Introduction to the Standard Model

New Horizons in Lattice Field Theory
IIP Natal, March 2013

Rogerio Rosenfeld
IFT-UNESP

Lecture 1: Motivation/QFT/Gauge Symmetries/QED/QCD
Lecture 2: QCD tests/Electroweak sector/Symmetry Breaking
Lecture 3: Successes/Shortcomings of the Standard Model
Lecture 4: Beyond the Standard Model

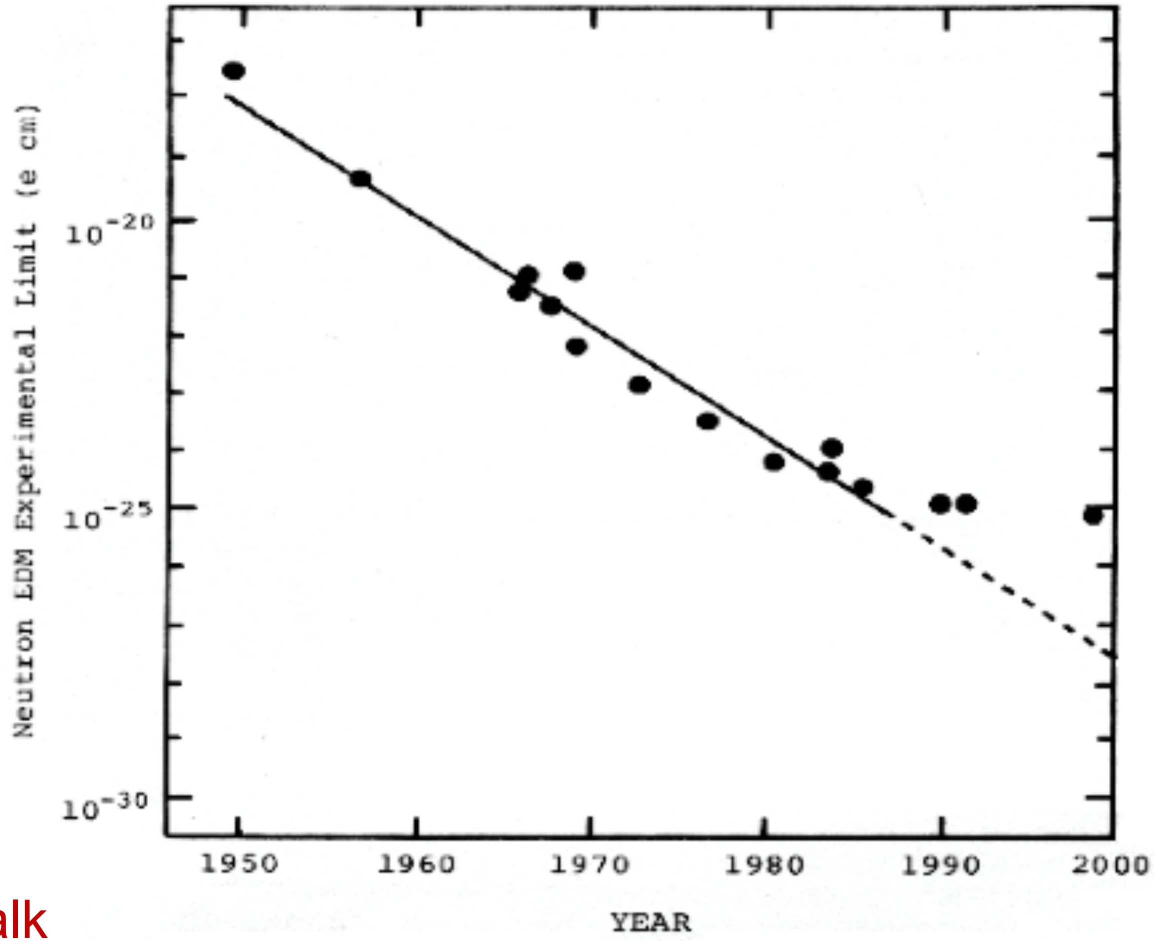
QCD lagrangian

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}(\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu}) + \sum_{q=u,d,s,c,b,t} \bar{q}(i\,\not{\mathbf{D}} - m_q)q$$

$$+\theta\ \mathrm{Tr}\left(\mathbf{F}_{\mu\nu}\tilde{\mathbf{F}}^{\mu\nu}\right)$$

θ term is allowed by gauge symmetries but violates CP.
It arises from the vacuum structure of QCD.
It can also affect perturbative calculations (jets ang. dist.)
Strong bounds from neutron electric dipole moment.
Strong CP problem.

Solution may be related to Dark Matter in the universe!



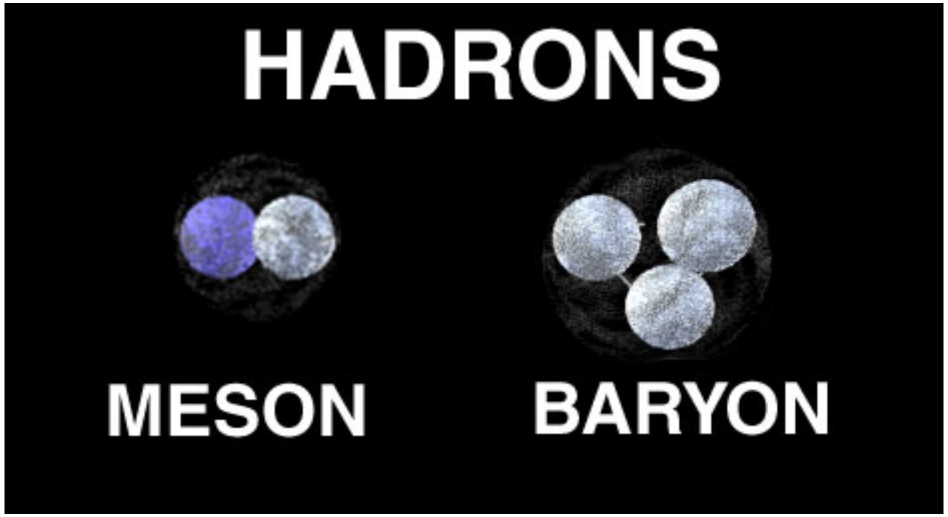
M. Romalis talk

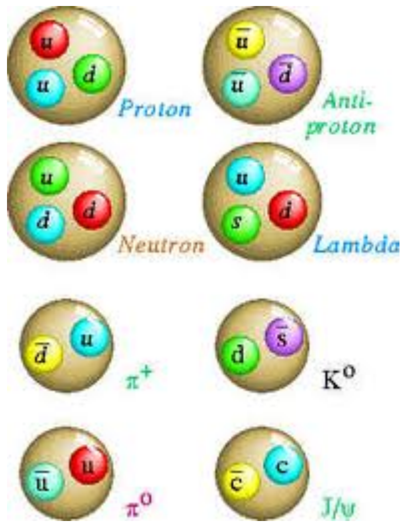
	Neutron	^{199}Hg
QCD $\bar{\theta}$	$\bar{\theta} = (1.9 \pm 2.8) \times 10^{-10}$ QCD sum rules	$\bar{\theta} = (1.2 \pm 0.7) \times 10^{-10}$

Hadrons: mesons ($q\bar{q}$) and baryons (qqq or $\overline{qqq}$) color neutral (color singlet) states.

$$M = \frac{1}{\sqrt{2}}(q^a \bar{q}'_a)$$

$$B = \frac{1}{\sqrt{6}}(\epsilon_{abc} q^a q'^b q''^c)$$





Strong interactions among hadrons is a residual effect analogue to the van der Waals force between electrically neutral molecules.

Exotic states such as glueballs, di-meson molecule, pentaquarks ... NOT SEEM YET

Factorization theorem: separation between scales.

short distance scales, perturbative region, where everything can be computed in terms of quarks and gluons

long distance scales, where non-perturbative effects are dominant and one needs inputs from measurements of hadronic structure.

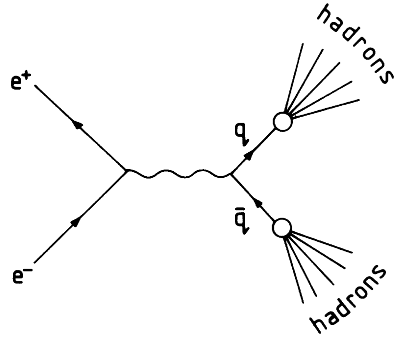
Transition from an unconfined phase of quarks and gluons (quark-gluon plasma) to a confined phase (hadrons) is being studied in the collision of heavy ions at RHIC and LHC.

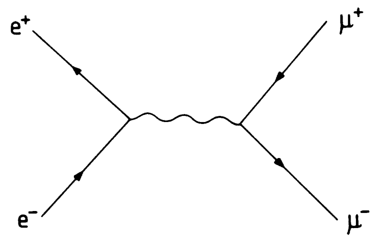
QCD Tests and Evidences for color

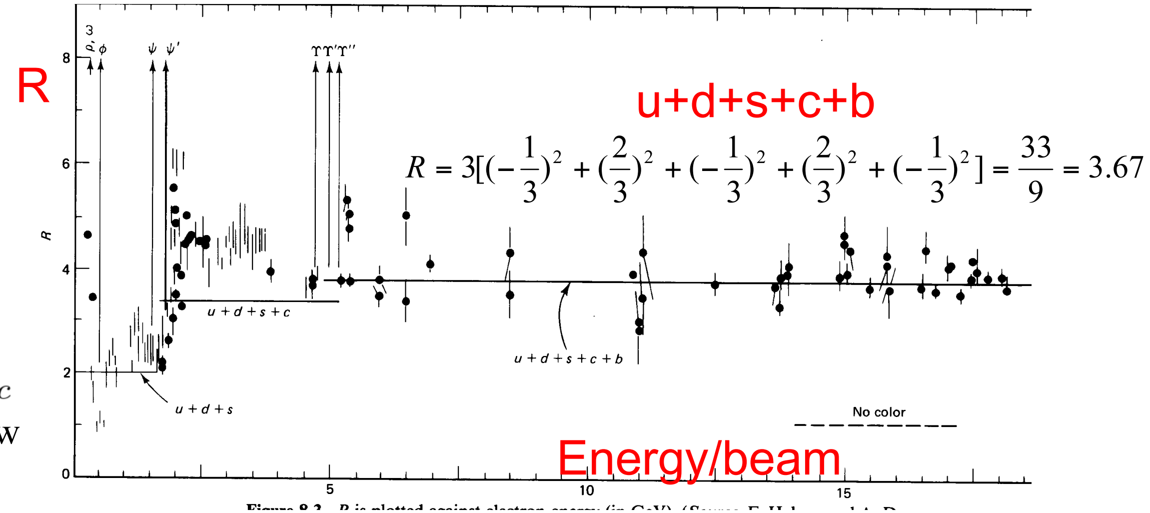
- Δ^{++} : (uuu) state with J=3/2 – totally symmetric state – needs color to make it antisymmetric to obey Pauli's principle

- Electron-positron annihilation into hadrons

$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \sum_i Q_i^2$$





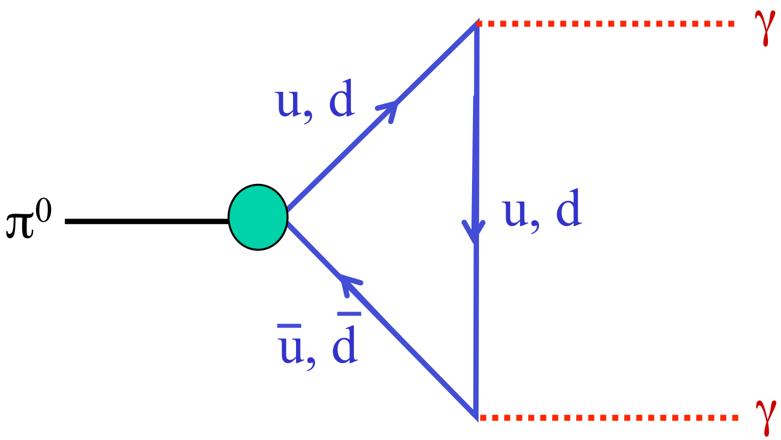


u+d+s

$$R = N_c\left[\left(\tfrac{2}{3}\right)^2 + \left(\tfrac{-1}{3}\right)^2 + \left(\tfrac{-1}{3}\right)^2\right] = \tfrac{2}{3}N_c$$

New

- Decay $\pi^0 \rightarrow \gamma\gamma$

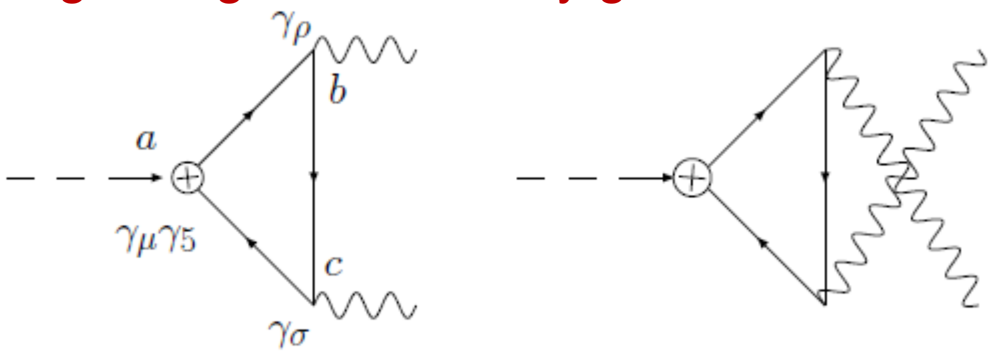


Triangle Diagram
Each color contributes one amplitude. Three colors changes the decay rate by 9.

Anomaly cancellation in the SM needs 3 colors

Peskin&Schroeder, chapt. 19
Laenen, NIKHEF lectures 2006

Triangle diagrams that may generate anomalies



Result is proportional to Tr[t_a,{t_b,t_c}], where t_a, are generator associated with the vertices of the triangle.

Example for SU(2)-SU(2)-U(1) $\mathrm{Tr}\left[Q\{\tau^b,\tau^c\}\right] = \frac{1}{2}\delta^{bc}\mathrm{Tr}\left[Q\right]$

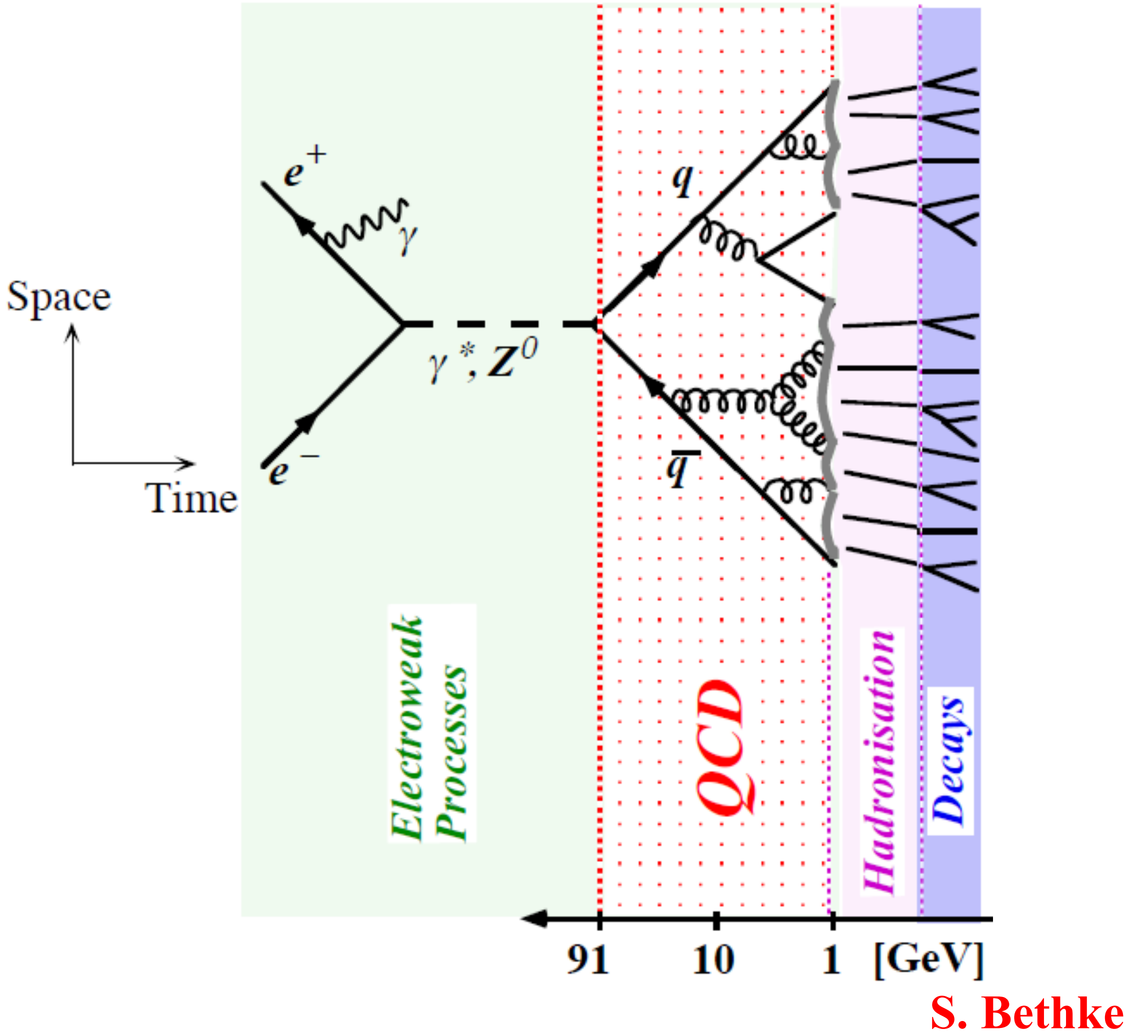
$$\mathrm{Tr}\left[Q\right] = 3 \times \left(\frac{2}{3} - \frac{1}{3}\right) + (0-1)$$

color

SU(2)-SU(2)-SU(2)

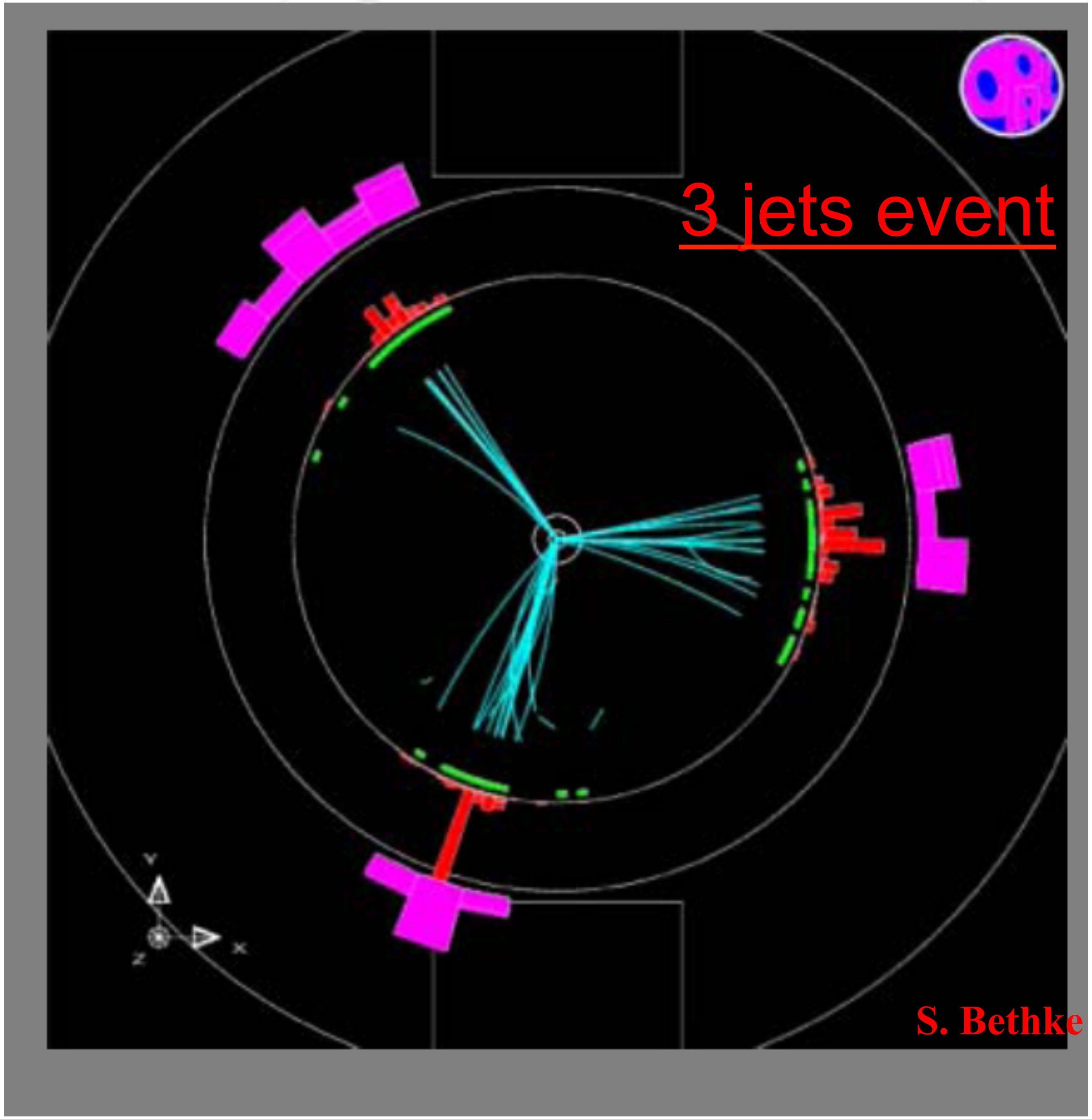
$$\mathrm{Tr}\left[\tau^a\{\tau^b,\tau^c\}\right] = \frac{1}{2}\delta^{bc}\mathrm{Tr}\left[\tau^a\right] = 0$$

• Jets in QCD: quarks and gluons produced at high energies radiate more quarks and gluons and hadronize; hadrons produced in “jets” – “spray” of particles - Factorization at work

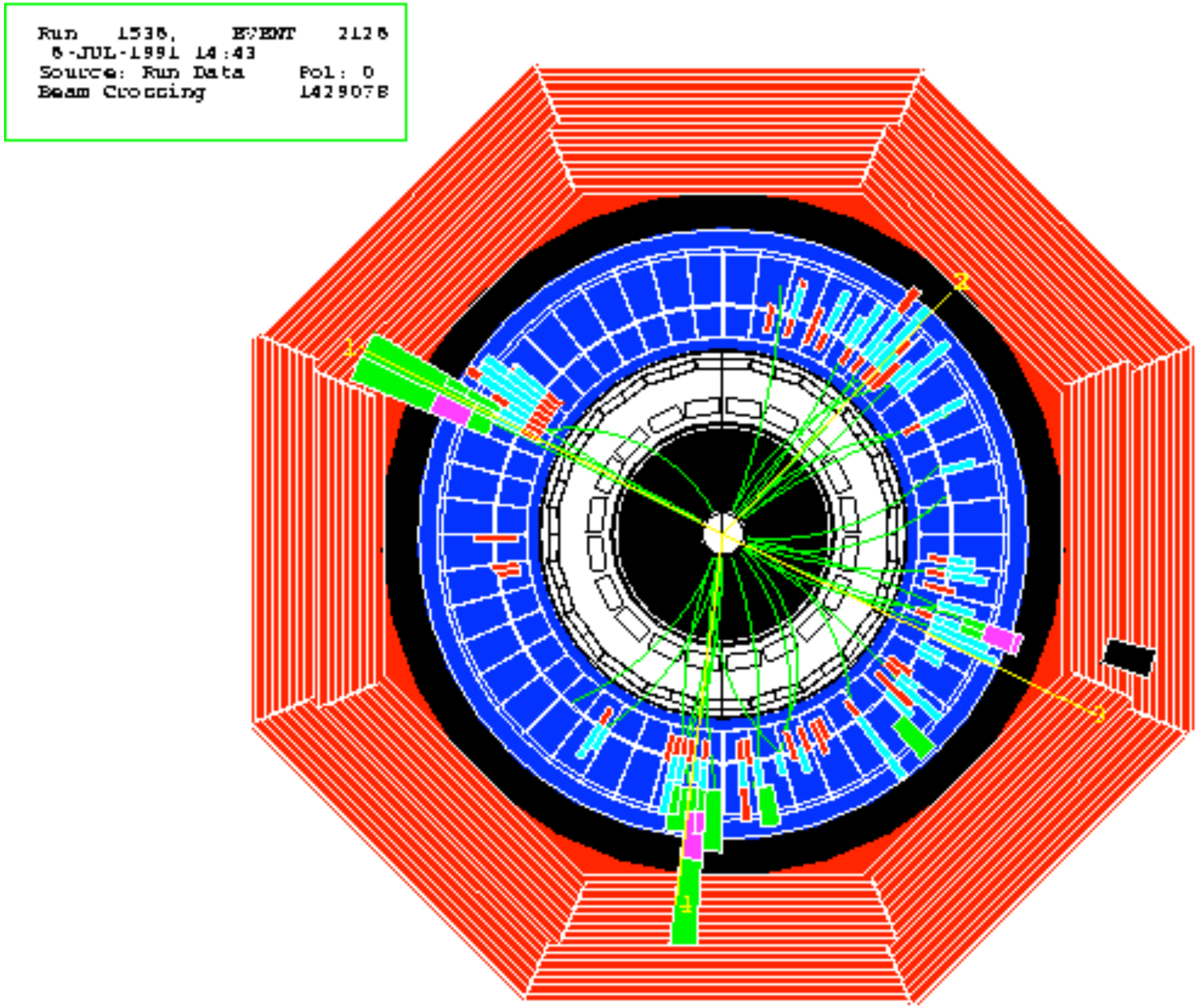


Hadronic event recorded at 205.4 GeV c.m. @ LEP

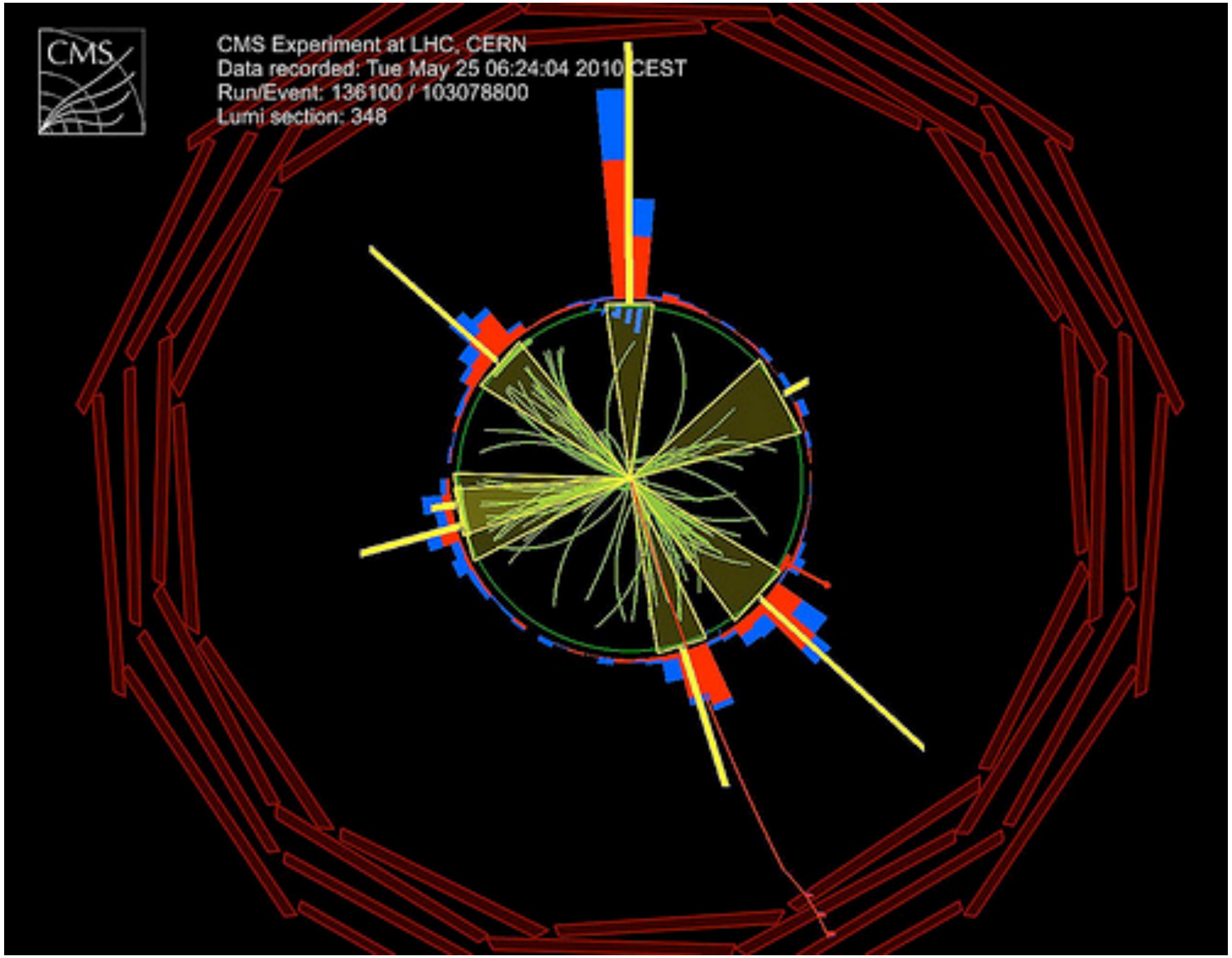
(August 1989 - November 2000)



4 jets event at Stanford Large Detector ~ mid 1990's

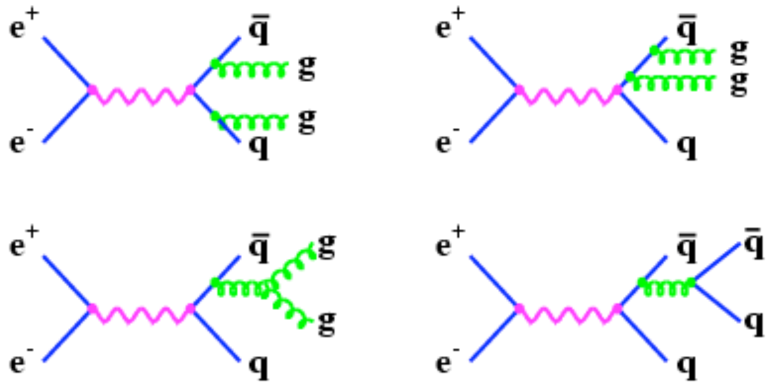


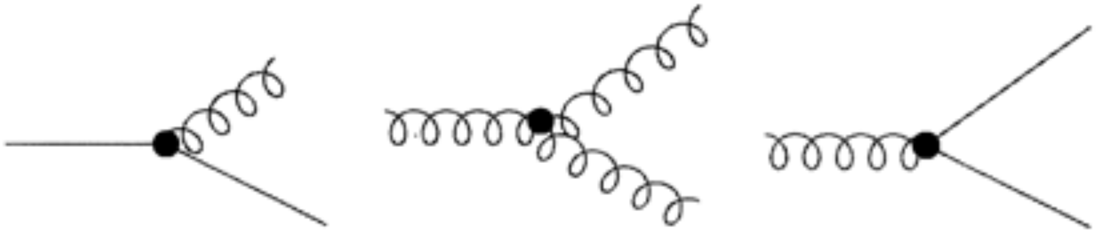
CMS experiment at the LHC: display of a multi-jet event at 7 TeV



Definition of what is a jet is tricky
(infrared and collinear singularities)

• QCD non-abelian nature tested 3 and 4 jets events

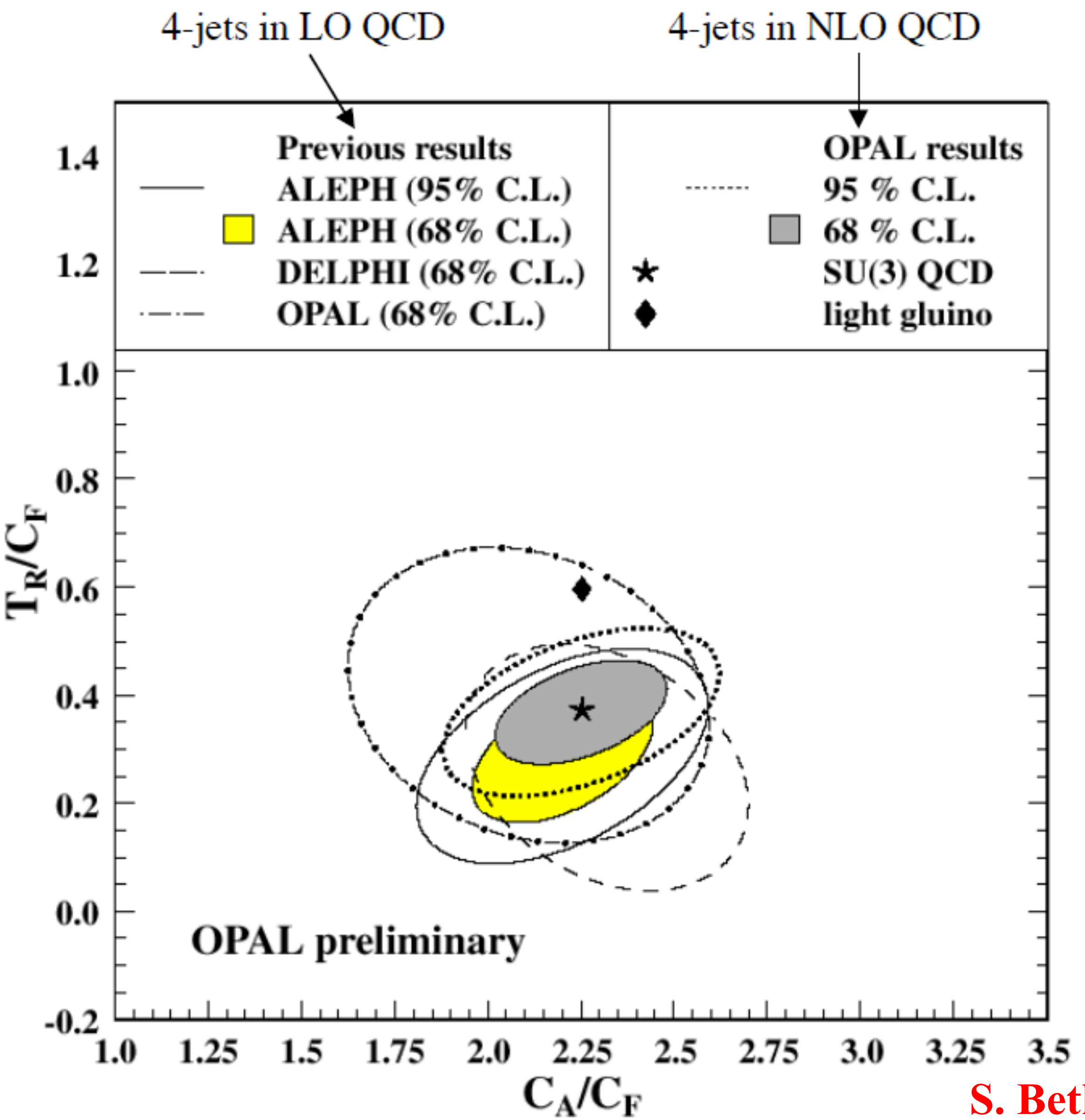




Gluon radiation	Triple gluon vertex	Gluon splitting
$q \to qg$	$g \to gg$	$g \to q\bar{q}$
$C_F = \frac{4}{3}$	$C_A = 3$	$T_F = \frac{1}{2}$

The "color factors" C_F, C_A, T_F specify the relative probabilities of the three processes.

- QCD non-abelian nature in 4 jets events



QCD:
$C_A=3$
$C_F=4/3$
$T_R=1/2$

Abelian QCD:
$C_A=0$
$C_F=1$
$T_R=6$

S. Bethke

- Running of coupling constant

The so-called beta function describes the running of coupling constants (quantum effects – loops)

$$\beta(g) \equiv \frac{dg(\mu)}{d\ln\mu}$$

For QED at 1-loop

$$\beta(e) = \frac{e^3}{12\pi^2} \quad \text{or} \quad \beta(\alpha) = \frac{2\alpha^2}{3\pi}$$

Electric charge is larger at larger energies!

- Running of coupling constant

Fine structure function grows with energy:

$$\alpha(\Lambda) = \frac{\alpha(\mu)}{1 - \frac{2}{3\pi}\alpha(\mu)\ln\left(\frac{\Lambda}{\mu}\right)}$$

Landau pole

- Running of coupling constant

For QCD at 1-loop

gluons

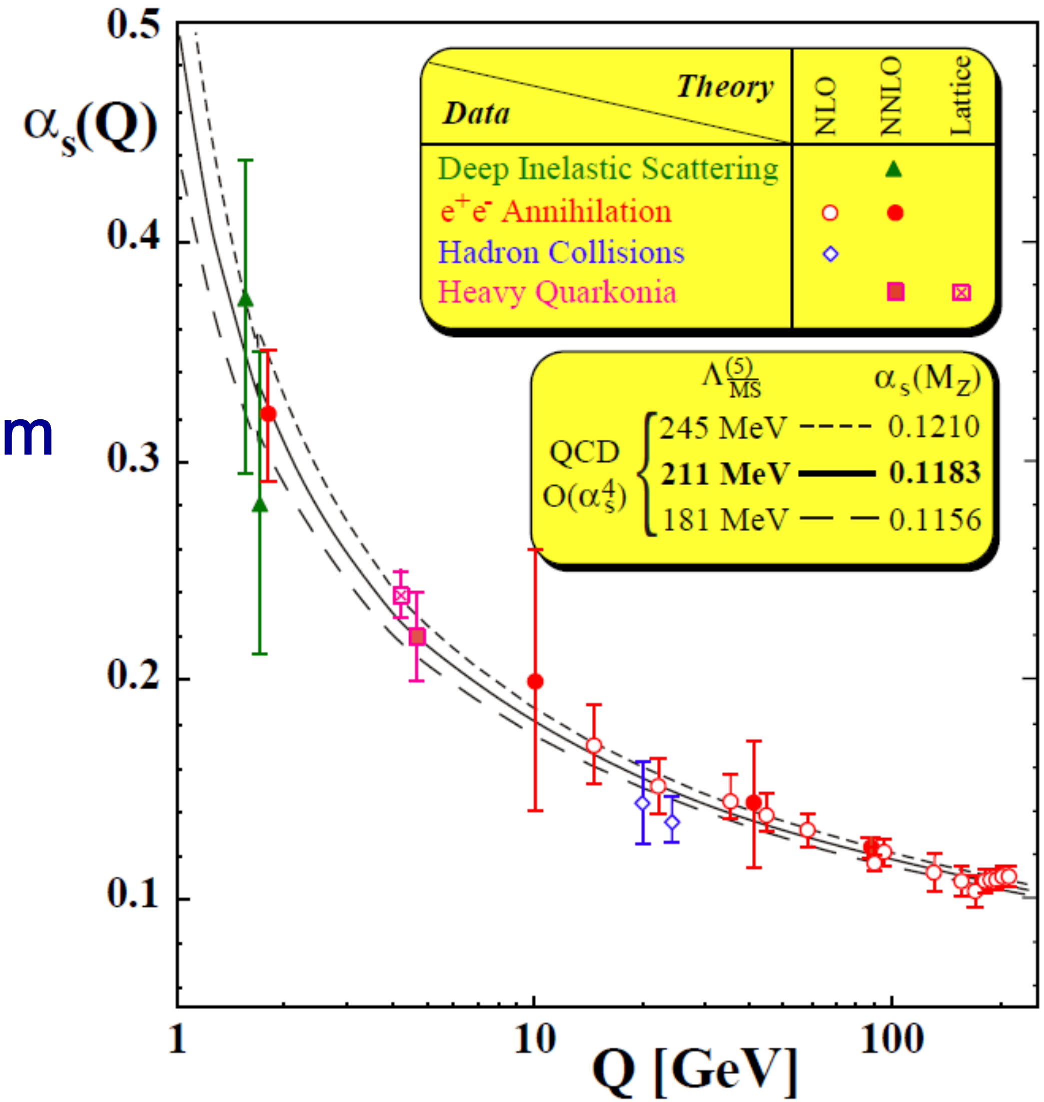
quarks

$$\beta(\alpha_s) = -\left(11 - \frac{2}{3} n_f\right) \frac{\alpha_s^2}{2\pi}$$

Asymptotic freedom

• Running of coupling constant

Asymptotic freedom
2004 Nobel prize



2002 world average result: $\alpha_s(M_Z) = 0.1883 \pm 0.0027$

- Quarkonium spectra

Potential models based on short-distance gluon exchange and a linear $b\bar{b}$ confining potential at large distances have reproduced reasonably well the spin-triplet nS, nP , and 1D bottomonium spectra below flavor threshold.

arXiv:1212.6552

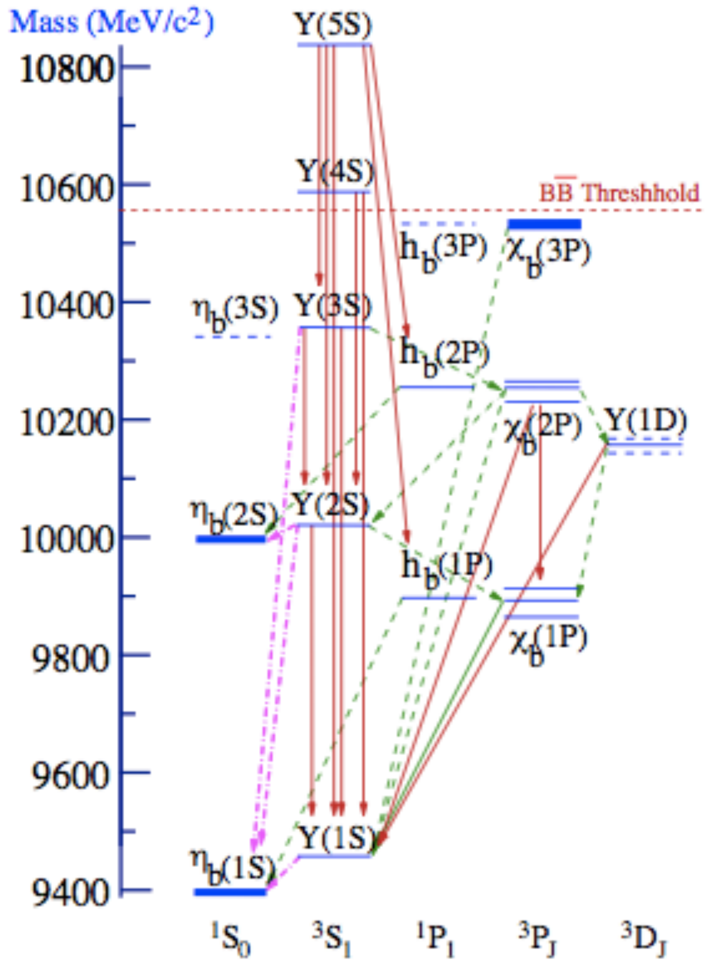
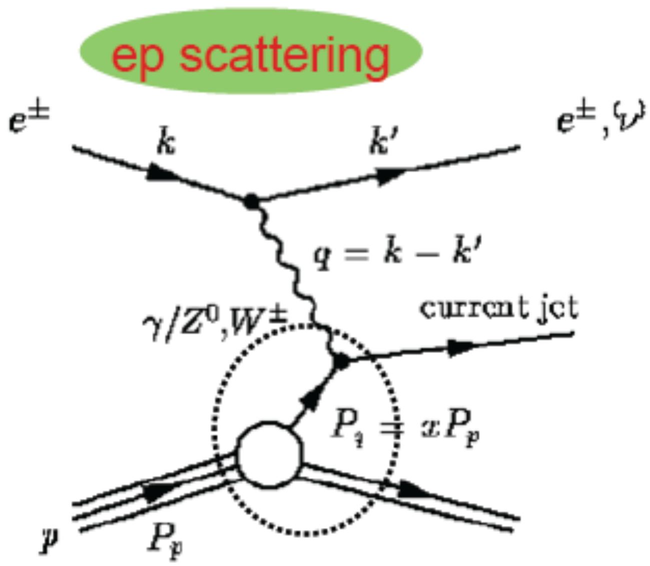


Figure 1: Current knowledge of the bottomonium system. Solid lines correspond to known states while dashed lines are predicted ones. The thicker lines indicate the range of measured masses for newly discovered states. (Solid, dashed, dot-dashed) arrows denote (hadronic, electric dipole [E1], and magnetic dipole [M1]) transitions, respectively.

• Deep inelastic scattering: “seeing” the structure of the proton (SLAC, HERA)



• New techniques for computing multiparton scattering amplitudes in QCD using spinor products and recursion relations are being rapidly developed (Peskin, 1101.2414)

• Hadron spectrum form lattice QCD:

S. Durr et al, Science 322, 1224 (2008)

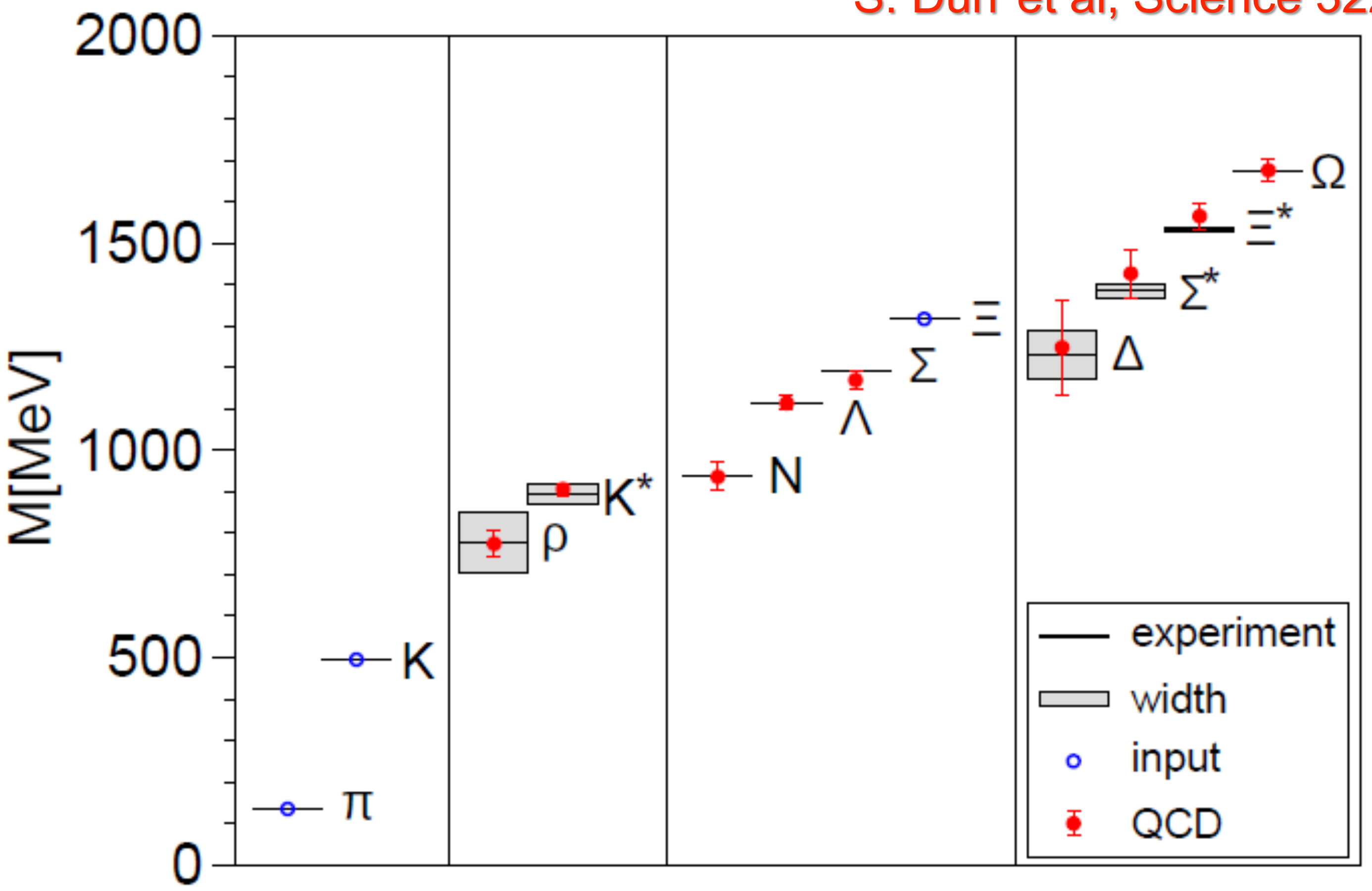


Figure 3: The light hadron spectrum of QCD. Horizontal lines and bands are the experimental values with their decay widths. Our results are shown by solid circles. Vertical error bars represent our combined statistical (SEM) and systematic error estimates. π, K and Ξ have no error bars, because they are used to set the light quark mass, the strange quark mass and the overall scale, respectively.

Conclusion:

There are several evidences that QCD is the theory describing the strong interactions.

It is one of the pillars of the Standard Model and a prototype to model strongly coupled New Physics

Nagging problems:

- nonperturbative regime hard to describe with analytical techniques
- why there is no CP violation in QCD?
- what is the origin of quark masses? Electroweak sector!

Building the Standard Model

The Standard Model comprises the strong, weak and electromagnetic interactions.

We have already study how QCD describes successfully the strong interactions.

The weak and electromagnetic forces are “unified” in the so-called electroweak interaction, which is the focus of this lecture.

Electroweak model

Huge amount of experimental and theoretical work was done to arrive at the correct symmetries and particles.

Quarks and leptons can interact via electroweak forces.

Weak forces are chiral: can tell left from right components
Parity is violated.

$$\psi = \psi_R + \psi_L$$

$$\psi_{R,L} \equiv \frac{1 \pm \gamma_5}{2} \psi$$

Electroweak theory is described by a gauge theory based on the direct product of 2 gauge groups: $SU(2)_L \times U(1)_Y$

- the corresponding quantum numbers are called weak isospin (I) and hypercharge (Y)

- there are 2 independent couplings g and g' (unification?)

- left-handed fermions are doublets under $SU(2)_L$ (I=1/2)

- right-handed fermions are singlets under $SU(2)_L$ (I=0)

Electroweak quantum numbers for the first generation of fermions:

$$\begin{pmatrix} u \\ d \end{pmatrix}_L (Y = 1/3) \quad , \quad u_R(Y = 4/3), \;\; d_R(Y = -2/3),$$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L (Y = -1) \quad , \quad e_R(Y = -2).$$

The other generations follow the same pattern.

In the SM there are no right-handed neutrinos.

Eletric charge:

$$q = \frac{Y}{2} + I_3$$

Interaction of fermions with gauge fields occur through the covariant derivative in the kinetic term:

$$\mathcal{L}_F = \overline{\psi}\, \not{\mathbf{D}} \psi$$

with

3 weak isospin gauge bosons

hypercharge gauge boson

$$\mathbf{D}_\mu = \partial_\mu - ig\frac{\tau^i W^i_\mu}{2} - ig'\frac{Y}{2}B_\mu$$

weak isospin coupling constant

hypercharge coupling constant

fermion hypercharge

Interaction of fermions with gauge fields is:

$$\bar{\psi}\gamma_\mu\psi = \bar{\psi}_R\gamma_u\psi_R + \bar{\psi}_L\gamma_\mu\psi_L$$

$$\mathcal{L}_F = \sum_{R=e_R,u_R,d_R,\ldots} \bar{R} i\gamma^\mu \left(\partial_\mu - ig'\frac{Y}{2}B_\mu\right) R + \sum_{L=\begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \ldots} \bar{L} i\gamma^\mu \left(\partial_\mu - ig\frac{\tau^i W^i_\mu}{2} - ig'\frac{Y}{2}B_\mu\right) L$$

Important observation: fermion mass terms and mass terms for gauge bosons violate $SU(2)_L \times U(1)_Y$:

$$\bar{\psi}\psi = \bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R$$

Need a mechanism to break the symmetry!

Take a step back: global symmetries

Global symmetry G can be realized in two ways depending on the transformation of the vacuum state.

1. Linear realization (Wigner-Weyl mode)

- vacuum is invariant under G: $G|0\rangle = |0\rangle$
- physical states classified in multiplets of G
- symmetry is linearly realized for any field: $\Psi \longrightarrow g \cdot \Psi$

$$g \in G$$

2. Non-linear realization (Nambu-Goldstone mode)

- vacuum Φ_0 is NOT invariant under G: there are degenerate and inequivalent vacuum states: $G|0\rangle = |0\rangle_g \neq |0\rangle$

- physical states are not classified in multiplets of G: symmetry is "hidden"

- symmetry is non-linearly realized: $\Psi \to F(g, \Psi) \cdot \Psi$

- there exists massless scalar fields: Nambu-Goldstone bosons

Goldstone theorem: heuristic "proof"

Consider a set of N scalar fields $\vec{\phi} = (\phi_1, \phi_2, \ldots, \phi_N)$

described by an O(N) invariant lagrangian

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \vec{\phi}\right) \cdot \left(\partial^\mu \vec{\phi}\right) - V(|\vec{\phi}|)$$

Vacuum is given by minimum of potential $\frac{\partial V}{\partial \phi_i} = 0 \text{ at } \vec{\phi} = \vec{\phi}_0$

Mass matrix is $$M_{ij}^2 = \frac{\partial^2 V}{\partial\phi_i \partial\phi_j} \text{ at } \vec{\phi} = \vec{\phi}_0$$

Field transformation $$\vec{\phi} \rightarrow \vec{\phi}' = G\vec{\phi} = e^{i\alpha_a T^a}\vec{\phi}$$

Infinitesimal transformation $$\delta\phi_i = i\alpha_a T_{ij}^a \phi_j$$

State is left invariant by a generator when

$$T^a \vec{\phi} = 0$$

Variation of the potential must vanish (O(N) invariance):

$$\delta V = \frac{\partial V}{\partial \phi_i}\delta\phi_i = i\alpha_a \frac{\partial V}{\partial \phi_i} T^a_{ij}\phi_j = 0$$

Taking a derivative:

$$\frac{\partial^2 V}{\partial \phi_i \partial \phi_k}\left(T^a_{ij}\phi_j\right) + \frac{\partial V}{\partial \phi_i}\left(T^a_{ij}\delta_{jk}\right) = 0$$

In the vacuum: $M^2_{ik}\left(T^a_{ij}\phi_j\right) = 0$

In the vacuum: $$M^2_{ik}\left(T^a_{ij}\phi_j\right) = 0$$

Either the generator T^a annihilates the vacuum (unbroken generator) or it doesn't (broken generator), in which case there must be a massless state. This is an example of the Goldstone theorem.

Broken global symmetries usually implies in massless scalars!

Conclusion can be avoided in models with local symmetries.

Spontaneous Symmetry Breaking

Simple example: abelian Higgs model

charged scalar field

$V(\phi)$

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + (\partial_\mu - ieA_\mu)\,\phi\,(\partial^\mu + ieA^\mu)\,\phi^* - \lambda\left(\phi\phi^* - \frac{v^2}{2}\right)^2$$

potential describing self-interaction

Lagrangian is invariant under the usual local U(1) symmetry of EM

$$\phi'(x) = \exp[i\alpha(x)]\phi(x)$$
$$A'_\mu(x) = A_\mu(x) - \frac{1}{e}\partial_\mu\alpha(x)$$

Vacuum: configuration with minimum energy

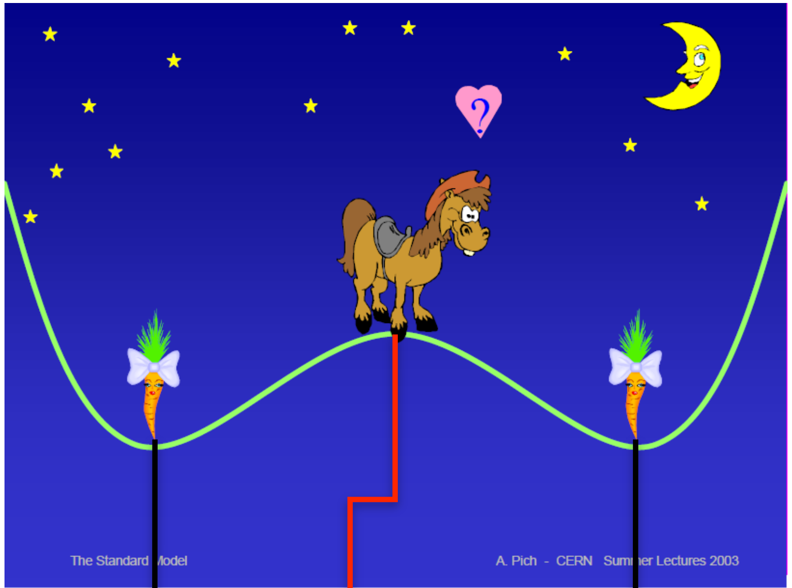
$$|\phi_0| = \frac{v}{\sqrt{2}}$$

Lagrangian is gauge invariant but vacuum configuration changes by a phase.

This generates spontaneous symmetry breaking (symmetry is realized in the Nambu-Goldstone mode)

Spontaneous symmetry breaking à la Higgs

A. Pich



False vacuum

True vacuum (degenerate states)

Choosing one of the possible vacua breaks the symmetry

It costs energy to climb up the hill::
Massive scalar field (Higgs boson)

It doesn't cost energy to move along valley
(shift symmetry: massless fields
(Nambu-Goldstone bosons)

"eaten" by gauge bosons →

Write

$$\phi(x) = \underbrace{\rho(x)}_{\text{radial part}} \exp[i \underbrace{\pi(x)/}_{\text{angular part}} v]$$

Under a gauge transformation

$$\rho' = \rho, \qquad \pi' = \pi - v\alpha(x)$$

Can eliminate angular part by choosing a gauge parameter

$$\alpha(x) = \frac{\pi(x)}{v}$$

This particular choice of gauge fixing is called unitary gauge.

Introduce a physical real scalar field h(x) by:

$$\rho(x) \equiv \frac{1}{\sqrt{2}}(h(x) + v)$$

Lagrangian is given by (exercise):

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}\partial_\mu h(x)\partial^\mu h(x) + \underbrace{\frac{1}{2}e^2v^2A_\mu A^\mu}_{\text{photon mass}} - \underbrace{\lambda v^2 h(x)^2}_{\text{scalar mass}} - \frac{\lambda}{4}h(x)^4 - \lambda v h(x)^3 + \frac{1}{2}e^2 h(x)^2 A_\mu A^\mu + e^2 v h(x) A_\mu A^\mu$$

photon mass $\quad m_A = ev$

"Higgs" mass $\quad m_h = 2\sqrt{\lambda}v$

Angular field $\pi(x)$ disappears in this gauge:
it is an unphysical field!

Counting degrees of freedom:

- before symmetry breaking
  2 (complex scalar field) + 2 (massless vector field) = 4
- after symmetry breaking
  1 (real scalar field h(x)) + 3 (massive vector field) = 4

One says that the angular field is "eaten" by gauge field providing its longitudinal component.

Equivalence theorem: $\mathcal{M}(A_{L_1} A_{L_2} \dots A_{L_n}) = \mathcal{M}(\pi_1 \pi_2 \dots \pi_n) + \mathcal{O}\left(\frac{m_A}{E}\right)$

Comments:

• Gauge field propagator is gauge dependent:

$$D^{\mu\nu}(p) = \frac{-i}{p^2 - m_A^2}\left(g^{\mu\nu} - (1-\xi)\frac{p^\mu p^\nu}{p^2 - \xi m_A^2}\right)$$

$\xi = 1$ (Feynman-'t Hooft gauge), $\xi = 0$ (Landau gauge) and $\xi \to \infty$ (unitary gauge)

• In non-unitary gauge, $\pi(x)$ must be taken into account

• If there were no gauge bosons (global symmetry), field $\pi(x)$ would be a physical massless field: Nambu-Goldstone boson.

The Higgs Sector of the SM

We know that fermions have masses and that the weak interactions are short ranged. Must find a mechanism to generate masses for fermions and weak gauge bosons.

Introduce a new sector of the theory to accomplish that: the Higgs sector. Define a complex scalar doublet, the Higgs doublet:

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

I=1/2
Y=1

A little history

My Life as a Boson
Lecture by Peter Higgs (University of Edinburgh)

MCTP, Ann Arbor, Michigan, 21 May 2001
duration: 53:59, 13 slides

See also www.kcl.ac.uk/nms/depts/physics/news/events/MyLifeasaBoson.pdf

My Life as a Boson

— The Story of "The Higgs"

Plan of Talk

1) 1960-66 from { Nambu, Goldstone }
   to { Brout & Englert, Higgs }

2) 1967-71 from { Weinberg, Salam }
   to { Veltman, 't Hooft }

3) 1972-present My Life as a Boson

1960 P.W.H. appointed to Edinburgh Lectureship

July 1960 First Scottish Universities Summer School in Physics, "Dispersion Relations"

Participants included

Dr. N. Cabibbo	Rome
Dr. S.L. Glashow	CERN
Mr. M.J.G. Veltman	Utrecht
Mr. D.W. Robinson	Oxford
Mr. K. Hepp	Zürich

8

1964 ACCIDENTAL BIRTH OF A BOSON

TH. 16 July — Phys. Rev. Letters (22 June), containing Gilbert's paper reaches Edinburgh.

F. 24 July — Broken Symmetries, Massless Particles and Gauge Fields (P.W.H.) sent to Physics Letters editor at CERN.

ACCEPTED

F. 31 July — Broken Symmetries and the Masses of Gauge Bosons (P.W.H.) sent to Physics Letters editor at CERN.

REJECTED

August — Paper revised by adding (inter alia) "It is worth noting that an essential feature of this type of theory is the prediction of incomplete multiplets of scalar and vector bosons"

31 August — Revised paper received by Physical Review Letters.

ACCEPTED

Referee (Nambu) draws to attention of PWH the paper by F. Englert & R. Brout, Broken Symmetry and the Mass of Gauge Vector Mesons (received by Phys. Rev. Letters 26 June, published 31 August)

Volume 12, number 2 PHYSICS LETTERS 15 September 1964

BROKEN SYMMETRIES, MASSLESS PARTICLES AND GAUGE FIELDS

P. W. HIGGS
Tait Institute of Mathematical Physics, University of Edinburgh, Scotland

Received 27 July 1964

VOLUME 13, NUMBER 16 PHYSICAL REVIEW LETTERS 19 OCTOBER 1964

BROKEN SYMMETRIES AND THE MASSES OF GAUGE BOSONS

Peter W. Higgs
Tait Institute of Mathematical Physics, University of Edinburgh, Edinburgh, Scotland
(Received 31 August 1964)

BROKEN SYMMETRY AND THE MASS OF GAUGE VECTOR MESONS*

F. Englert and R. Brout
Faculté des Sciences, Université Libre de Bruxelles, Bruxelles, Belgium
(Received 26 June 1964)

GLOBAL CONSERVATION LAWS AND MASSLESS PARTICLES*

G. S. Guralnik,† C. R. Hagen,‡ and T. W. B. Kibble
Department of Physics, Imperial College, London, England
(Received 12 October 1964)

As was done in the abelian Higgs model, will introduce a potential for the Higgs doublet self-interaction in such a way that the minimum energy configuration breaks the gauge symmetry:

$$\mathcal{L}_I = -\lambda \left(\Phi\Phi^\dagger - \frac{v^2}{2} \right)^2$$

Energy is minimum at $\langle \Phi\Phi^\dagger \rangle = \frac{v^2}{2}$

Write Higgs doublet as:

$$\Phi = e^{i\frac{\xi(\mathbf{x})\cdot\tau}{2v}} \left(\begin{array}{c} 0 \\ \frac{v+H(x)}{\sqrt{2}} \end{array} \right)$$

and work in unitary gauge $\Phi \to e^{-i\frac{\xi(\mathbf{x})\cdot\tau}{2v}}\Phi = \left(\begin{array}{c} 0 \\ \frac{v+H(x)}{\sqrt{2}} \end{array} \right)$

where H(x) is the physical Higgs real scalar field.

Lagrangian for the Higgs doublet in unitary gauge:

$$\mathcal{L}_\Phi = (\mathbf{D}_\mu \Phi)^\dagger (\mathbf{D}^\mu \Phi) + \mathcal{L}_I$$

$$= \left| \left\{ \partial_\mu - \frac{ig}{2} \begin{bmatrix} W_\mu^3 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & -W_\mu^3 \end{bmatrix} - \frac{ig'}{2} B_\mu \right\} \begin{bmatrix} 0 \\ \frac{v+H(x)}{\sqrt{2}} \end{bmatrix} \right|^2$$

$$-\lambda \left(\left(\frac{v+H(x)}{\sqrt{2}} \right)^2 - \frac{v^2}{2} \right)^2$$

This lagrangian presents several features we will discuss in the following.

Canonical kinetic term for the Higgs scalar with a mass (exercise)

$$m_H = 2\sqrt{\lambda}v$$

There are W W H, B B H, W W H H, B B H H, H H H and H H H H interactions.
Exercise: compute WWH, HHH and HHHH interactions.

Mass terms for some gauge fields are generated!

Mass term for gauge fields:

$$\mathcal{L}_{m,YM} = \frac{v^2}{8}\left\{g^2[(W^1)^2 + (W^2)^2] + (gW^3 - g'B)^2\right\}$$

Define charged gauge fields as $W^{\pm}_{\mu} \equiv (W^1_{\mu} \mp iW^2_{\mu})/\sqrt{2}$

$$\mathcal{L}_{m,W^{\pm}} = \frac{g^2 v^2}{8}\left(|W^+_{\mu}|^2 + |W^-_{\mu}|^2\right)$$

Mass of charged gauged bosons:

$$M_W = \frac{gv}{2}$$

Comment: Fermi theory of weak interactions- based on a non-renormalizable 4-fermion interaction

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} \bar{\nu}_\mu \gamma^\alpha (1-\gamma^5) \mu \bar{e} \gamma_\alpha (1-\gamma^5) \nu_e$$

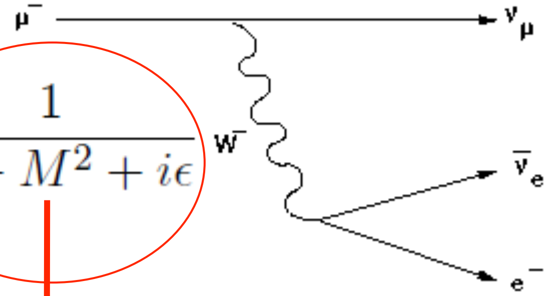
Muon width

$$\Gamma = \frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3}(1 - \frac{8m_e^2}{m_\mu^2})$$

$2.19712(8) \times 10^{-6}$ s

$G_F = 1.16637(1) \times 10^{-5}$ GeV^{-2}

$$-\frac{g}{2\sqrt{2}} \bar{\nu}_\mu \gamma^\mu (1-\gamma^5) W_\mu^+ \mu + h.c$$



$$\frac{1}{k^2 - M^2 + i\epsilon}$$

$$\frac{1}{M_W^2}$$

The mass of the charged gauge bosons is related to the Fermi decay constant:

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{1}{2v^2}$$

which fixes the value of the Higgs vev in the model:

$$v = 2^{-1/4} G_F^{-1/2} = 246 \text{ GeV}$$

Linear combination of neutral fields $gW_\mu^3 - g'B_\mu$ also acquires a mass.

weak (Weinberg) mixing angle

Define:

$$Z_\mu \equiv W_\mu^3 \cos\theta_W - B_\mu \sin\theta_W$$

where

$$\cos\theta_W \equiv \frac{g}{(g^2+g'^2)^{1/2}} \quad \left[\sin\theta_W = \frac{g'}{(g^2+g'^2)^{1/2}}\right]$$

we can write

$$\mathcal{L}_{m,YM} = \frac{v^2}{8}\left\{(gW^3 - g'B)^2\right\} = \frac{v^2(g^2+g'^2)}{8} Z^2$$

and the Z-boson mass is

$$M_Z = \frac{\sqrt{(g^2+g'^2)}v}{2} = M_W / \cos\theta_W$$

Orthogonal combination

$$A_\mu = B_\mu \cos\theta_W + W_\mu^3 \sin\theta_W$$

remains massless and can be identified with the photon.

Counting degrees of freedom:

- before symmetry breaking

4 (complex scalar field doublet) + 8 (4 massless vector field) = 12

- after symmetry breaking

1 (real scalar field h(x)) + 9 (3 massive vector field) + 2 (1 massless vector field) = 12

Again angular fields are “eaten” by gauge fields providing their longitudinal component.

Kinetic terms for gauge fields:

$$\mathcal{L}_G = -\frac{1}{4} W^i_{\mu\nu} W^{\mu\nu i} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

with

$$W^i_{\mu\nu} \equiv \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g\epsilon_{ijk} W^j_\mu W^k_\nu$$

$$B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu$$

After writing in terms of physical fields $W^{+/-}$, Z and A one finds trilinear and quartic couplings: $W^+ W^- Z$, $W^+ W^- A$, $W^+ W^- W^+ W^-$, $W^+ W^- ZZ$, $W^+ W^- ZA$, $W^+ W^- AA$.
These couplings have been measured at LEP, Tevatron and LHC.

Comment:
In unitary gauge only the physical fields are present.
It is possible to work in the R_ξ gauge, where the gauge boson propagators are given by

$$D^{\mu\nu}(p) = \frac{-i}{p^2-m_V^2}\left(g^{\mu\nu} - (1-\xi)\frac{p^\mu p^\nu}{p^2-\xi m_V^2}\right)$$

V=W,Z

and one has to take into account the contribution of non-physical NG boson in the calculation of processes.

Fermion masses

Fermion masses are forbidden by gauge symmetries. They are generated by introducing a coupling between fermions and the Higgs boson: Yukawa coupling

Scalar doublet with Y=-1: $\tilde{\Phi} = i\tau_2 \Phi^* = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix}$

$SU(2)_L \times U(1)_Y$ invariant interaction (1 generation):

$$\mathcal{L}_Y = -g_u \left[\bar{Q}_L \tilde{\Phi} u_R + \bar{u}_R \tilde{\Phi}^\dagger Q_L\right] - g_d \left[\bar{Q}_L \Phi d_R + \bar{d}_R \Phi^\dagger Q_L\right] - g_e \left[\bar{L}_L \Phi e_R + \bar{e}_R \Phi^\dagger L_L\right]$$

Yukawa coupling constants

In unitary gauge

$$\Phi = \left(\begin{array}{c} 0 \\ \frac{v+h(x)}{\sqrt{2}} \end{array} \right) \text{ and } \tilde{\Phi} = \left(\begin{array}{c} \frac{v+h(x)}{\sqrt{2}} \\ 0 \end{array} \right)$$

and (for 1 generation)

$$\begin{aligned} \mathcal{L}_Y \quad = \quad & -\frac{g_u v}{\sqrt{2}} \left(\bar{u}_L u_R + \bar{u}_R u_L\right) - \frac{g_d v}{\sqrt{2}} \left(\bar{d}_L d_R + \bar{d}_R d_L\right) - \frac{g_e v}{\sqrt{2}} \left(\bar{e}_L e_R + \bar{e}_R e_L\right) - \\ & \frac{g_u}{\sqrt{2}} h(x) \left(\bar{u}_L u_R + \bar{u}_R u_L\right) - \frac{g_d}{\sqrt{2}} h(x) \left(\bar{d}_L d_R + \bar{d}_R d_L\right) - \\ & \frac{g_e}{\sqrt{2}} h(x) \left(\bar{e}_L e_R + \bar{e}_R e_L\right) \end{aligned}$$

Comments:

• fermion mass is given by

$$m_f = \frac{g_f v}{\sqrt{2}}$$

• fermion coupling to Higgs boson is proportional to its mass:

$$g_f = \frac{\sqrt{2} m_f}{v}$$

• when including more than one generation, can have fermion mass matrices

Fermion mass matrices

• defining U'=(u',c',t') and D'=(d',s',b') as weak eigenstates SSB results in a general mass term

$$\mathcal{L}_m = -[\overline{U'}_R \mathcal{M}_U U'_L + \overline{D'}_R \mathcal{M}_D D'_L + \mathrm{h.c.}]$$

Fermion mass matrices
general 3x3 complex matrices

• convenient to work with mass eigenstates U=(u,c,t) and D=(d,s,b), where the mass matrix is diagonal.

• diagonalize mass matrices with unitary transformations on left and right-handed quarks:

$$Q'_L = L_Q Q_L; \quad Q'_R = R_Q Q_R \qquad (Q = U, D)$$

$$R_Q^+ \mathcal{M}_Q L_Q = L_Q^+ \mathcal{M}_Q^+ R_Q = \Lambda_Q$$

Diagonal mass matrix

Fermion interactions: charged currents

$$\mathcal{L}_F^{W^\pm} = -\frac{g}{\sqrt{2}}[\overline{U'}_L \gamma^\mu W_\mu^{(+)} D'_L + \text{h.c.}]$$

Weak eigenstates

In terms of mass eigenstates U=(u,c,t) and D=(d,s,b)

$$\mathcal{L}_F^{W^\pm} = -\frac{g}{\sqrt{2}}[\overline{U}_L \gamma^\mu W_\mu V D_L + \text{h.c.}]$$

Cabbibo-Kobayasi-Maskawa (CKM) matrix: $V \equiv L_U^+ L_D$

$$\begin{bmatrix} |d'\rangle \\ |s'\rangle \\ |b'\rangle \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} |d\rangle \\ |s\rangle \\ |b\rangle \end{bmatrix}$$

Can have flavor nondiagonal interactions! (eg, $W^+ \to t\bar{d}$)

Experimental values for the magnitudes of the CKM elements

$$\begin{pmatrix} 0.97428 \pm 0.00015 & 0.2253 \pm 0.0007 & 0.00347^{+0.00016}_{-0.00012} \\ 0.2252 \pm 0.0007 & 0.97345^{+0.00015}_{-0.00016} & 0.0410^{+0.0011}_{-0.0007} \\ 0.00862^{+0.00026}_{-0.00020} & 0.0403^{+0.0011}_{-0.0007} & 0.999152^{+0.000030}_{-0.000045} \end{pmatrix}$$

CKM matrix is unitary by construction. For the SM with three generations, it is described by 3 mixing angles and a CP violating phase.

Standard parametrization:

$$\begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{bmatrix}$$

Best measured values:

$\theta_{12} = 13.04 \pm 0.05°$, $\theta_{13} = 0.201 \pm 0.011°$, $\theta_{23} = 2.38 \pm 0.06°$, and $\delta_{13} = 1.20 \pm 0.08$

Fermion interactions: neutral currents

Consider first the case of a single quark family

$$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, \quad u_R, \quad d_R$$

Lagrangian with neutral interactions is

$$\mathcal{L}_F^{W^3,B} = -ig'\frac{Y_{u_R}}{2}\bar{u}_R\gamma_\mu u_R B^\mu - ig'\frac{Y_{Q_L}}{2}\bar{u}_L\gamma_\mu u_L B^\mu - igI_3\bar{u}_L\gamma_\mu u_L W^{3,\mu}$$

In order to find interactions with physical fields must write B and W^3 fields in terms of A and Z fields:

$$W_\mu^3 = Z_\mu \cos\theta_W + A_\mu \sin\theta_W$$
$$B_\mu = -Z_\mu \sin\theta_W + A_\mu \cos\theta_W$$

Exercise- show that the interaction of fermions with A is given by

$$\mathcal{L}_F^A = -i\left[g'\cos\theta_W \frac{Y_{u_R}}{2}\bar{u}_R\gamma_\mu u_R + g'\cos\theta_W \frac{Y_{Q_L}}{2}\bar{u}_L\gamma_\mu u_L + g\sin\theta_W I_3 \bar{u}_L\gamma_\mu u_L\right] A^\mu$$

and similarly for the d-quark.

In order to identify A with the photon field we set

$$e = g'\cos\theta_W = g\sin\theta_W = \frac{gg'}{\sqrt{g^2+g'^2}}$$

and

$$q_u = \frac{Y_{u_R}}{2} = \frac{Y_{Q_L}}{2} + I_3$$

This "explains" the quantum number assignements for leptons.

Interaction of fermions with Z gauge boson:

$$\mathcal{L}_F^Z = -i(g^2 + {g'}^2)^{1/2} \bar{f}\,[(1-\gamma_5)a_L + (1+\gamma_5)a_R]\,\gamma_\mu f Z^\mu$$

where $a_L = (I_3 - q\sin^2\theta_W)$ and $a_R = q\sin^2\theta_W$

Z coupling is flavor diagonal: absence of flavor changing neutral currents (FCNC) at tree level! (many good BSM models have been killed because of this)

Parameters of the Standard Model

Gauge field sector: g_1, g_2 and g_3

Higgs sector: v and λ

Matter sector: Masses of quarks and leptons: 9

(but actually from Higgs) Quark mixing: 3 angles

Phase (CP violation): 1

Strong CP violation: 1

Total: 19 parameters

Summary of Standard Model

$$\mathcal{L}_{\mathrm{SM}} = \mathcal{L}_{\mathrm{gauge}} + \mathcal{L}_{\mathrm{fermion}} + \mathcal{L}_{\mathrm{Yukawa}} + \mathcal{L}_{\mathrm{Higgs}}$$

Summary of Standard Model