Introduction to the Standard Model

New Horizons in Lattice Field Theory
IIP Natal, March 2013

Rogerio Rosenfeld
IFT-UNESP

Some references

Books:

- Quigg: Gauge Theories of Strong, Weak, and Electromagnetic Interactions
- Halzen and Martin: Quarks and Leptons
- Peskin and Schroeder: An Introduction to Quantum Field Theory
- Donoghue, Golowich and Holstein: Dynamics of the Standard Model
- Barger and Phillips: Collider Physics

Lectures and review articles:

- Rosenfeld: http://www.sbfisica.org.br/~evjaspc/xvi/
- Hollik: arXiv:1012.3883
- Buchmuller and Ludeling: arXiv:hep-ph/0609174
- Rosner's Resource Letter: arXiv:hep-ph/0206176
- Quigg: arXiv:0905.3187
- Altarelli: arXiv:1303.2842

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School on Particle Physics in the LHC Era

April 1 - 12, 2013
Campus of IFT - UNESP - São Paulo, Brazil

With the recent results from the 2011-2012 LHC runs, including the discovery of a new Higgs-like particle, it is timely to review what was learned and what can be learned in the near future about the High Energy Frontier. The purposes of the School are to provide students with an in-depth understanding of the Standard Model, to familiarize them with aspects of collider physics, to explore what may lay beyond the Standard Model, and to give them the necessary tools to test different models with the newest data.

Lecturers

Chris Quigg (Fermilab)
Standard Model

Joseph Lykken (Fermilab)
Collider Physics

Zackaria Chacko (University of Maryland) and Marcela Carena (Fermilab)
Beyond SM @ LHC - Weakly coupled extensions

Eduardo Pontón (ICTP-SAIFR and IFT-UNESP)
Beyond SM @ LHC - Strongly coupled extensions

Alexander Belyaev (University of Southampton and Rutherford Lab)
Numerical Tools for Physics @ LHC

Organizers:
Marcela Carena (Fermilab), **Giacomo Cacciapaglia** (Lyon), **Aldo Deandrea** (Lyon), **Rogerio Rosenfeld** (IFT-UNESP)

The online application form and more information can be found at:

www.ictp-saifr.org/LHC

The application deadline is **February 8, 2013.**

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Standard model of the electroweak interactions: a success history told in Nobel Prizes

- 1906: Electron (J. J. Thomson, 1897)
- 33: QED (Dirac)
- 36: Positron (Anderson, 1932)
- 57: Parity violation (Lee and Yang, 56)
- 65: QED (Feynman, Schwinger and Tomonaga)
- 69: Eightfold way (Gell-Mann, 63)
- 74: Charm (Richter and Ting, 74)
- 79: SM (Glashow, Weinberg and Salam, 67-68)
- 80: CP violation (Cronin and Fitch, 64)
- 84: W&Z (Rubbia and Van der Meer, 83)
- 88: b quark (Lederman, Schwartz and Steinberger, 77)
- 90: Quarks (Friedman, Kendall and Taylor, 67-73)
- 95: Neutrinos (Reines, 56); Tau (Perl, 77)
- 99: Renormalization (Veltman and 't Hooft, 71)
- 02: Neutrinos from the sky (Davis and Koshiba)
- 04: Asymptotic freedom (Gross, Politzer and Wilczek)
- 08: CP violation (KM) and SSB (Nambu)

The Nobel Prize in Physics 2008

Y. Nambu

"for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics"

M. Kobayashi

T. Maskawa

"for the discovery of the origin of the broken symmetry which predicts the existence of at least three families of quarks in nature"

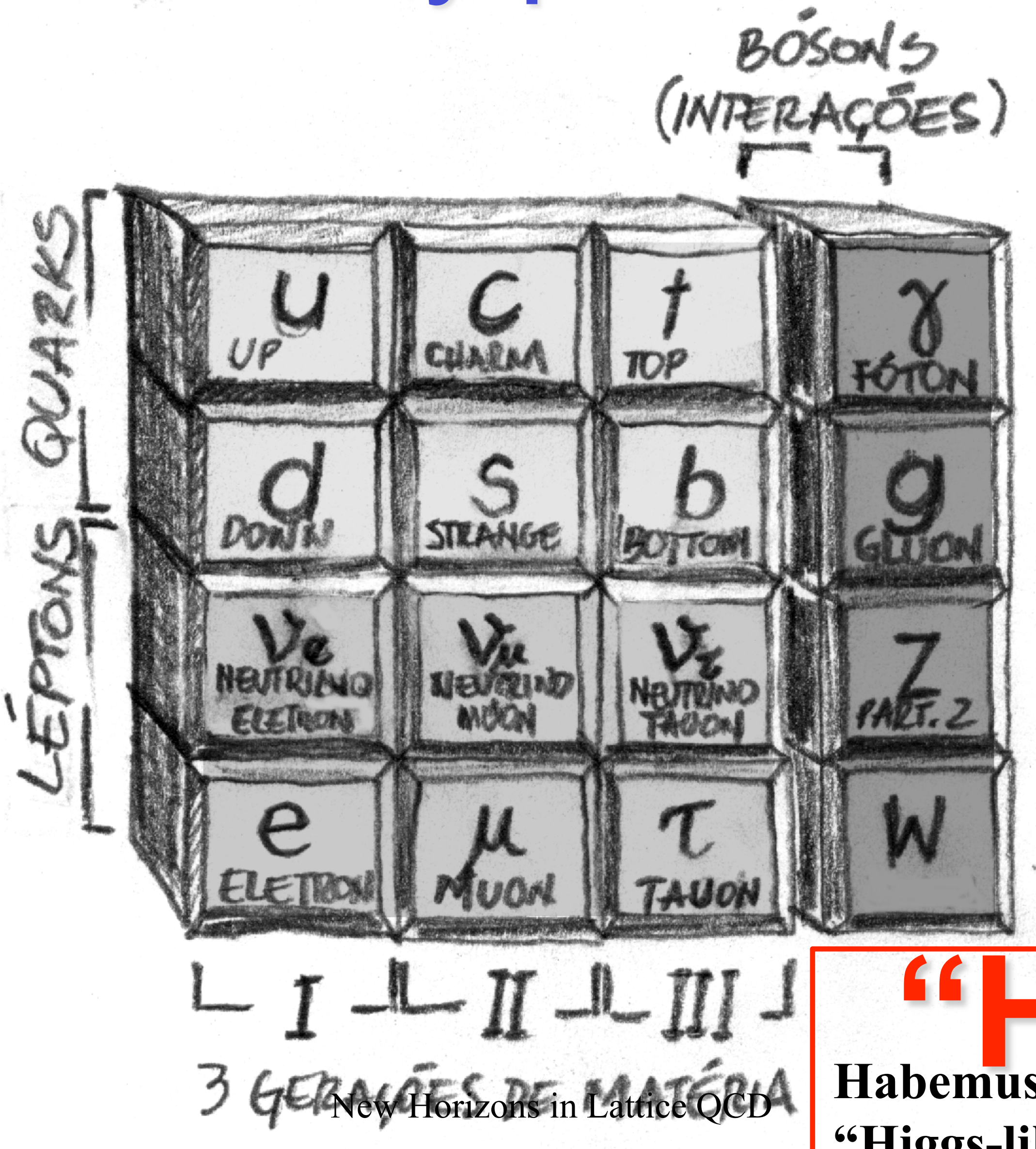
20xx: Higgs ??

Standard model:
$SU(3)_c x SU(2)_L x U(1)_Y$ gauge theory

Theoretical basis: $\left\{ \begin{array}{l} \text{Renormalizable quantum field theory} \\ + \\ \text{Gauge symmetries} \\ + \\ \text{Spontaneous symmetry breaking} \end{array} \right.$

Higgs mechanism

Elementary particles today



"H"

Habemus Higgs? July 2012
"Higgs-like" boson

Conventions

Natural units: $\hbar = c = 1$

$$x = (x^\mu) = (x^0, \vec{x}), \quad x^0 = t\,,$$

$$k = (k^\mu) = (k^0, \vec{k}\,), \quad k^0 = E = \omega_k = \sqrt{\vec{k}^{\,2} + m^2}$$

$$\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$$

$$a \cdot b = a_\mu b^\mu = a^0 b^0 - \vec{a} \cdot \vec{b}$$

$$\partial_\mu \partial^\mu = \frac{\partial^2}{\partial t^2} - \nabla^2$$

Classical Field Theory

Local field theory is defined by a lagrangian density (functional of fields and derivatives):

$$\mathcal{L} \equiv \mathcal{L}\left[\phi(x,t), \partial_\mu \phi(x,t)\right]$$

Field equations (Euler-Lagrange equations) are obtained by extremizing the action $S = \int d^4x \mathcal{L}$:

$$\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right] = 0$$

Example: free real scalar field $\phi(x,t)$ with mass m

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{m^2}{2}\phi^2$$

Exercise: derive the field equation for a free real scalar field(Klein-Gordon equation):

$$\left(\partial_\mu \partial^\mu + m^2\right)\phi = 0$$

Particular solution of KG equation

$$\phi^{part.}(x,t) = e^{\pm i(kx-\omega_k t)} \qquad \omega_k^2 = k^2 + m^2$$

General solution of KG equation is a superposition (Fourier expansion)

$$\phi(x,t) = \int \frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k}\left[a_k e^{i(kx-\omega_k t)} + a_k^* e^{-i(kx-\omega_k t)}\right]$$

Quantum Field Theory

The Lagrangian encodes the particle content of the theory via fields and their interactions. Local, Lorentz-invariant, hermitian Lagrangian lead to an unitary S-matrix.

The interactions are usually determined by symmetry or invariance principles, such as gauge symmetry.

The standard model Lagrangian contain three types of fields:

- Fermion fields describing “matter”
- Vector fields describing the interactions
- Scalar fields describing the electroweak breaking sector

Quantum Field Theory

Promote fields to operators and impose canonical equal time commutation relations:

$$\left[\hat{\phi}(x,t),\hat{\Pi}(x',t)\right]=i\delta^3(x-x'),\quad \left[\hat{\Pi}(x,t),\hat{\Pi}(x',t)\right]=0,\quad \left[\hat{\phi}(x,t),\hat{\phi}(x',t)\right]=0$$

Canonical conjugate to field $\Pi=\delta\mathcal{L}/\delta\dot{\phi}=\dot{\phi}$

This procedure promotes the Fourier coefficients to operators

$$\hat{\phi}(x,t)=\int\frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k}\left[\hat{a}_k e^{i(kx-\omega_k t)}+\hat{a}_k^\dagger e^{-i(kx-\omega_k t)}\right]$$

$$\left[\hat{a}_k,\hat{a}_{k'}^\dagger\right]=(2\pi)^3 2\omega_k\delta^3(k-k'),\quad [\hat{a}_k,\hat{a}_{k'}]=0,\quad \left[\hat{a}_k^\dagger,\hat{a}_{k'}^\dagger\right]=0$$

Field: infinite set of harmonic oscillators labelled by k.

Exercise: show that the classical hamiltonian

$$H = \frac{1}{2}\int d^3x \left[\dot{\phi}^2 + (\nabla\phi)^2 + m^2\phi^2\right]$$

is promoted to a hamiltonian operator

$$\hat{H} = \int \frac{d^3k}{(2\pi)^3}\frac{1}{2k_0}k_0\left[\hat{a}_k^\dagger \hat{a}_k + \frac{1}{2}\right]$$

$$k_0 = +\sqrt{k^2+m^2}$$

vacuum energy (harmless?)

Particle interpretation:

$\hat{a}_k^\dagger \hat{a}_k$ counts number of particles with momentum k and energy k_0

Vacuum state: $\hat{a}_k|0\rangle = 0$

1-particle state: $|k\rangle = \hat{a}_k^\dagger|0\rangle$

•
•
•

Also:

$\hat{a}_k|k'\rangle = 2k^0\delta^3(\vec{k} - \vec{k'})|0\rangle$

Comment:

particles can be created and destroyed in QFT – particle number is not well defined.

Intuitively this happens due to uncertainty principle when:

$$\Delta p > m$$

Procedure can be carried out for other fields:

Complex scalar fields

$$\hat{\phi}(x,t) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k} \left[\hat{a}_k e^{i(kx-\omega_k t)} + \hat{b}_k^\dagger e^{-i(kx-\omega_k t)} \right]$$

Independent coefficients
(2 degrees of freedom, particle-antiparticle)

Fermion fields (Dirac equation)

$$\hat{\psi}(x) = \int \frac{d^3k}{(2\pi)^3} \frac{m}{k_0} \sum_{\alpha=1,2} \left[\hat{b}_\alpha(k) u^\alpha(k) e^{-ikx} + \hat{d}_\alpha^\dagger(k) v^\alpha(k) e^{ikx} \right]$$

$u^{1,2}(k)$ and $v^{1,2}(k)$ are the positive and negative energy spinors, which are solutions to the equations $(\gamma \cdot k - m)u(p) = 0$ and $(\gamma \cdot k + m)v(p) = 0$, where γ_μ are the usual Dirac matrices (4 degrees of freedom)

Vector fields

$$\hat{A}_\mu(x) = \int \frac{d^3k}{(2\pi)^3 2k_0} \sum_{\lambda=0,1,2,3} \left[\varepsilon_\mu^{(\lambda)} \hat{a}^{(\lambda)}(k) e^{-ikx} + \varepsilon_\mu^{(\lambda)*} \hat{a}^{(\lambda)\dagger}(k) e^{ikx}\right],$$

where $\varepsilon_\mu^{(\lambda)}$ are the four polarization 4-vectors.

Orthonormal basis: $\varepsilon_\mu(\vec{k})^{(\lambda)} \cdot \varepsilon_\mu(\vec{k})^{(\lambda')*} = g^{\lambda\lambda'}$

Lorentz condition (follows from eq. of motion for massive vector field):

$$k \cdot \varepsilon_\mu(\vec{k})^{(\lambda)} = 0$$

Three polarization vectors for a massive field.
Two polarization vectors for a massless field (no longitudinal degree of freedom).

Interactions and the S-matrix

There is more to life than free fields:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_I$$

Evolution operator in the interaction picture

Evolution of a state:

$$|t\rangle = U(t, t_0)|t_0\rangle$$

S(cattering) matrix (unitary):

$$S = \lim_{\substack{t_0 \to -\infty \\ t \to +\infty}} U(t, t_0)$$

S-matrix is computed perturbatively from interactions:

$$S = 1+i\int d^4x\ \mathcal{L}_I(x)+\frac{(i)^2}{2!}\int d^4x_1\int d^4x_2\ T\left[\mathcal{L}_I(x_1)\mathcal{L}_I(x_2)\right]+\cdots = T\left[e^{i\int d^4x\ \mathcal{L}_I(x)}\right]$$

T: time ordered product

Transition matrix $\quad S = \mathbf{1} + \mathcal{T}$

Scattering amplitude is defined as:

$$\mathcal{T} = \underbrace{(2\pi)^4\delta^4\left(\sum_i p_i\right)}_{\text{Conservation of energy and momentum}}\ \underbrace{\mathcal{M}}_{\text{scattering amplitude}}$$

Example: 4 different real scalar fields with interaction

$$\mathcal{L}_I = -g\phi_A\phi_B\phi_C\phi_D$$

Want to compute the transition amplitude of a boson A with momentum p_A scattering on a boson B with momentum p_B producing bosons C and D with momenta p_C and p_D: $A + B \to C + D$.

$$\mathcal{T}_{AB\to CD} = \langle p_C, p_D|S|p_A, p_B\rangle$$

Lowest order:

$$\mathcal{T}^{(1)}_{AB\to CD} = \int d^4x \langle p_C, p_D| - ig\hat{\phi}_A(x)\hat{\phi}_B(x)\hat{\phi}_C(x)\hat{\phi}_D(x)|p_A, p_B\rangle$$

Exercise: show that

$$\mathcal{T}^{(1)}_{AB\to CD} = -ig(2\pi)^4\delta^4(p_C + p_D - p_A - p_B)$$

energy-momentum conservation

Finally, the scattering amplitude is defined as:

$$\mathcal{T}_{AB\to CD} = (2\pi)^4\delta^4(p_C + p_D - p_A - p_B)\mathcal{M}_{AB\to CD}$$

conservation of energy and momentum

scattering amplitude

Lowest order:

$$\mathcal{M}_{AB\to CD} = ig$$

Scattering amplitudes are used to compute cross sections and decay rates of particles which are observed in experiments.

Higher contribution (3rd order in g):

$$\mathcal{T}^{(3)} = \int d^4x_1 \int d^4x_2 \int d^4x_3 \langle p_C, p_D| \frac{(-ig)^3}{3!} \quad (1)$$
$$T\,[\,\hat{\phi}_A(x_1)\hat{\phi}_B(x_1)\hat{\phi}_C(x_1)\hat{\phi}_D(x_1)\hat{\phi}_A(x_2)\hat{\phi}_B(x_2)\hat{\phi}_C(x_2)\hat{\phi}_D(x_2)$$
$$\hat{\phi}_A(x_3)\hat{\phi}_B(x_3)\hat{\phi}_C(x_3)\hat{\phi}_D(x_3)\,]\,|p_A, p_B\rangle$$

Usual Field Theory methods to compute:

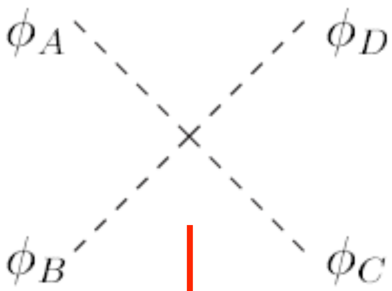
- Wick’s theorem (time ordered and normal ordered products)
- Sum over all possible “contractions” of 2 field operators

Old-fashioned way!

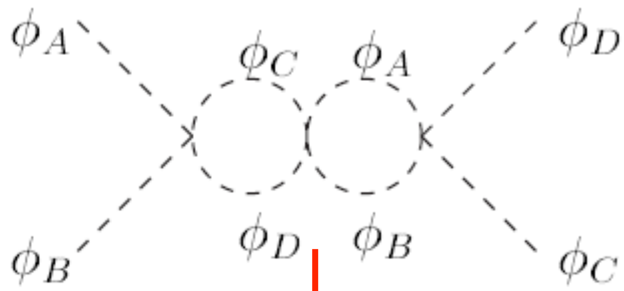
Quantum Field Theory for the “masses”: Feynman rules

In order to compute scattering amplitudes one should:

- Draw all possible diagrams at a given order in perturbation theory;
- For each vertex associate a factor $-ig$;
- For each propagator, associate a factor $\frac{i}{p^2-m^2+i\epsilon}$;
- Impose 4-momentum conservation at each vertex;
- Integrate over each undetermined momentum $\int \frac{d^4p}{(2\pi)^4}$



tree-level



2-loop level (why not 1-loop?)

Loop diagrams are usually infinite. The theory diverges at higher orders. A theory is said renormalizable when all the divergences can be absorbed in the definition of the parameters (masses, coupling constants) of the theory.

As a consequence of the renormalization procedure, masses and coupling are not constants but depend on an energy scale. They are called **running parameters**. New energy scales are introduced in the theory via running!

Steps to construct and test a model:

- Postulate a set of elementary particles
- Construct a Lagrangian with interactions (symmetries)
- Derive Feynman rules
- Calculate processes as precisely as possible
- Measure parameters of model
- Make predictions for new processes
- Compare with experiments
- If agreement is found, you have a good model
- If not, back to 1st step...

Only works in weak coupling regimes!

Can not use perturbation theory to study strongly coupled models: new methods are necessary

- Lattice computations
- Schwinger-Dyson equations
- Chiral perturbation theory
(low energy effective lagrangians)
- QCD sum rules
- 1/N expansions

....

Complication in strongly coupled theories:

observed particles do not corresponds to fields in the original lagrangian!

What is a particle?

In these lectures we will deal only with weak coupling (easy part?):
Electroweak theory, high energy QCD
Collider physics

What Is An Elementary Particle?

by STEVEN WEINBERG

WHEN A STRANGER, hearing that I am a physicist, asks me in what area of physics I work, I generally reply that I work on the theory of elementary particles. Giving this answer always makes me nervous. Suppose that the stranger should ask, "What is an elementary particle?" I would have to admit that no one really knows.

Let me declare first of all that there is no difficulty in saying what is meant by a *particle*. A particle is simply a physical system that has no continuous degrees of freedom except for its total momentum. For instance, we can give a complete description of an electron by specifying its momentum, as well as its spin around any given axis, a quantity that in quantum mechanics is discrete rather than continuous. On the other hand, a system consisting of a free electron and a free proton is not a particle, because to describe it one has to specify the momenta of both the electron and the proton—not just their sum. But a bound state of an electron and a proton, such as a hydrogen atom in its state of lowest energy, is a particle. Everyone would agree that a hydrogen atom is not an *elementary* particle, but it is not always so easy to make this distinction, or even to say what it means.

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Basic question: what are the particles and interactions of Nature?

It took more than 50 years to find out that the symmetries that ultimately determine the interactions fall in the class of local gauge symmetries.

Quantum electrodynamics (QED) is a paradigm for local gauge symmetries and we will study it next.

Gauge symmetries

Dirac lagrangian for a free fermion of mass m:

$$\mathcal{L}_{\mathrm{free}} = \overline{\psi}(i\,\partial\!\!\!/ - m)\psi$$

Lagrangian is invariant under a global phase (gauge) transformation:

$$\psi(x) \to \psi'(x) \equiv \exp[i\alpha]\psi(x)$$

Promote global to local phase transformation:

$$\psi(x) \to \psi'(x) \equiv \exp[i\alpha(x)]\psi(x)$$

One must modify the free lagrangian in order to restore local phase (or gauge) symmetry by introducing a vector field through a so-called covariant derivative that will substitute the normal derivative:

coupling constant

$$\partial_\mu \to D_\mu \equiv \partial_\mu + ieA_\mu$$

gauge vector field

Local gauge invariant lagrangian:

$$\mathcal{L} = \overline{\psi}(i \not{D} - m)\psi = \underbrace{\overline{\psi}(i \not{\partial} - m)\psi}_{\text{free lagrangian}} - \underbrace{e\overline{\psi} \not{A}(x)\psi}_{\text{interaction}}$$

Both vector and fermion fields change under a local gauge transformation:

$$\psi(x) \to \psi'(x) \equiv \exp[i\alpha(x)]\psi(x)$$

$$A_\mu(x) \to A'_\mu(x) = A_\mu(x) - \frac{1}{e}\partial_\mu \alpha(x)$$

Exercise: check that $D_\mu \psi \to e^{i\alpha(x)} D_\mu \psi$

In order to complete the lagrangian one must introduce a kinetic term for the gauge field:

$$\mathcal{L} = \underbrace{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\text{kinetic term for gauge field (equations of motion = Maxwel's equations)}} + \overline{\psi}(i\not{\partial} - m)\psi - e\overline{\psi}\not{A}\psi$$

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$$

A mass term for the photon violates gauge symmetry:

$$m_\gamma A_\mu A^\mu$$

Symmetry → Dynamics

One can derive electrodynamics requiring local gauge symmetry! It implies in a massless photon and a conserved electric charge (Noether's theorem).

In this simple case the symmetry form a so-called U(1) group, whose elements are unimodular complex numbers (phases) that commute with each other (abelian group).

Million dollar question:
What symmetries determine the dynamics of the strong and weak interactions?

Non-abelian symmetries

Yang and Mills (1954) tried to describe the dynamics of strong interactions between protons and neutrons grouping them into a single entity called a nucleon:

$$\Psi(x) = \begin{pmatrix} p(x) \\ n(x) \end{pmatrix}$$

"Isospin" doublet

Motivation: protons and neutrons have same properties under strong interactions

Free lagrangian

$$\mathcal{L}_{\mathrm{free}} = \bar{\Psi}(i \not\partial - \underbrace{m}_{\text{nucleon mass}})\Psi$$

is invariant under global rotations in this inner isospin space:

$$\Psi \to \Psi' = \exp(i\vec{\alpha} \cdot \underbrace{\vec{\sigma}}_{\text{Pauli matrices}}/2)\Psi(x)$$

Rotations are determined by three numbers $\vec{\alpha}$

Dynamics is determined by promoting global symmetry to a local symmetry. Let us study the general case of an n-component fermion multiplet:

$$\Psi(x) = \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \\ \vdots \\ \psi_n(x) \end{pmatrix}$$

with a transformation

$$\Psi \to \Psi' = \mathbf{U}(\alpha_i)\Psi = \exp(i\alpha_i \mathbf{T}^i)\Psi(x)$$

The nxn complex matrices U must be unitary (for global invariance) and we will also require them to have determinant 1. This fixes the symmetry group to be SU(N) which has n^2 -1 elements.

The n^2 -1 nxn complex hermitian and traceless matrices T are called the generators of the group and obey the commutation relations

$$[\mathbf{T}_i, \mathbf{T}_j] = i\underbrace{c_{ijk}}_{\text{structure constants of the group}}\mathbf{T}_k$$

Normalization:

$$\mathrm{Tr}\,[\mathbf{T}_i\mathbf{T}_j] = \delta_{ij}/2$$

Example for isospin (SU(2)): $\mathbf{T}_i = \sigma_i, \quad i = 1,3 \quad c_{ijk} = \epsilon_{ijk}$

Non-abelian gauge fields

Introduce an nxn complex matrix valued vector field

$$\mathbf{A}_\mu(x) \equiv A^i_\mu(x)\mathbf{T}_i$$

and the corresponding field strength tensor

$$\mathbf{F}_{\mu\nu} = F^i_{\mu\nu}\mathbf{T}_i$$

$$\mathbf{F}_{\mu\nu} = \partial_\mu \mathbf{A}_\nu - \partial_\nu \mathbf{A}_\nu - ig[\mathbf{A}_\mu, \mathbf{A}_\nu]$$

$$F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g c_{ijk} A^j_\mu A^k_\nu$$

Again one can obtain invariance under a local gauge transformation by introducing a covariant derivative

$$\partial_\mu \to \mathbf{D}_\mu \equiv \partial_\mu - ig\mathbf{A}_\mu(x)$$

The lagrangian

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}(\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu}) + \bar{\Psi}(i\,\not{\mathbf{D}} - m)\Psi$$

is invariant under the local gauge transformations (ex.: show it)

$$\Psi(x) \to \Psi'(x) = U(x)\Psi(x)$$

$$\mathbf{A}_\mu \to \mathbf{A}'_\mu = U\mathbf{A}_\mu U^{-1} - \frac{i}{g}(\partial_\mu U)U^{-1}$$

This lagrangian is the basis for the construction of the SM

Comments:

• Non-abelian part of field strength tensor introduces self-couplings among gauge fields which are responsible for the remarkable property of asymptotic freedom

$$g c_{ijk} (\partial_\mu A^i_\nu) A^{\mu j} A^{\nu k}$$

$$\frac{g^2}{4} c_{ijk} c_{imn} A^{\mu j} A^{\nu k} A^m_\mu A^n_\nu$$

• Gauge symmetry forbids mass terms for the gauge fields – one must find a mechanism to generate mass for short range interactions such as the weak force.

Quantum Chromo Dynamics - QCD

Strong interactions are described by a local SU(3) gauge symmetry.

Fermions called quarks interact strongly through the exchange of a massless vector gauge field called gluon.

Each quark comes in three different colors transforming as the fundamental representation of SU(3):

$$q(x) = \begin{pmatrix} q_R \\ q_G \\ q_B \end{pmatrix}$$

There are six different types (or flavors) of quarks:

u, d, s, c, b, t

QCD lagrangian is given by

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}(\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu}) + \sum_{q=u,d,s,c,b,t} \bar{q}(i \not{\mathbf{D}} - m_q)q$$

$$\mathbf{F}_{\mu\nu} = F^i_{\mu\nu}\lambda_i/2 \qquad \mathbf{D}_\mu = \partial_\mu - ig_s\mathbf{A}_\mu(x) \qquad \mathbf{A}_\mu(x) = A^i_\mu(x)\lambda_i/2$$

8 3x3 matrices:
Gell-Mann matrices

QCD coupling constant

8 gluons

Comments:

- Electric charges of quarks are fractionary: +2/3 for u,c and t
  -1/3 for d,s and b

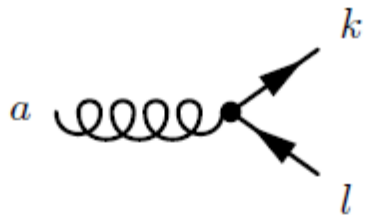
- One must introduce a gauge fixing term for proper quantization. This may introduce unphysical fields (ghosts) which must be taken into account in virtual processes.

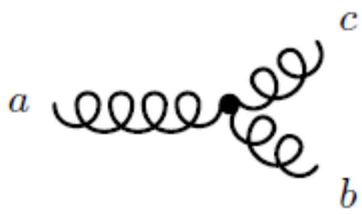
- In analogy to QED, one defines $\alpha_s \equiv \frac{g_s^2}{4\pi}$

- Coupling constant depends on energy scale, being large at low energies and small at large energies (asymptotic freedom).

- The interaction is so strong at low energies that quarks and gluons are always confined in hadrons. Perturbation techniques are not applicable!

QCD Feynman rules

- Quark-Gluon-Vertex: 

$ig_S(T_a)_{kl}\gamma^\mu$
- Quark-Propagator:  $i\frac{\not{q}+m}{q^2-m^2+i\epsilon} \equiv \frac{i}{\not{q}-m+i\epsilon}$
- Gluon-Propagator:  $\frac{-ig_{\mu\nu}}{q^2+i\epsilon}$ Feynman gauge
- Triple-Gluon-Vertex: 

- Quartic-Gluon-Vertex: 

(Complicated expressions)

+ ghosts