Mean Lives of the 2p and 3p Levels in Atomic Hydrogen*

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Radiation from hydrogen atoms in excited states produced by the beam-foil method was studied as a function of time using a vacuum ultraviolet monochromator. Measured values of the mean lives of the 2p and 3p levels in hydrogen are presented and compared with theory. The effects of cascades from levels of higher n were observed and have been included in the mean-life calculations.

I. INTRODUCTION

Attempts to measure atomic mean lives began with Wein1 but the amount of accurate experimental data on this subject is still surprisingly meager. The past few years have seen an increase in activity in this field, thanks in part to the development of the beam-foil excitation technique2 which is directly applicable to this research. Recently, mean lives of excited states formed by beam-foil excitation have been investigated in nitrogen,3 oxygen,4 hydrogen,5 neon,6 helium,7 and the iso-electric sequence of lithium-like ions.8

We have used the beam-foil excitation technique to measure the mean lives of the 2p and 3p levels of hydrogen. Cascades from higher levels into the 2p and 3p levels were observed and their effect on the rate of decay of the 2p and 3p levels has been determined. Hydrogen has been chosen for these initial experiments because its wave functions are exactly known and allow all observed effects to be amenable to exact calculation.

II. THEORY

The intensity of a spectral line (in photons/sec) resulting from a radiative electric dipole transition between the atomic levels at energies E and E' and described by the quantum numbers nlj and n'l'j' (E nlj > E n'l'j') is given by

$$I_{n'l'j'}^{nlj}(t) = A (nlj \rightarrow n'l'j') N_{nlj}(t),$$

(1)

where the Einstein coefficient, A (nlj \rightarrow n'l'j'), is the probability per unit time that the transition will occur and N_{nlj}(t) is the population of the upper level. If the contributions to the population of the upper level due to cascades from higher levels are negligible,

$$N_{nlj}(t) = N_{nlj}^0 e^{-\alpha_{nlj} t},$$

(2)

where N_{nlj}^0 is the population of the level |nlj⟩ at t=0 and \( \alpha_{nlj} = \sum_{n'l'j'} A (nlj \rightarrow n'l'j') \) is the reciprocal of the mean life of the level |nlj⟩. Therefore, we have

$$I_{n'l'j'}^{nlj}(t) = A (nlj \rightarrow n'l'j') N_{nlj}^0 e^{-\alpha_{nlj} t}.$$  

(3)

It has been shown that A (nlj \rightarrow n'l'j'), and therefore \( \alpha_{nlj} \), is independent of j, m_j and j', m_{j'}. Dropping the subscript j, we have

$$I_{n'l'}^{nl}(t) = A(nl \rightarrow n'l') N_{nl}^0 e^{-\alpha_{nl} t} = I_n e^{-\alpha_n t}.$$  

(4)

When cascades cannot be neglected, the expression for I_{n'l'}^{nl} becomes more complicated. If the levels n_{l'i}, i = 2 \cdots M, cascade into the level n_{l'i}, Eq. (2) must

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1 W. Wein, Ann. Physik 60, 39 (1919); 66, 16 (1921); 73, 32 (1924); 83, 1 (1927).
be replaced by

$$N_i(t) = N_i^0 e^{-\alpha_1 t} + \sum_{i=2}^{M} \beta_{1i} N_i^0 (e^{-\alpha_1 t} - e^{-\alpha_i t}),$$  \hspace{1cm} (5)$$

where the subscripts 1 and i represent n_{d1} and n_{di}, respectively, and

$$\beta_{1i} = \lambda (n_{d1} \rightarrow n_{di}) / (\alpha_1 - \alpha_i).$$  \hspace{1cm} (6)$$

We note that if the 1/\alpha_i are large compared to 1/\alpha_1, the effect of cascades is very small\textsuperscript{11} and

$$N_i(t) = N_i^0 e^{-\alpha_1 t},$$  \hspace{1cm} (7)$$

In any method of detection employing the counting of photons emitted during atomic transitions from \( |nl \rangle \) to \( |n'\ell' \rangle \), the intensity of the spectral line is related to the actual counting rate \( c(i) \) in the absence of background, by

$$c(i) = k I_{n'\ell} \cdot \alpha_i = k A (n l \rightarrow n'\ell) N_i(t),$$  \hspace{1cm} (8)$$

where \( k \) is a constant which depends on geometrical factors and detection efficiency.

In the beam-foil excitation technique, ions traveling with a well-defined velocity \( v \) in a well-defined beam are put into various excited levels on passing through the foil. The emergent particles, which are atoms or ions, generally radiate as they travel downstream from the foil, and the change in the intensity of radiation as a function of distance \( x \) from the foil may be used to determine the mean lives of the excited levels.

Equation (5) may be written

$$N_i(t) = \sum_{i=1}^{M} K_i e^{-\alpha_i t},$$

with

$$K_i = (N_i^0 - \sum_{i=2}^{M} \beta_{1i} N_i^0) / \alpha_i,$$

and

$$K_i = \beta_{1i} N_i^0, \hspace{1cm} \text{if} \ i \neq 1;$$

or, using \( t = x/v \),

$$N_i(x) = \sum_{i=1}^{M} K_i e^{-\alpha_i x/v},$$

where \( K_i \) is independent of \( x \). Therefore,

$$c(x) = k A (n l \rightarrow n'\ell) \sum_{i=1}^{M} K_i e^{-\alpha_i x/v}.$$  \hspace{1cm} (10)$$

The general form of \( c(x) \) is shown in Fig. 1.

Since a finite length of the radiating beam is observed by the detector, the total counting rate is given by the integral of \( c(x) dx \) over this length. If the center of the

\textsuperscript{11} This also depends on the reasonable assumption that the higher excited states are not as profusely populated as the lower states. Relative intensity measurements of \( \text{Ly}_\alpha \) and \( \text{Ly}_\beta \) indicate that the populations fall rapidly with increasing \( n \).
counts were normalized to equal amounts of charge collected in the Faraday cup in order to ensure that the decrease in intensity with distance was due to decay of the atoms in the beam from excited states and not to fluctuations in the beam current.

The carbon foil was translated upstream from the entrance slit of the spectrometer by means of a precision screw of 32 threads per inch. Moving the foil upstream increased the distance between the observed section of the beam and the point of initial excitation. This allowed us to measure the decay of intensity of the light from the beam as a function of distance from the exciter foil. The counting rates observed were determined to be the sum of three contributions:

1. the dark current of the photomultiplier,
2. the background light caused by collisions between beam particles and the residual gas atoms in the target chamber,
3. the signal from the beam caused by decays of atoms initially excited by the foil.

In the measurement of the Lyman $\alpha$ and Lyman $\beta$ light, the two sources of background were determined separately. The background due to the photomultiplier dark current was determined by closing the exit slit of the spectrometer and counting for 100 sec. This background was determined to an accuracy of $\pm 2\%$ and amounted to less than 10% of the total signal for Lyman $\alpha$ and less than 30% for Lyman $\beta$. The background from random excitations was measured by rotating the foil out of the beam, counting for 100 sec and subtracting the photomultiplier dark current. This source of background amounted to less than 4% of the total signal.

We emphasize that all measurements of intensity from both Lyman $\alpha$ and Lyman $\beta$ were made with the same foil. From other experiments, we have learned that the nature of the light emitted by this beam is a function of the thickness of the foil and perhaps of the nature of its preparation. It cannot be assumed that excitation equilibrium exists in foils with the thickness ($\sim 10 \mu g/cm^2$) we used.

### IV. RESULTS

#### A. Lyman $\alpha$ Measurements

The decay of Lyman $\alpha$ was observed over 8.27 cm. The beam velocity in this case was $7.92 \times 10^8$ cm/sec, which gave an observation time of $1.04 \times 10^{-8}$ sec or about 6.5 theoretical mean lives.

Figure 3 shows a semilog plot of the decrease in counting rate from Lyman $\alpha$ light as a function of distance downstream from the foil. This composite curve was decomposed using a version of the "Malik" curve-fitting program. These data were first fitted to functions of several exponentials but the fits were poor. In the case of the two exponentials, the value for the mean life of the second term was on the order of $2 \times 10^{-4}$ sec. The best fit to the data was obtained when the data were fitted to one exponential plus a constant. A constant is acceptable since the states which can cascade into the $2p$ level are long-lived and our data in the region beyond 6.5 cm are too poor to provide discrimination between mean lives of $\approx 2 \times 10^{-7}$ sec. The computer fit yields

$$1/\tau_{2p} = 1/\tau_{3p} = (1.600 \pm 0.004) \times 10^{-9} \text{ sec}.$$  

#### B. Lyman $\beta$ Measurements

The decay of intensity of Lyman $\beta$ was measured over a distance of 3.92 cm. The beam velocity was $4.74 \times 10^8$ cm/sec. Therefore, the decay of the $3p$ states was

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viewed for $5.9 \times 10^{-9}$ sec or about 1.1 theoretical mean lives.

Figure 4 shows the results of observations of the decrease in intensity of Lyman $\beta$ light. Cascade effects are again apparent but are too weak to warrant analysis by the use of "Malik." The mean life of the 3$p$ state was determined by a least-squares fit to the first six points shown in Fig. 3. The solid line represents this decay. The least-squares fit yields

$$1/\alpha_{3p} = \tau_{3p} = (5.58 \pm 0.13) \times 10^{-9} \text{ sec}.$$ 

The experimental results are shown in Table I, along with the theoretical values of $\tau_{2p}$ and $\tau_{3p}$ from Condon and Shortley.\textsuperscript{13}

We note that the velocities used in this experiment were determined to better than 0.5% and that there was essentially no error in the location of the position of the foil. In the data used to determine the mean life of the 3$p$ level the statistics on the experimental points are better than 2%. The 3% deviation of our measured value for $\tau_{3p}$ from the theoretical value given by Condon and Shortley cannot, therefore, be accounted for by any experimental error. This deviation is interpreted as being caused by the effect of cascades on the decrease of intensity of Ly$\beta$ radiation.

We have considered the effect of small extraneous electric fields on the mean lives of the 2$p$ and 3$p$ levels.

\textsuperscript{13} Reference 9, p. 136, Table 5.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
Spectral line & Transition & Experimental mean life (10$^{-8}$ sec) & Theoretical mean life (10$^{-8}$ sec) \\
\hline
1215.6 Å Ly$\alpha$ & 1s-2p & 1.600±0.004 & 1.60 \\
1025.7 Å Ly$\beta$ & 1s-3p & 5.58 ±0.13 & 5.40 \\
\hline
\end{tabular}
\caption{Summary of results of Lyman $\alpha$ and Lyman $\beta$ measurements.}
\end{table}

These fields might arise from accumulations of charge on nearby surfaces in the target chamber or from electrons or ions in the beam itself. Our calculations, which follow the treatment of the Stark effect in hydrogen by Bethe and Salpeter\textsuperscript{14} show that although mixing of these states occurs for very small fields, stray fields would have to be greater than 30 V/cm before our measurements would be affected. At 30 V/cm, the state that reduces to pure 2$p$ for zero electric field has a mean life of $1.606 \times 10^{-9}$ sec and the mean life of the state that reduces to pure 2s for zero field is $3.75 \times 10^{-7}$ sec. Specific studies\textsuperscript{15} of the effect of electric fields on the intensity of light radiated downstream from the foil confirm the fact that any fields present in these experiments are less than 5 V/cm.

V. SUMMARY

Experimental mean lives for the 2$p$ and 3$p$ levels in hydrogen have been determined. The measured value for $\tau_{3p}$ agrees with theory and the 3% deviation of the measured value of $\tau_{3p}$ from the theoretical value is attributed to cascade effects.

The general technique outlined here may be extended to yield data on the relative initial populations of decaying states and those states which cascade into them. Such work is in progress.

The accuracy with which we were able to determine these mean lives demonstrates the elegance of the thin-foil excitation technique when applied to measurements of mean lives.

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