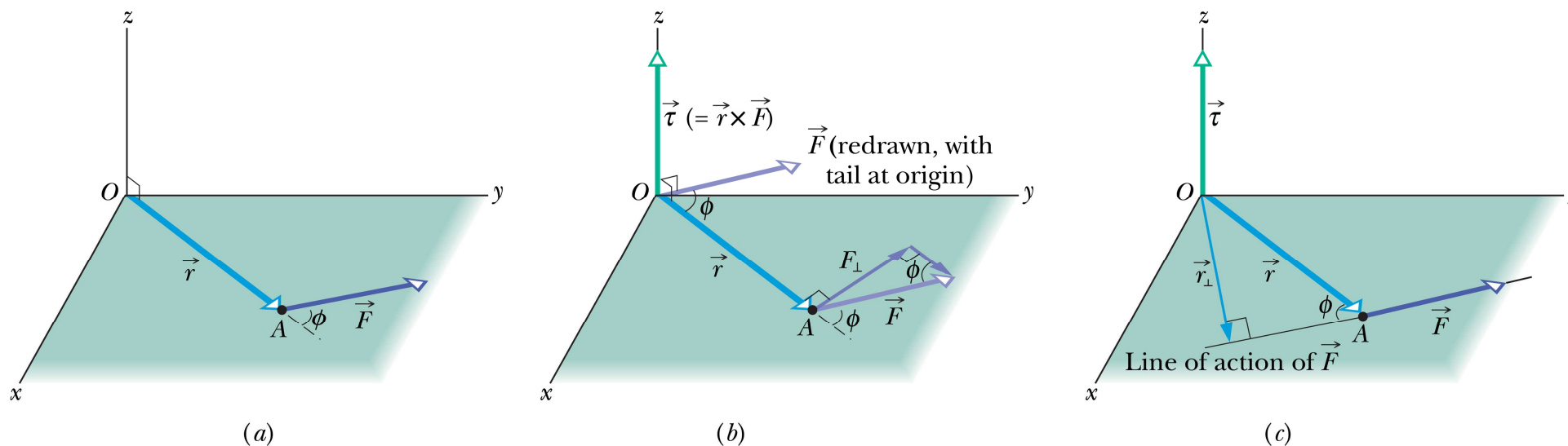
Torque e Momento angular



Partícula movendo-se num círculo no plano xy



Revisão de torque



- (a) Uma força age sobre uma partícula em A
- (b) Essa força produz um torque sobre a partícula em relação à origem
- (c) Regra da mão direita: o vetor torque aponta no sentido + Z

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Momento angular

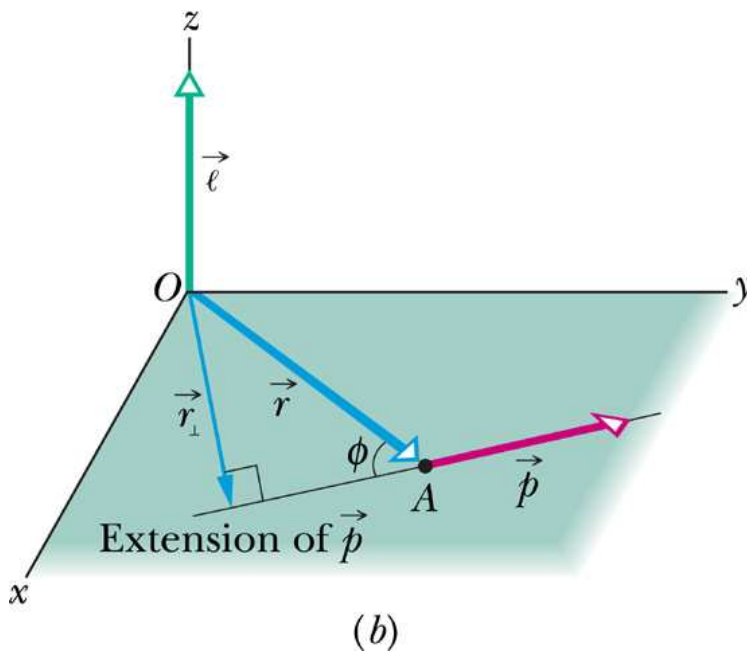
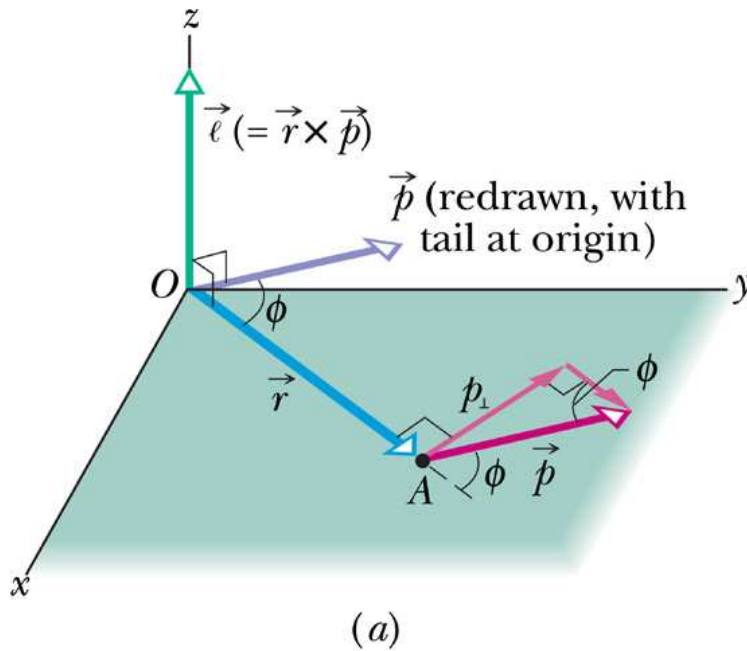
A partícula em A possui momento linear
 $P = mv$ com o vetor P no plano xy

A partícula possui momento angular
 $L = r \times P$ em relação à origem O

Regra da mão direita: o vetor momento angular aponta no sentido $+Z$

- O módulo de $L = rP_{\perp} = rmv_{\perp}$
- O módulo de L é dado também por
- $L = r_{\perp}P = r_{\perp}mv$

Halliday, Resnick, Walker
Fundamentos da Física



Exemplo: primeira roda gigante (1893)



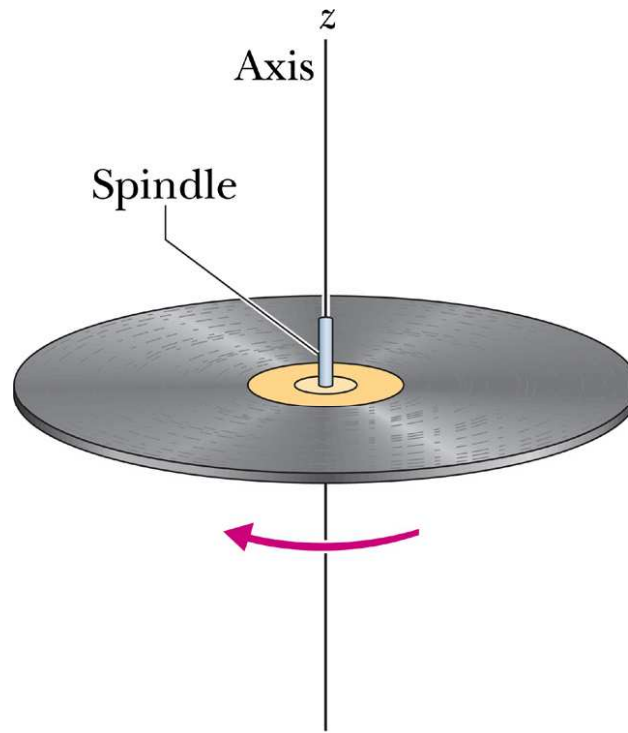
Halliday, Resnick, Walker
Fundamentos da Física



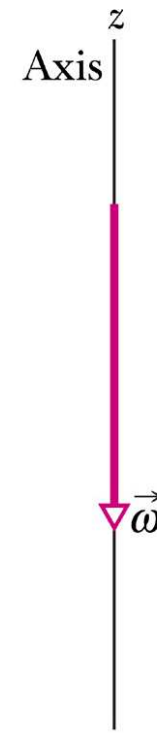
(a)



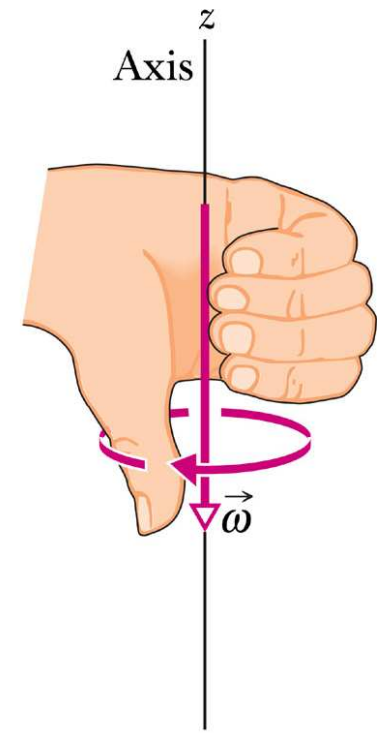
(b)



(a)



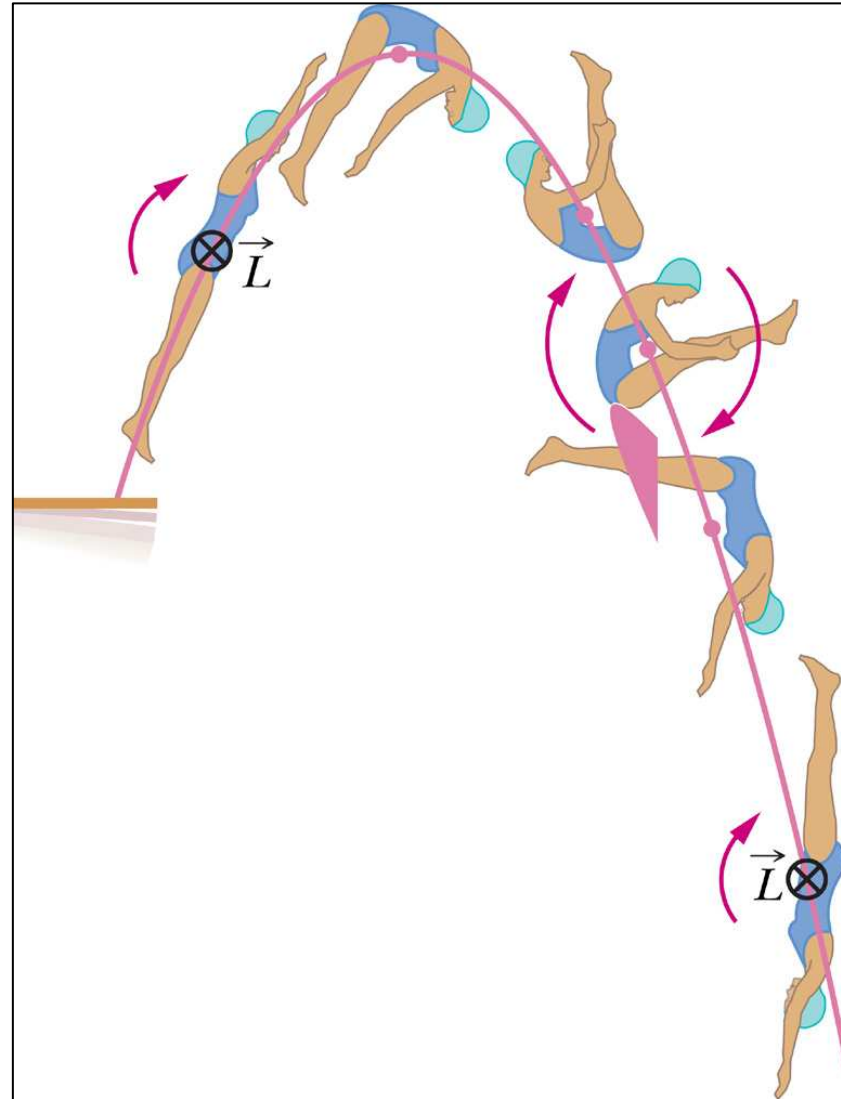
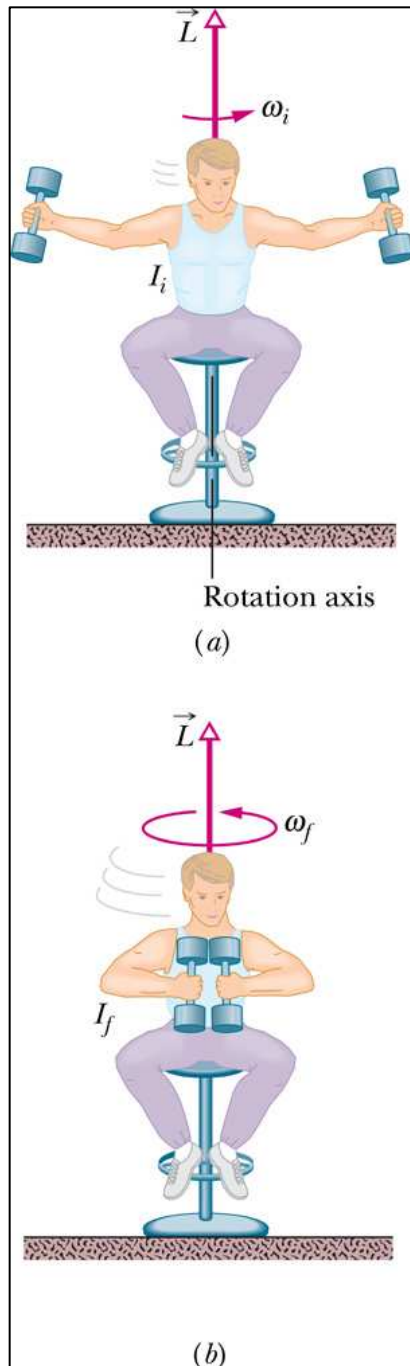
(b)



(c)

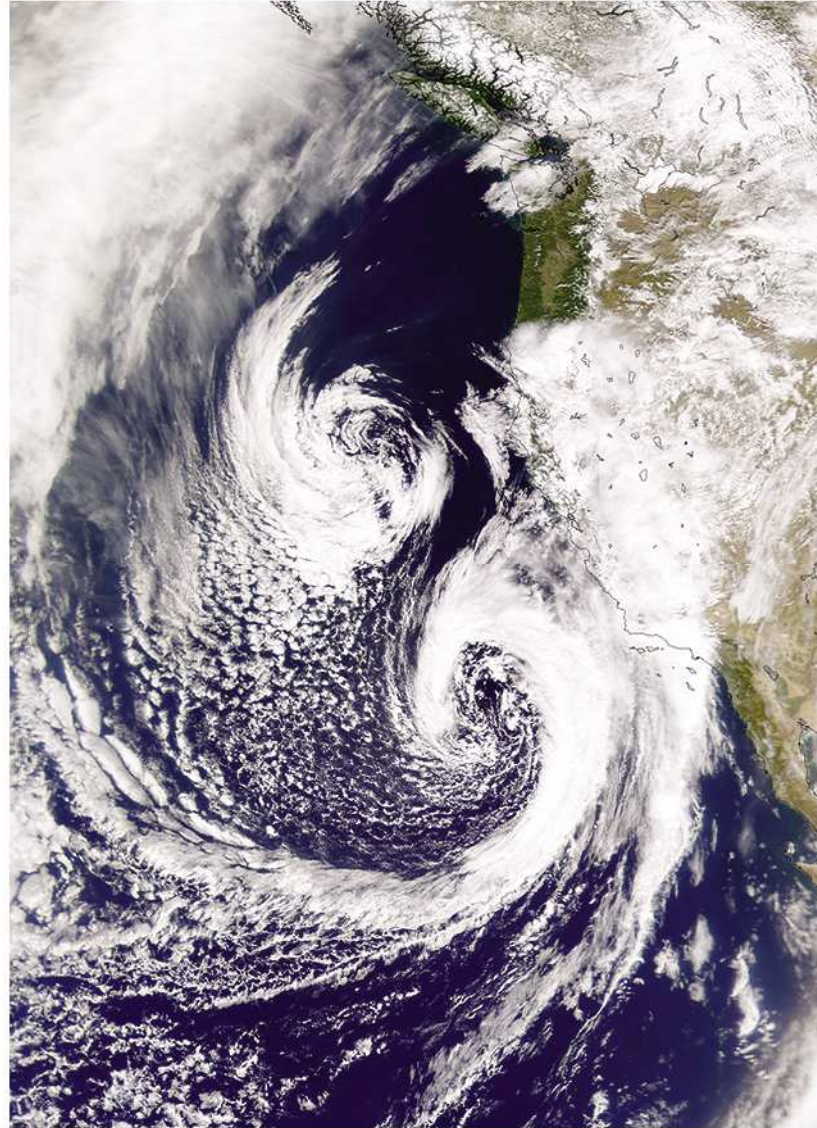
Patinadora Sasha Cohen em movimento de (a) traslação e (b) de rotação em torno de um eixo vertical
Ref: Halliday, Resnick, Walker, *Fundamentos da Física*

Conservação do Momento Angular



Halliday, Resnick, Walker, *Fundamentos da Física*





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