QCD and Big Bang Nucleosynthesis

New Horizons in Lattice Field Theory

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International Institute of Physics

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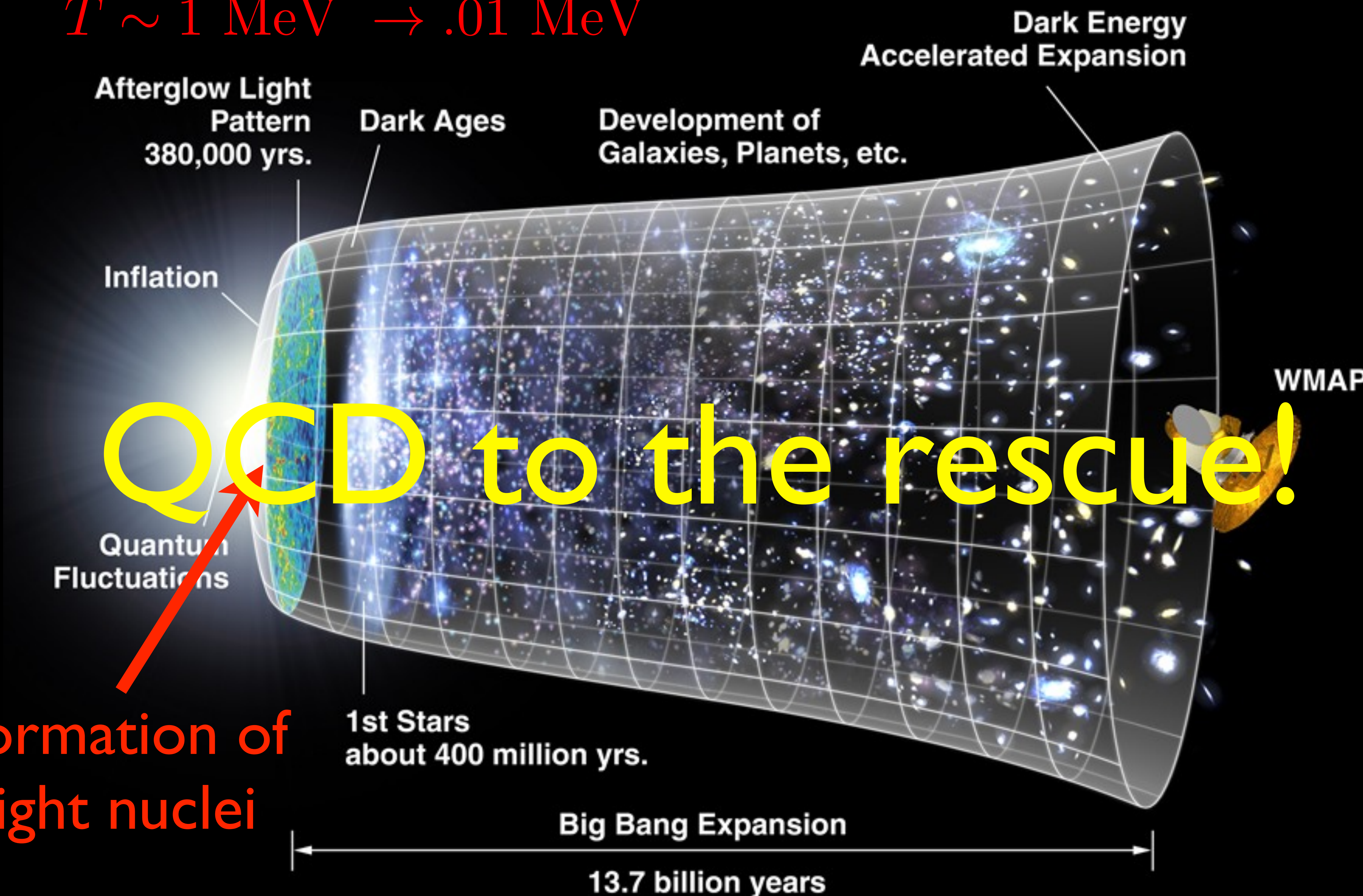
$T \sim 10$ billion $K \to .1$ billion K $\qquad$ $t \sim 1$ sec $\to 15$ min

$T \sim 1$ MeV $\to .01$ MeV

QCD to the rescue!



Formation of light nuclei

Big Bang Nucleosynthesis (BBN)

- What is Big Bang Nucleosynthesis?

  Describes our understanding of the evolution of the early universe from a time approximately one second after the Big Bang to approximately 15 minutes after the Big Bang.

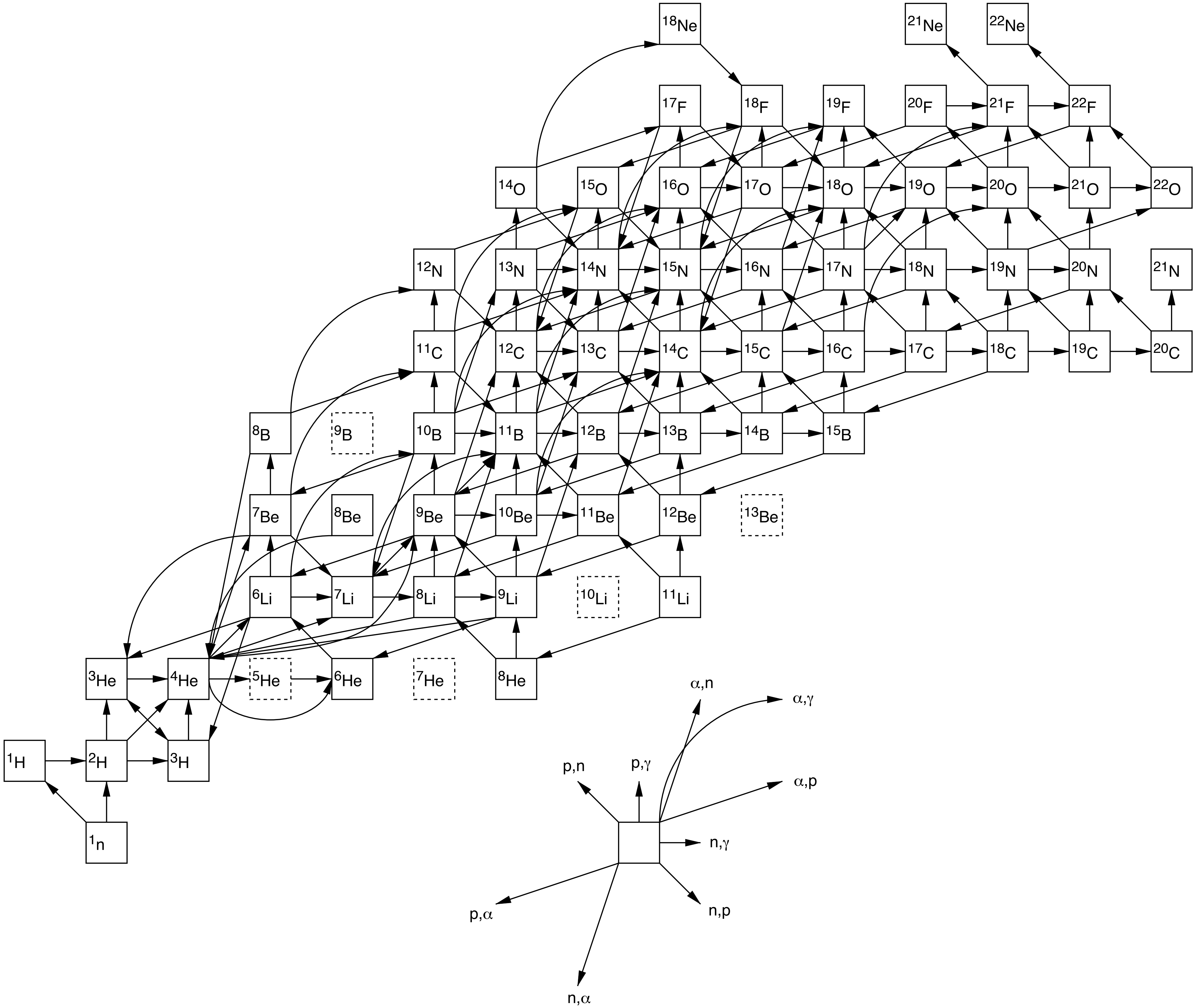
  At this time, the only relevant degrees of freedom in the universe are protons, neutrons, electrons and photons

  A chain of coupled nuclear reactions produces the primordial abundance of light nuclei H, D, ^{3}He, ^{4}He, ^{7}Li

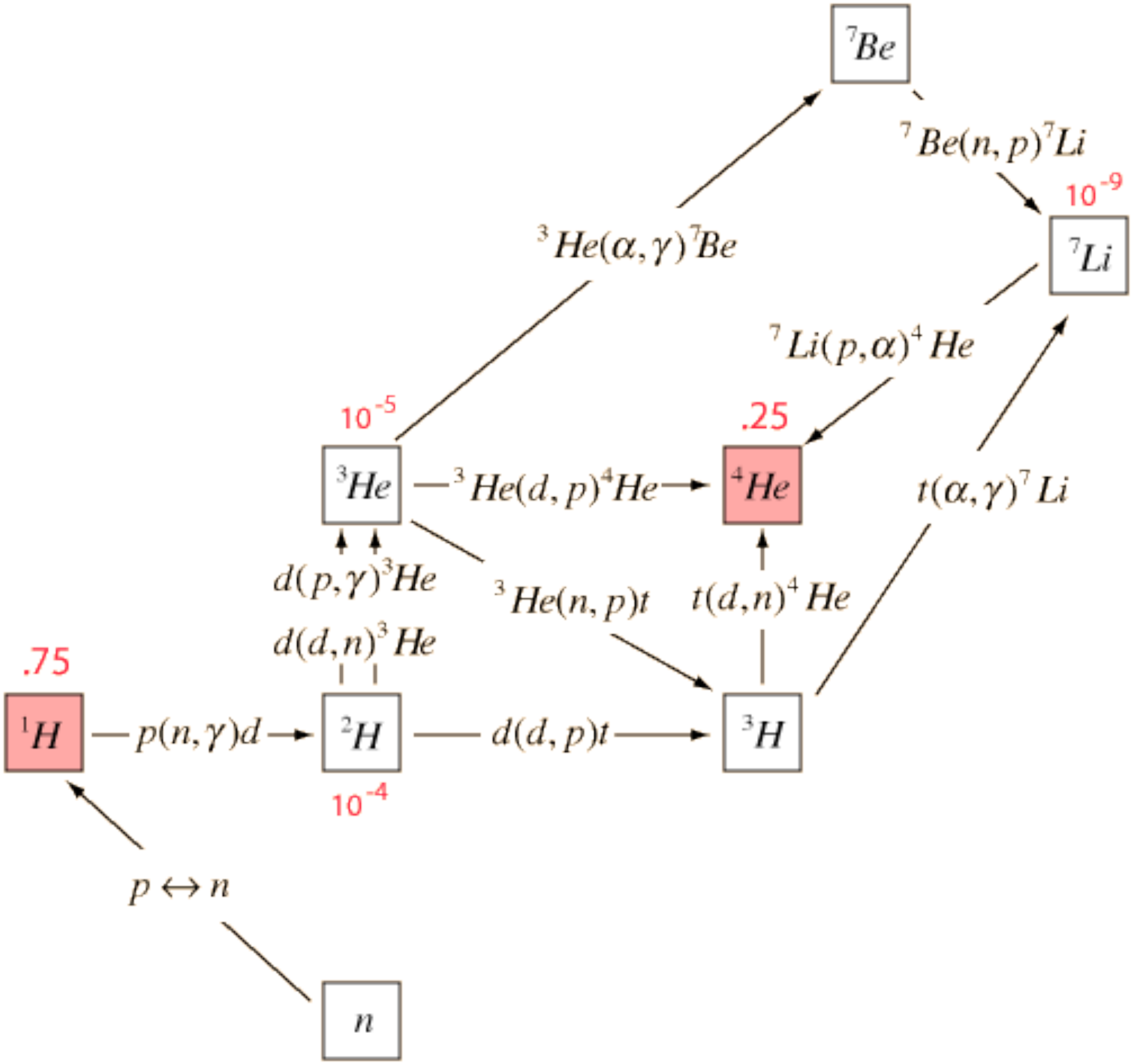
  Given the measured nuclear reactions, the only input/output to our understanding of BBN is the primordial ratio of baryons to photons

$$\eta \equiv \frac{X_N}{X_\gamma}$$

Big Bang Nucleosynthesis (BBN)



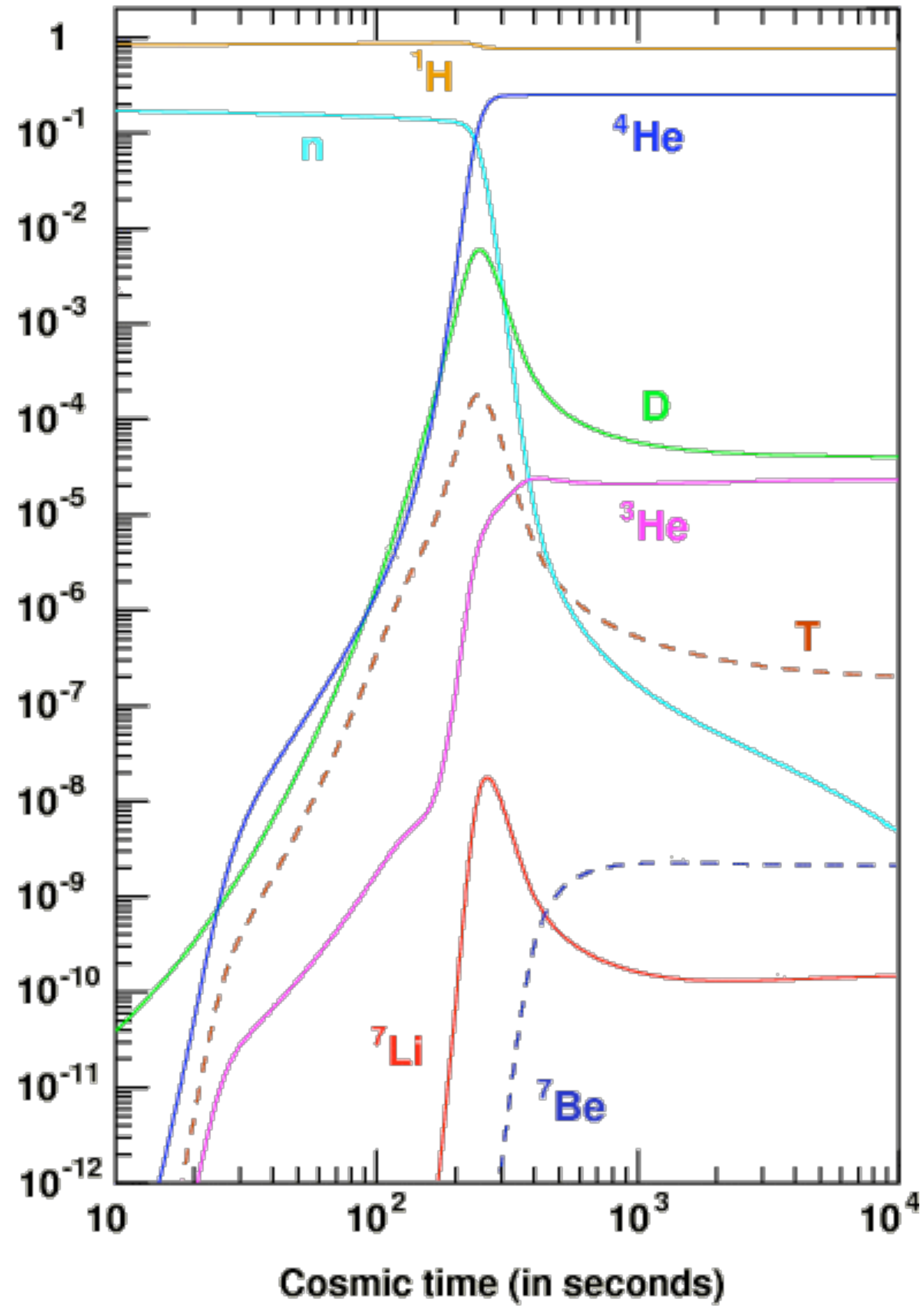
Big Bang Nucleosynthesis (BBN)



Big Bang Nucleosynthesis (BBN)

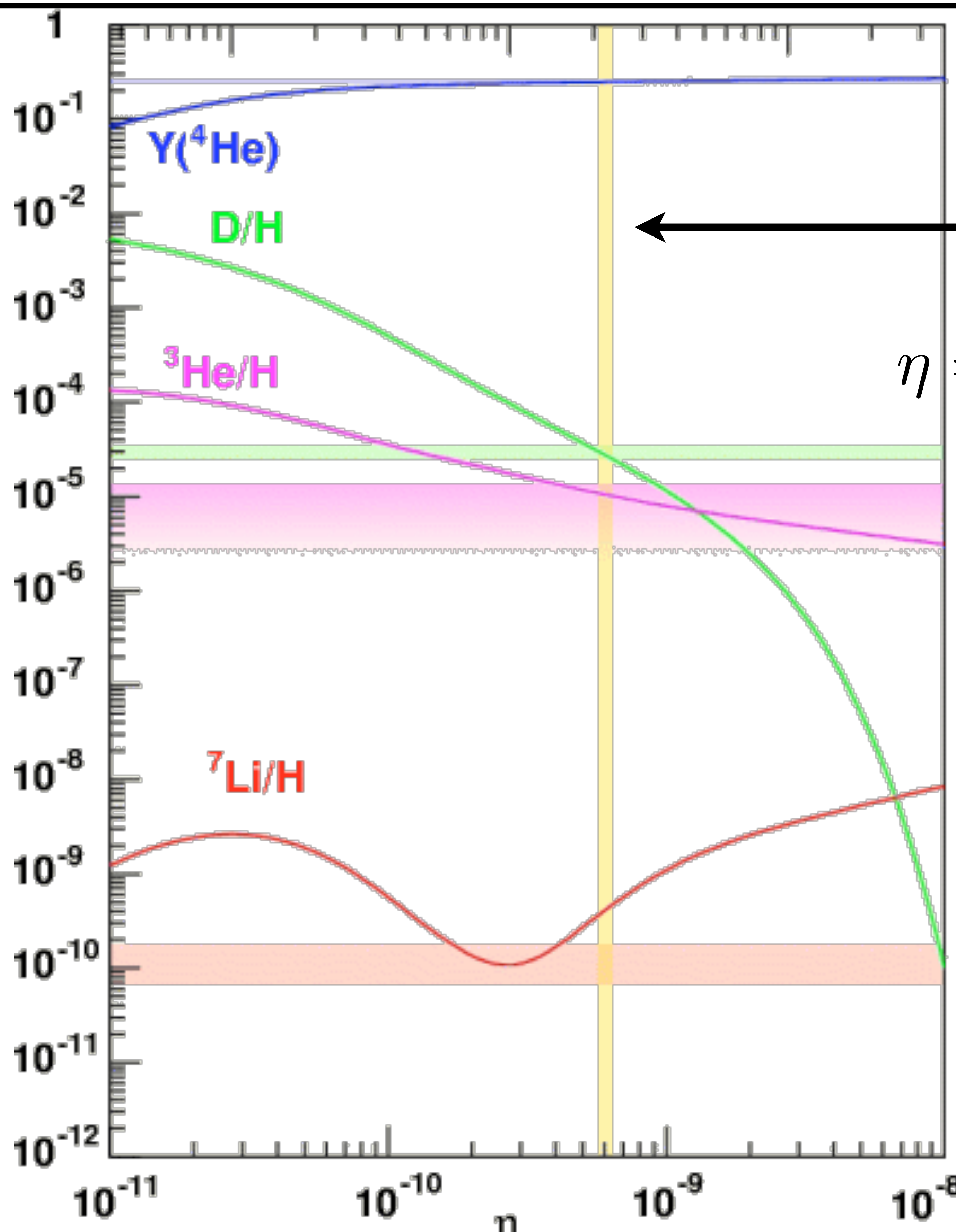
Abundance of light nuclear elements versus cosmic time after Big Bang.

Something special is happening around 3 min.



Big Bang Nucleosynthesis (BBN)

~75% H
~25% ^{4}He



CMB

$$\eta = 6.19(15) \times 10^{-10}$$

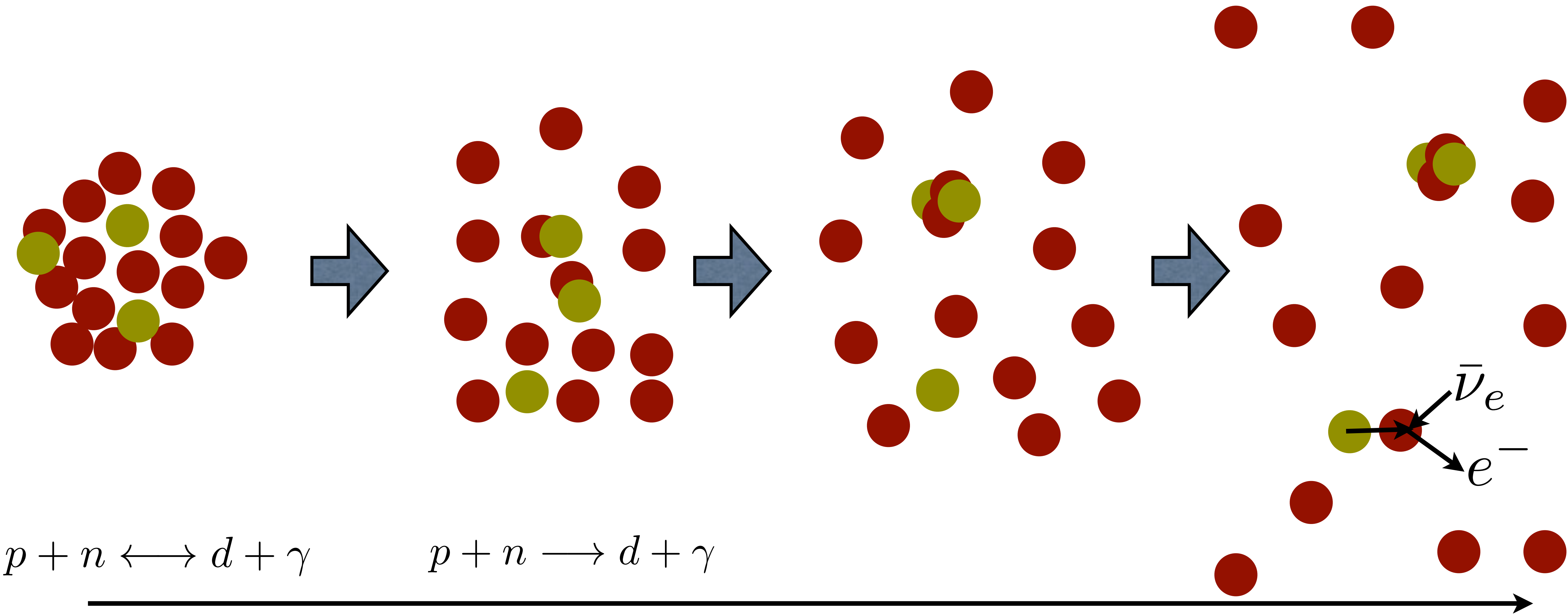
$$\eta \equiv \frac{X_N}{X_\gamma}$$

Big Bang Nucleosynthesis (BBN)

"The violent Universe: the Big Bang"
review by Keith A. Olive
arXiv:1005.3955

The number of active neutrino species in the early universe is an adjustable parameter in BBN. If one changes $n_\nu = 3 \rightarrow \{2, 4\}$ the predicted abundances from BBN no longer match the observed abundances G. Steigman arXiv:0807.3004

Big Bang Nucleosynthesis (BBN)



$t \sim 1$ sec	$t \sim 3$ min	$t \sim 3^+$ min	$t \sim 15$min
$T \sim 1$ MeV	$T \sim 0.1$ MeV	$T \sim 0.1^-$ MeV	$T \sim 0.01$ MeV

$$\frac{X_n}{X_p} = e^{-\frac{M_n - M_p}{T}}$$

Initial conditions

B_d

deuterium binding energy

τ_n

neutron lifetime

Big Bang Nucleosynthesis (BBN)

- M_n - M_p plays an extremely significant role in the evolution of the universe as we know it

Initial conditions for Big Bang Nucleosynthesis (BBN)

$$\frac{X_n}{X_p} = e^{-\frac{M_n - M_p}{T}}$$

- The neutron lifetime is highly sensitive to the value of this mass splitting

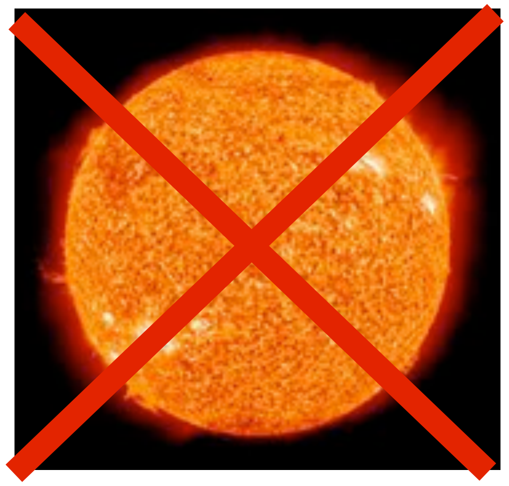
$$\frac{1}{\tau_n} = \frac{(G_F \cos\theta_C)^2}{2\pi^3} m_e^5 (1 + 3g_A^2)\, f\left(\frac{M_n - M_p}{m_e}\right)$$

Point Nucleons $f(a) \simeq \frac{1}{15}\left(2a^4 - 9a^2 - 8\right)\sqrt{a^2 - 1} + a \ln\left(a + \sqrt{a^2 - 1}\right)$

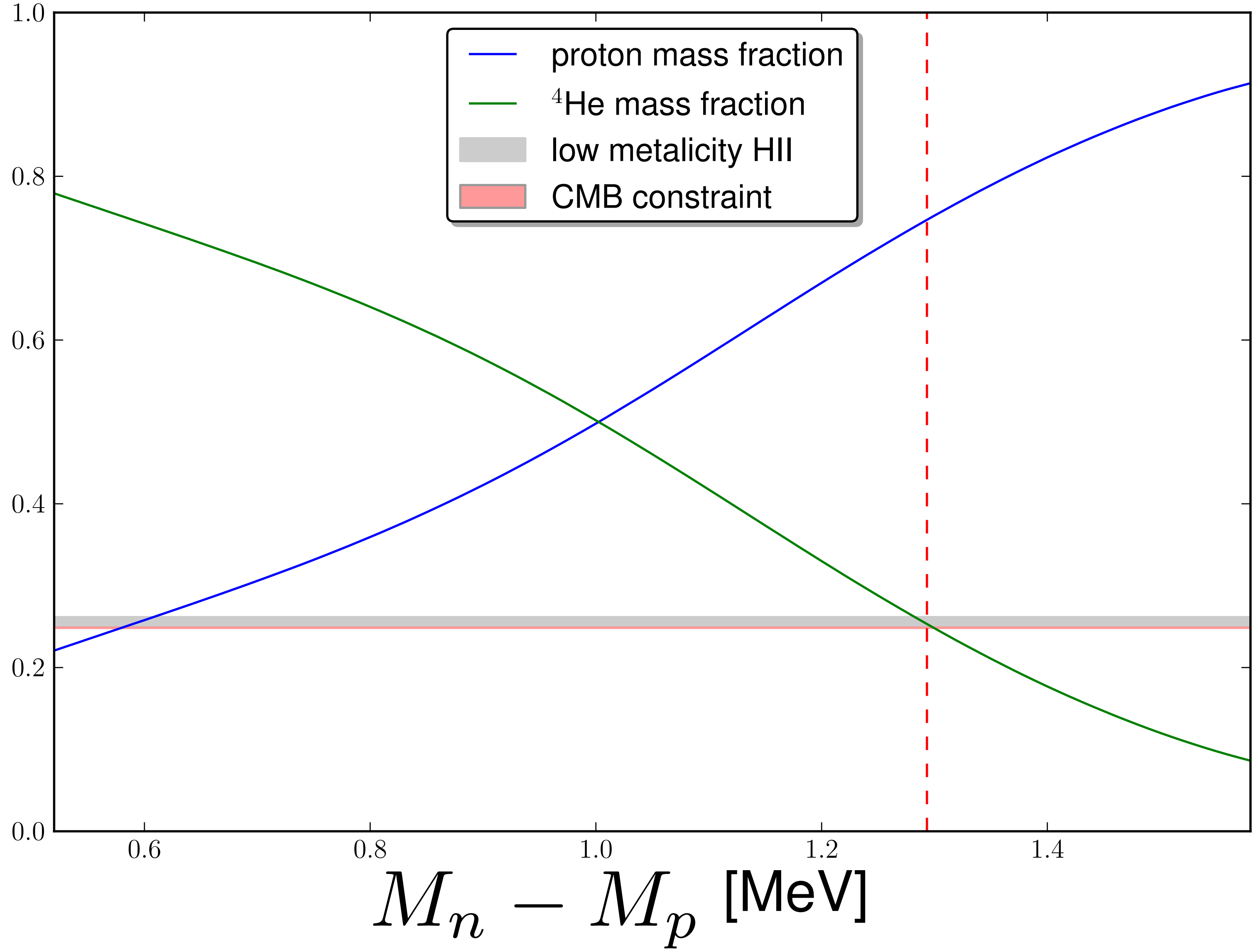
Griffiths "Introduction to Elementary Particles"

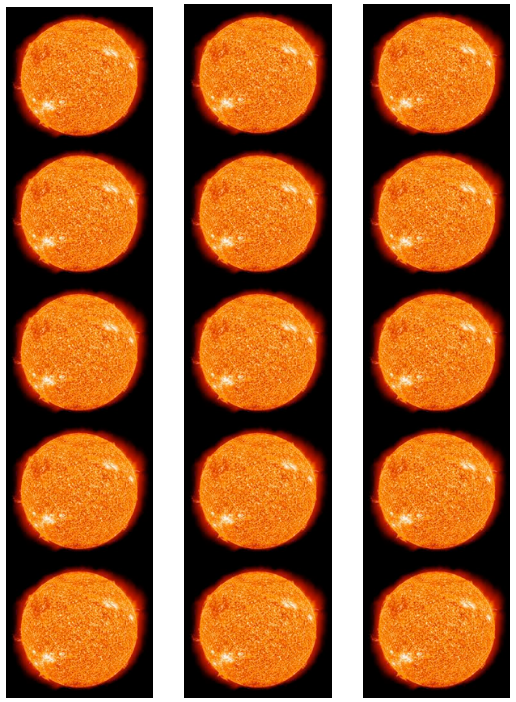
10% change in $M_n - M_p$ corresponds to ~100% change neutron lifetime

Big Bang Nucleosynthesis (BBN)



No Sun!





Too many suns?

M_n - M_p

- Nature: $M_n - M_p = 1.29333217(42)\ \mathrm{MeV}$ CODATA
- Standard Model has two sources of isospin breaking

$$\hat{Q} = \frac{1}{6}\mathbb{1} + \frac{1}{2}\tau_3 \qquad m_q = \hat{m}\mathbb{1} - \delta\tau_3$$

- Given only electro-static forces, one would predict

$$M_p > M_n$$

- The contribution from $m_d - m_u$ is comparable in size but opposite in sign

M_n - M_p

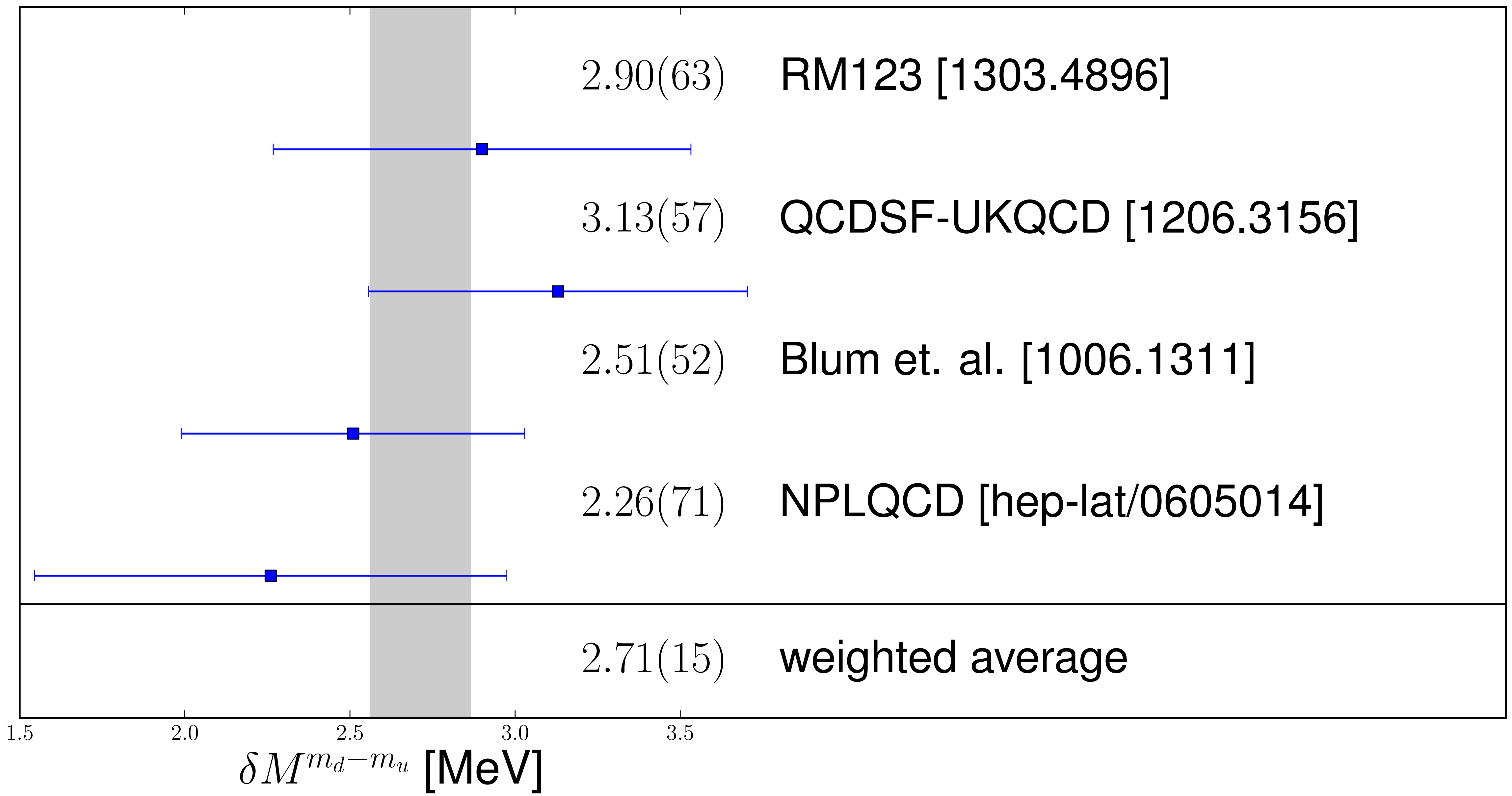
- We would like to understand the Neutron-Proton mass splitting from first principles
- $M_n - M_p = \delta M^\gamma + \delta M^{m_d - m_u}$ Separation only valid at LO in isospin breaking
- $\delta M^{m_d - m_u}$ Well understood from lattice QCD

  δM^γ Disparate scales relevant for QCD and QED make this a very challenging problem to solve with LQCD: large systematic uncertainties
- Alternative means to determine δM^γ

  Cottingham Formulation

M_n - M_p

- $\delta M^{m_d - m_u}_{LQCD} = 2.71(15)$ MeV



M_n - M_p

- $\delta M_{LQCD}^{m_d - m_u} = 2.71(15)\ \text{MeV}$
- $\delta M^{\gamma} = -0.76(30)\ \text{MeV}$

Gasser & Leutwyler
Nucl. Phys. B94 (1975)
Phys. Rept. 87 (1982) "Quark Masses"

central value from elastic contribution

uncertainty from estimates of inelastic contributions

- Experiment & lattice QCD

$$\delta M_{n-p}^{phys} - \delta M_{LQCD}^{m_d - m_u} = -1.42(15)\ \text{MeV}$$

Can we improve our understanding of these contributions?

Of course!

Electromagnetic Self Energy: Cottingham Formula

$$\delta M^{\gamma}_{p-n} = \alpha_{f.s.} \times f_{p-n}(QCD, QED)$$

Electromagnetic Self Energy: Cottingham Formula

Walker-Loud, Carlson, Miller PRL 108 (2012) [arXiv:1203.0254]

$$\delta M^{\gamma} = \delta M^{el} + \delta M^{inel} + \delta M^{sub} + \delta M^{ct}$$

δM^{el}	δM^{inel}	δM^{sub}	δM^{ct}
elastic	inelastic	unknown subtraction	counter-term renormalization
↑	↑	↑	↑
precisely determined	newly determined (precisely)	newly determined (imprecisely)	determined by J.C. Collins

$$\delta M^{\gamma}_{p-n} = 1.30(03)(47) \text{ MeV}$$

~~$\delta M^{\gamma}_{p-n} = 0.76(30)$ MeV~~ Gasser & Leutwyler

$$\delta M^{phys}_{p-n} - \delta M^{m_d - m_u}_{LQCD} = 1.42(15) \text{ MeV}$$ Experiment & LQCD

Electromagnetic Self Energy: Cottingham Formula

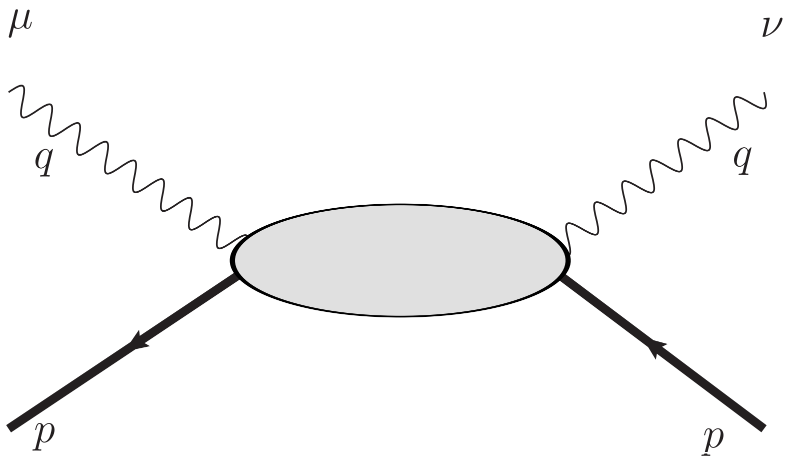
- Updating G&L result uncovered a "technical oversight"
  - The application of the Cottingham Formula requires the use of a subtracted dispersion integral.
  - Gasser & Leutwyler had an argument to evade the unknown subtraction function.
  - The argument was based on incorrect assumptions about QCD scaling violations predicted by the parton model
  - this has gone (mostly) unnoticed since 1982

Electromagnetic Self Energy: Cottingham Formula

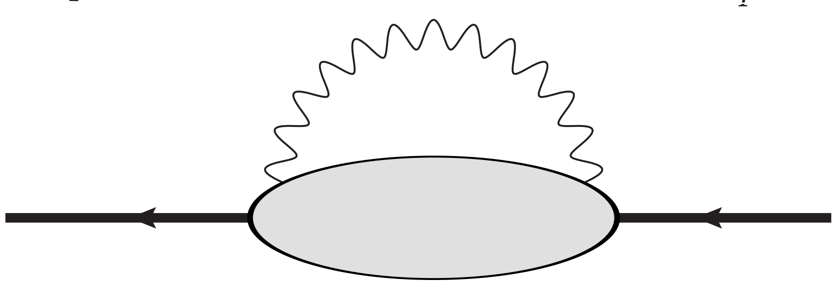
electromagnetic correction

determined from Compton Scattering

- Cini, Ferrari, Gato PRL 2 (1959)
- Cottingham Annals Phys 25 (1963)
- Gasser, Leutwyler Nucl. Phys. B94 (1975)
- Collins Nucl. Phys. B149 (1979)
- Gasser, Leutwyler Phys. Rept 87 (1982)
- AWL, C.Carlson, G.Miller PRL 108 (2012)



$$T_{\mu\nu} = \frac{i}{2}\sum_{\sigma}\int d^4\xi \ e^{iq\cdot\xi}\langle p\sigma|T\left\{J_\mu(\xi)J_\nu(0)\right\}|p\sigma\rangle$$



$$\delta M^{\gamma} = \frac{i}{2M}\frac{\alpha}{(2\pi)^3}\int_R d^4q \frac{T^{\mu}_{\mu}(p,q)}{q^2+i\epsilon}$$

$$\alpha = \frac{e^2}{4\pi}$$

Integral diverges and must be renormalized

Electromagnetic Self Energy: Cottingham Formula

electromagnetic correction

Cini, Ferrari, Gato	PRL 2 (1959)
Cottingham	Annals Phys 25 (1963)
Gasser, Leutwyler	Nucl. Phys. B94 (1975)
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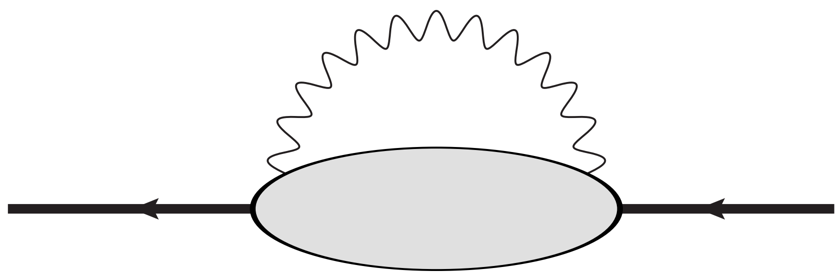


$$\delta M^\gamma = \frac{i}{2M}\frac{\alpha}{(2\pi)^3}\int_R d^4q \frac{T^\mu_\mu(p,q)}{q^2+i\epsilon}$$

- Wick rotate $q^0 \to i\nu$ variable transform $Q^2 = \mathbf{q}^2 + \nu^2$

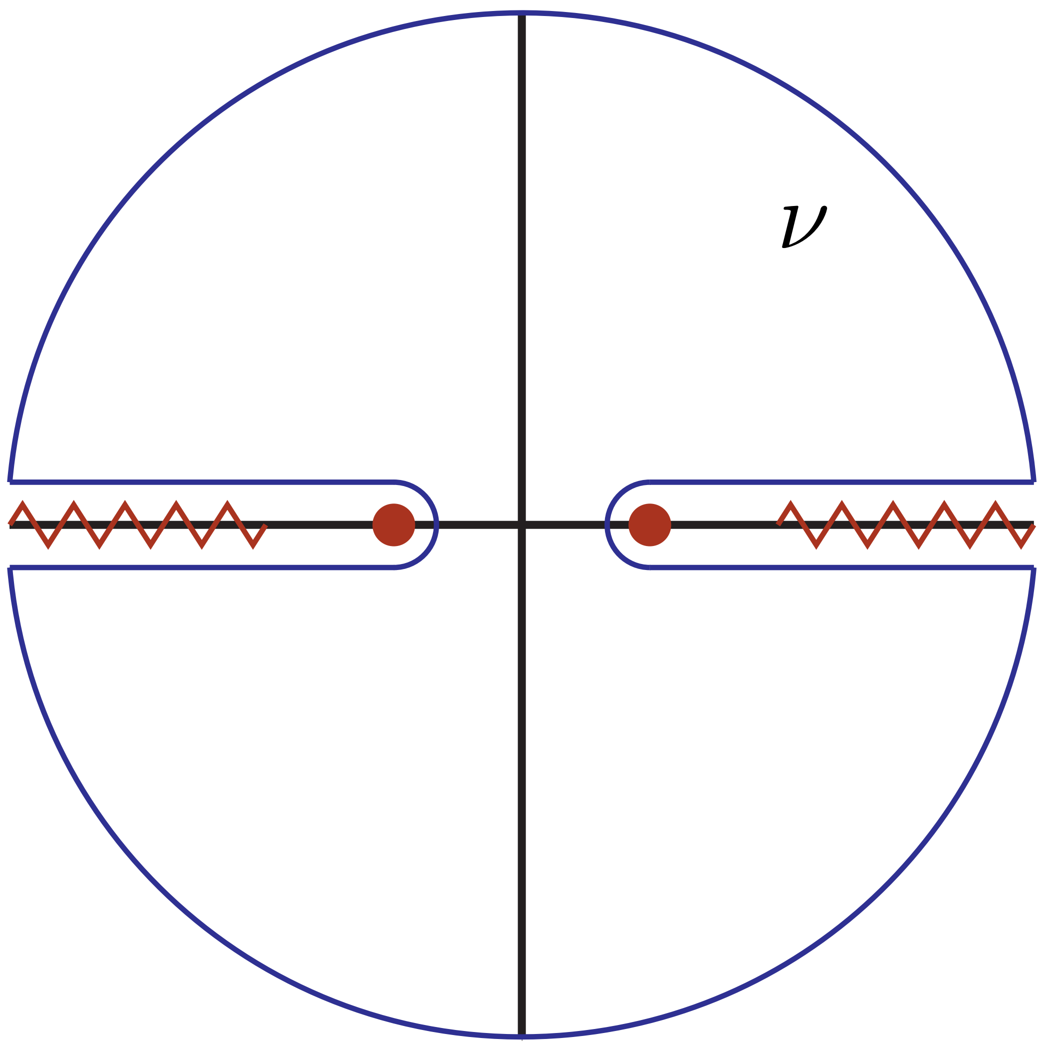
$$\delta M^\gamma = \frac{\alpha}{8\pi^2}\int_0^{\Lambda^2} dQ^2 \int_{-Q}^{+Q} d\nu \frac{\sqrt{Q^2-\nu^2}}{Q^2}\frac{T^\mu_\mu}{M} + \delta M^{ct}(\Lambda)$$

$$T^\mu_\mu = -3\,T_1(i\nu,Q^2) + \left(1-\frac{\nu^2}{Q^2}\right)T_2(i\nu,Q^2)\,, \qquad (7a)$$

$$= -3Q^2\,t_1(i\nu,Q^2) + \left(1+2\frac{\nu^2}{Q^2}\right)Q^2 t_2(i\nu,Q^2)\,. \qquad (7b)$$

use dispersion integrals to evaluate scalar functions $T_{1,2}(i\nu,Q^2)$ $[t_{1,2}(i\nu,Q^2)]$

Electromagnetic Self Energy: Cottingham Formula



dispersion integral = Cauchy contour integral

$$T_i(\nu, Q^2) = \frac{1}{2\pi} \oint d\nu' \frac{T_i(\nu', Q^2)}{\nu' - \nu}$$

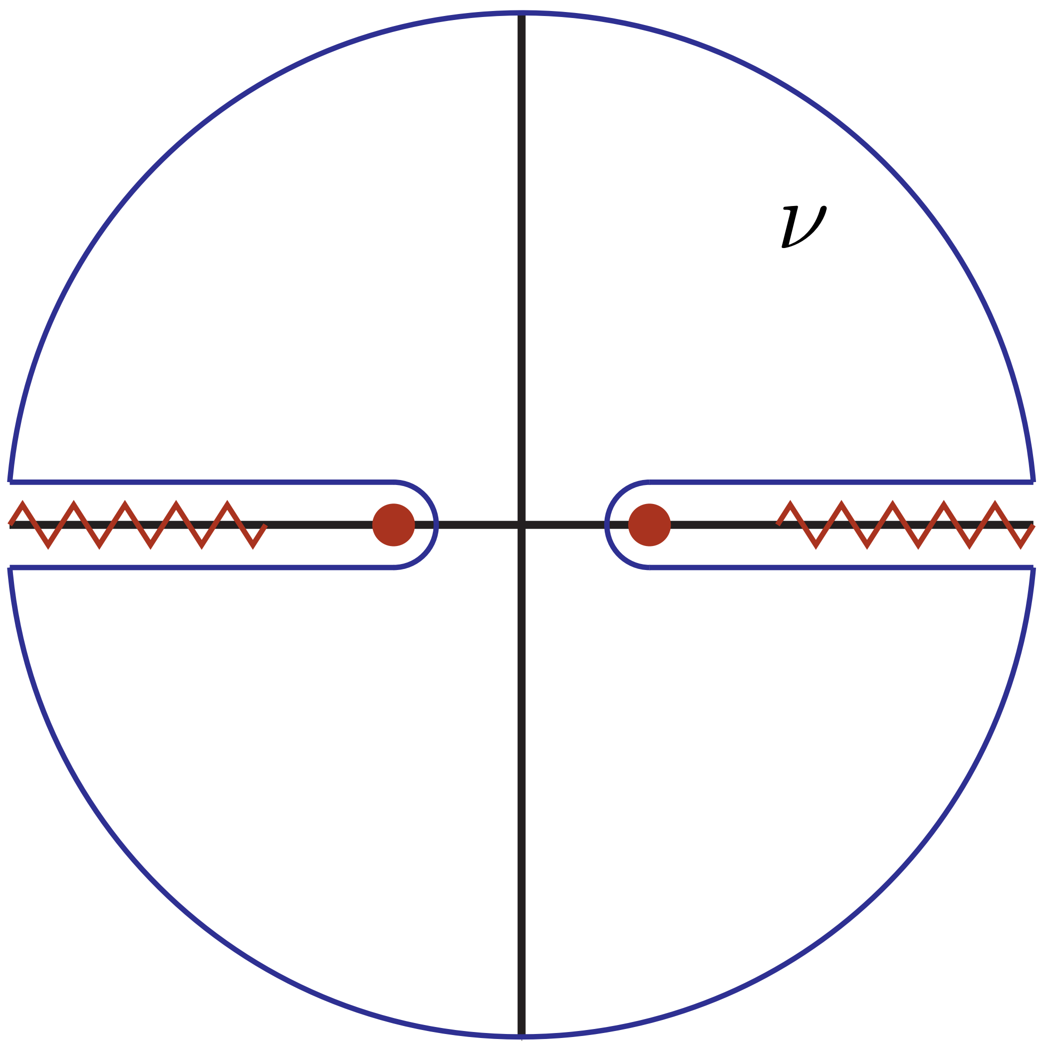
Crossing Symmetric

$$T_i(\nu, Q^2) = T_i(-\nu, Q^2)$$

$$T_i(\nu, Q^2) = \frac{1}{2\pi} \int_{\nu_t}^{\infty} d\nu' \frac{2\nu'}{(\nu')^2 - \nu^2} 2\mathrm{Im} T_i(\nu' + i\epsilon, Q^2)$$

(provided contour and infinity vanishes)

Electromagnetic Self Energy: Cottingham Formula



if contour at infinity does not vanish

subtracted dispersion integral

$$g(\nu) = \frac{T_i(\nu, Q^2)}{\nu^2}$$

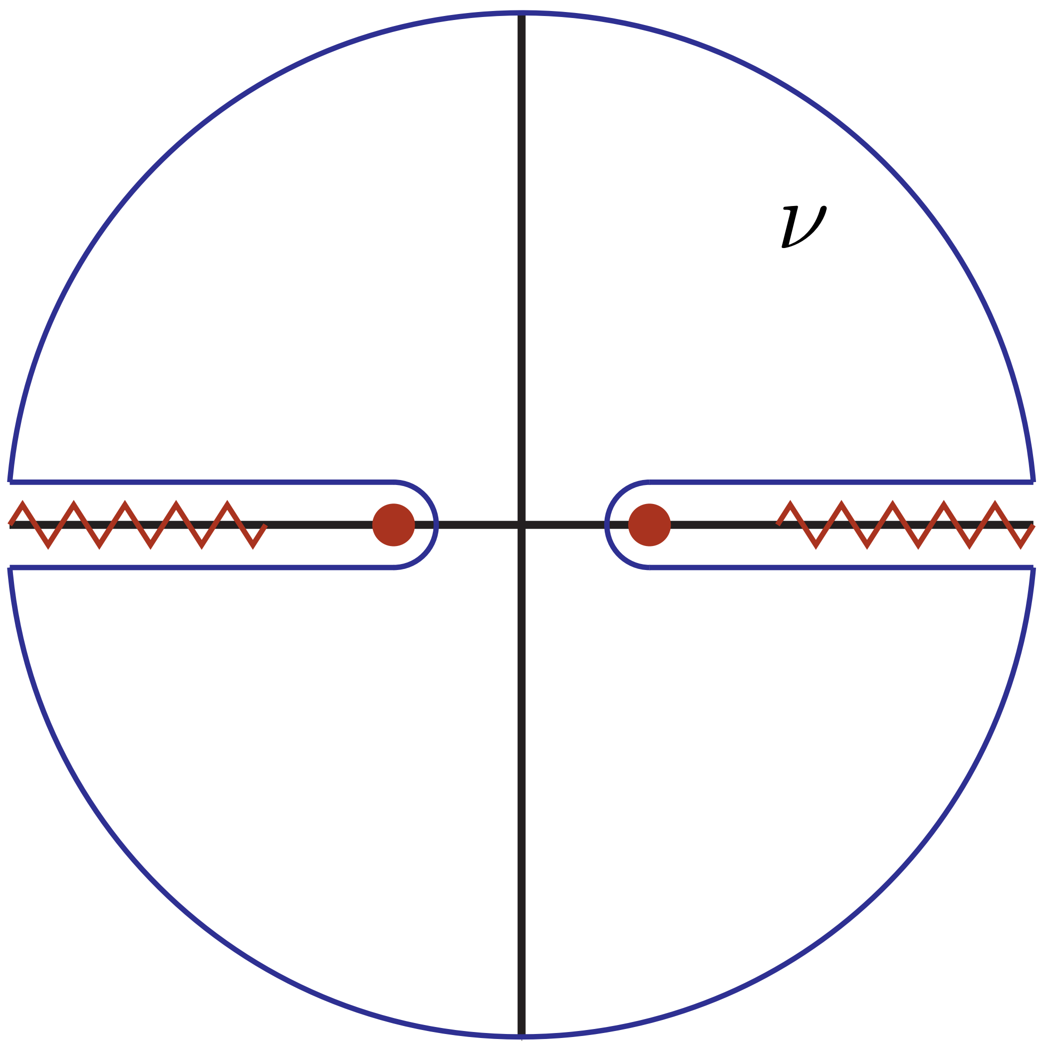
introduces new pole at $\nu = 0$
which you need to subtract

$$T_i(\nu, Q^2) = \frac{\nu^2}{2\pi} \int_{\nu_t}^{\infty} d\nu' \frac{2\nu'}{\nu'^2(\nu'^2 - \nu^2)} 2\mathrm{Im}T_i(\nu' + i\epsilon, Q^2) + T_i(0, Q^2)$$

measured experimentally

unknown function

Electromagnetic Self Energy: Cottingham Formula



It is known that

$$T_2(\nu, Q^2) \quad [t_2(\nu, Q^2)]$$

satisfies unsubtracted dispersion integral while

$$T_1(\nu, Q^2) \quad [t_1(\nu, Q^2)]$$

requires a subtraction

Regge behavior

$$\mathrm{Im} t_1 [T_1] \Big|_{p-n} \propto \nu^{1/2}$$

H. Harari: PRL 17 (1966)

H.D. Abarbanel S. Nussinov: Phys.Rev. 158 (1967)

Electromagnetic Self Energy: Cottingham Formula

Gasser and Leutwyler: Nucl Phys B94 (1975), Phys. Rept. 87 (1982)

at the time, introducing an unknown subtraction function would be disastrous for getting a precise value:

they provided an argument based upon the parton model to avoid the subtracted dispersive integral

$$\delta M^{\gamma}_{p-n} = 0.76(30)\ \mathrm{MeV}$$

central value: from elastic contribution

uncertainty: estimates of inelastic structure contributions

however, one can show assumptions they relied upon do not hold

one must face the subtraction function

Electromagnetic Self Energy: Cottingham Formula

Gasser and Leutwyler: Nucl Phys B94 (1975), Phys. Rept. 87 (1982)

how does the assumption break down?

$$\delta M^\gamma = \frac{\alpha}{8\pi^2}\int_0^{\Lambda^2} dQ^2 \int_{-Q}^{+Q} d\nu \frac{\sqrt{Q^2-\nu^2}}{Q^2}\frac{T_\mu^\mu}{M} + \delta M^{ct}(\Lambda)$$

$$T_\mu^\mu = -3\, T_1(i\nu, Q^2) + \left(1 - \frac{\nu^2}{Q^2}\right) T_2(i\nu, Q^2)\,, \qquad (7a)$$

$$= -3Q^2\, t_1(i\nu, Q^2) + \left(1 + 2\frac{\nu^2}{Q^2}\right) Q^2 t_2(i\nu, Q^2)\,. \quad (7b)$$

is there some motivation to pick t_i vs T_i ?

Electromagnetic Self Energy: Cottingham Formula

Gasser and Leutwyler: Nucl Phys B94 (1975), Phys. Rept. 87 (1982)

how does the assumption break down?

in the point limit (electron) $t_1(\nu, Q^2) = 0!$

for the nucleon (with motivated resummations) the elastic contribution is

$$t_1(\nu, Q^2) = \frac{2}{Q^2}\left[\frac{Q^4 \frac{G_M^2 - G_E^2}{1+\tau}}{(Q^2 - i\epsilon)^2 - 4M^2\nu^2} - \underbrace{\left(F_1^2 - \frac{G_E^2 + \tau G_M^2}{1+\tau}\right)}\right]$$

$$\tau = \frac{Q^2}{4M^2}$$

"Fixed-Pole" missed by unsubtracted dispersion relation

Electromagnetic Self Energy: Cottingham Formula

Gasser and Leutwyler: Nucl Phys B94 (1975), Phys. Rept. 87 (1982)

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$$\tau = \frac{Q^2}{4M^2}$$

numerically, this term is negligible

Electromagnetic Self Energy: Cottingham Formula

Gasser and Leutwyler: Nucl Phys B94 (1975), Phys. Rept. 87 (1982)

how does the assumption break down?

in the point limit (electron) $t_1(\nu, Q^2) = 0!$

real problem comes in the Regge limit: Q^2 fixed, $\nu \to \infty$

$$\mathrm{Im}t_1(\nu, Q^2) = \frac{\pi M \nu}{Q^4}\left[2xF_1(x, Q^2) - F_2(x, Q^2)\right] \qquad x = \frac{Q^2}{2M\nu}$$

in the strict DIS limit: Callan-Gross relation

$$2xF_1(x) - F_2(x) = 0$$

Gasser and Leutwyler relied on the assumption

Parton Model

$$2xF_1(x, Q^2) - F_2(x, Q^2) = \frac{H_1(x)}{\nu}$$

if this were true, their argument would go through, however...

Electromagnetic Self Energy: Cottingham Formula

Gasser and Leutwyler: Nucl Phys B94 (1975), Phys. Rept. 87 (1982)

how does the assumption break down?

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Zee, Wilczek and Treiman Phys.Rev. D10 (1974)

$$2xF_1(x) - F_2(x) = \frac{-32}{9}\frac{\alpha_s(Q^2)}{4\pi}F_2(x) \qquad \text{QCD}$$

This criticism first given by J.C. Collins: Nucl. Phys. B149 (1979)

Electromagnetic Self Energy: Cottingham Formula

Gasser and Leutwyler: Nucl Phys B94 (1975), Phys. Rept. 87 (1982)

how does the assumption break down?

in the point limit (electron) $t_1(\nu, Q^2) = 0!$

real problem comes in the Regge limit: $Q^2 \text{ fixed}, \nu \to \infty$

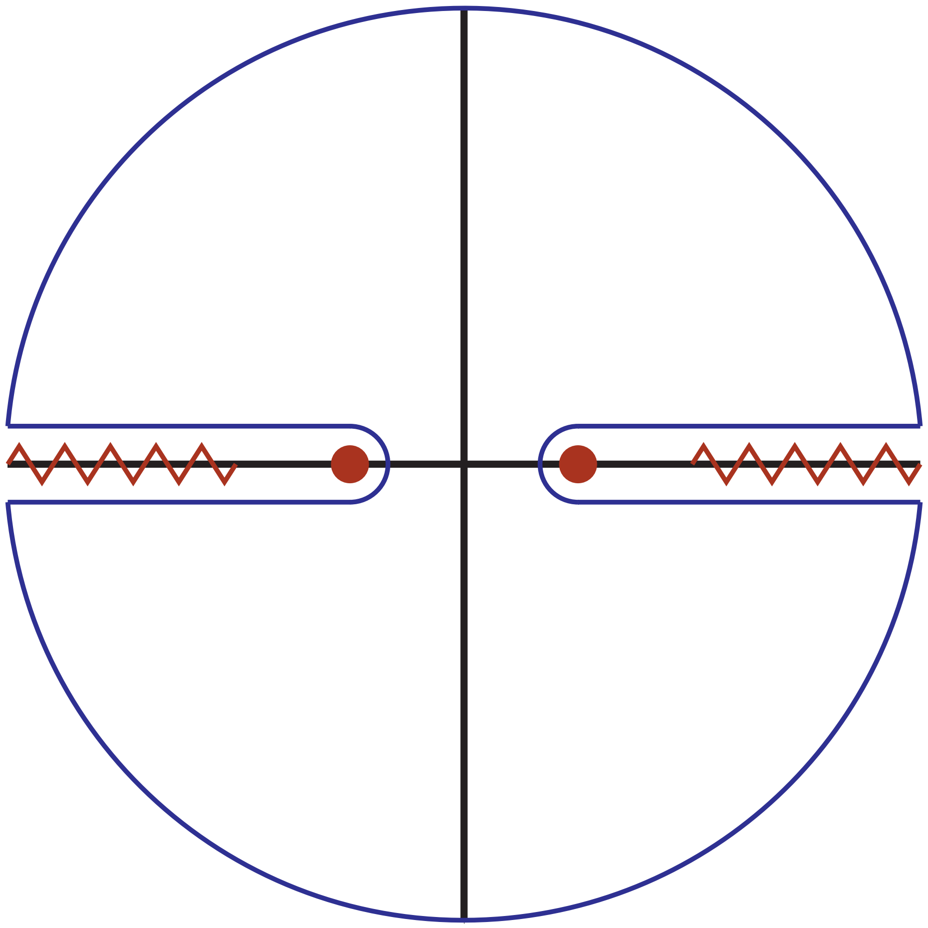
$$\lim_{x\to 0} F_2^{p-n}(x) \propto x^{1/2}$$

$$x = \frac{Q^2}{2M\nu}$$

$$\text{Im} t_1^{p-n}(\nu, Q^2) \propto \alpha_s(Q^2) \frac{\sqrt{M\nu}}{Q^3}$$

Electromagnetic Self Energy: Cottingham Formula

$$t_1(\nu, Q^2) = \frac{1}{2\pi} \int_{\nu_t}^{\infty} 2\nu' d\nu' \frac{2\mathrm{Im}t_1(\nu' + i\epsilon, Q^2)}{(\nu')^2 - \nu^2}$$

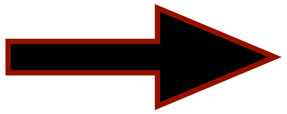


Regge Limit

fixed Q^2

$\nu \to \infty$

$$\mathrm{Im}t_1^{p-n}(\nu, Q^2) \propto \alpha_s(Q^2) \frac{\sqrt{M\nu}}{Q^3}$$



integrand at infinite contour scales as $1/\sqrt{\nu}$

subtracted dispersion integral is unavoidable

Electromagnetic Self Energy: Cottingham Formula

evaluation of various contributions

Electromagnetic Self Energy: Cottingham Formula

perform once subtracted dispersion integral for $T_1(t_1)$
and unsubtracted dispersion integral for $T_2(t_2)$

$$\delta M^{\gamma} = \delta M^{el} + \delta M^{inel} + \delta M^{sub} + \delta\tilde{M}^{ct}$$

$$\delta M^{el} = \frac{\alpha}{\pi}\int_0^{\Lambda_0^2} dQ \left\{ \frac{3\sqrt{\tau_{el}}G_M^2}{2(1+\tau_{el})} + \frac{\left[G_E^2 - 2\tau_{el}\, G_M^2\right]}{1+\tau_{el}} \left[(1+\tau_{el})^{3/2} - {\tau_{el}}^{3/2} - \frac{3}{2}\sqrt{\tau_{el}} \right] \right\}$$

$$\delta M^{inel} = \frac{\alpha}{\pi}\int_0^{\Lambda_0^2} \frac{dQ^2}{2Q} \int_{\nu_{th}}^{\infty} d\nu \left\{ \frac{3F_1(\nu,Q^2)}{M} \left[\frac{\tau^{3/2} - \tau\sqrt{1+\tau} + \sqrt{\tau}/2}{\tau} \right] + \frac{F_2(\nu,Q^2)}{\nu} \left[(1+\tau)^{3/2} - \tau^{3/2} - \frac{3}{2}\sqrt{\tau} \right] \right\},$$

$$\tau_{el} = \frac{Q^2}{4M^2}$$

$$\tau = \frac{\nu^2}{Q^2}$$

$$\delta M^{sub} = -\frac{3\alpha}{16\pi M}\int_0^{\Lambda_0^2} dQ^2\ T_1(0,Q^2)\,,$$

$$\delta\tilde{M}^{ct} = -\frac{3\alpha}{16\pi M}\int_{\Lambda_0^2}^{\Lambda_1^2} dQ^2 \sum_i C_{1,i}\langle \mathcal{O}^{i,0}\rangle\,,$$

OPE: operators and Wilson coeffic.
J.C. Collins: Nucl. Phys. B149 (1979)

Electromagnetic Self Energy: Cottingham Formula

elastic contribution: use well measured form factors

$$\delta M^{el} = \frac{\alpha}{\pi}\int_0^{\Lambda_0^2} dQ \left\{ \frac{3\sqrt{\tau_{el}} G_M^2}{2(1+\tau_{el})} + \frac{\left[G_E^2 - 2\tau_{el}\, G_M^2\right]}{1+\tau_{el}} \left[(1+\tau_{el})^{3/2} - {\tau_{el}}^{3/2} - \frac{3}{2}\sqrt{\tau_{el}} \right] \right\}$$

$$\delta M^{el}\Big|_{p-n} = 1.39(02)\ \text{MeV}$$

- insensitive to value of Λ_0 since form factors fall as $1/Q^4$
- uncertainty from Monte Carlo evaluation of parameters describing form factors

central values: $\Lambda_0^2 = 2\ \text{GeV}^2$

uncertainties: $1.5\ \text{GeV}^2 \leq \Lambda_0^2 \leq 2.5\ \text{GeV}^2$

Electromagnetic Self Energy: Cottingham Formula

inelastic terms: use modern knowledge of structure functions to improve determination of inelastic contributions

$$\delta M^{inel} = \frac{\alpha}{4\pi M} \int_0^{\Lambda_0^2} \frac{dQ^2}{Q} \int_{W_{th}^2}^{\infty} dW^2 \left\{ \frac{3F_1(\nu, Q^2)}{M} \left[\frac{\tau^{3/2} - \tau\sqrt{1+\tau} + \sqrt{\tau}/2}{\tau} \right] + \frac{F_2(\nu, Q^2)}{\nu} \left[(1+\tau)^{3/2} - \tau^{3/2} - \frac{3}{2}\sqrt{\tau} \right] \right\}$$

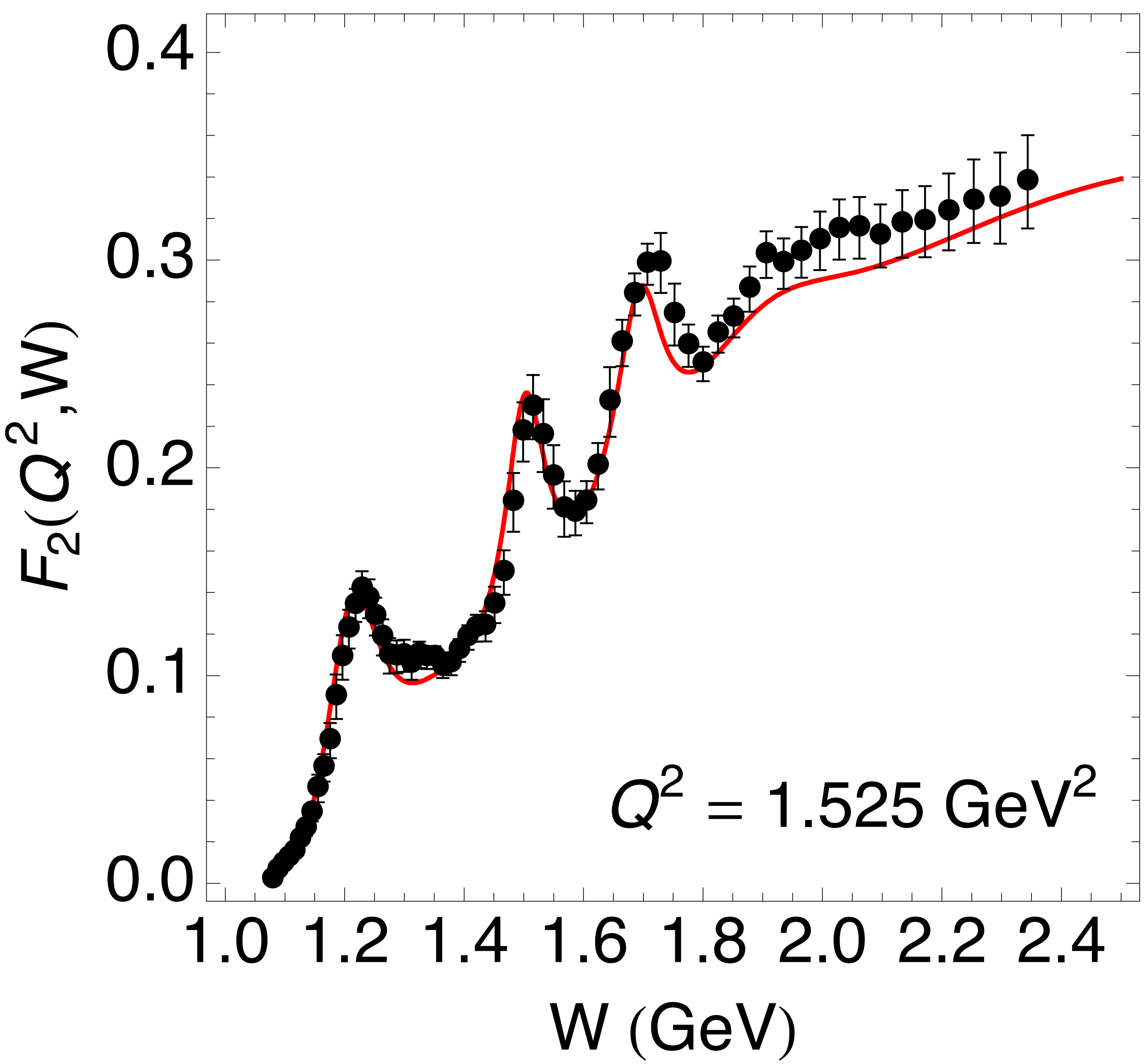
$$\delta M^{inel}\Big|_{p-n} = 0.057(16) \text{ MeV}$$

- contributions from two regions:
  - resonance region — Bosted and Christy: Phys.Rev. C77, C81
  - scaling region — Capella et al: PLB 337
  - Sibirtsev et al: Phys. Rev. D82
- uncertainty dominated by choice of transition between two regions

Electromagnetic Self Energy: Cottingham Formula

inelastic terms: use modern knowledge of structure functions to improve determination of inelastic contributions

$$\delta M^{inel} = \frac{\alpha}{4\pi M}\int_0^{\Lambda_0^2}\frac{dQ^2}{Q}\int_{W_{th}^2}^{\infty} dW^2 \left\{\frac{3F_1(\nu,Q^2)}{M}\left[\frac{\tau^{3/2}-\tau\sqrt{1+\tau}+\sqrt{\tau}/2}{\tau}\right] + \frac{F_2(\nu,Q^2)}{\nu}\left[(1+\tau)^{3/2}-\tau^{3/2}-\frac{3}{2}\sqrt{\tau}\right]\right\}$$



F_2 data from JLAB

resonance fit:

Bosted and Christy: Phys.Rev. C77, C81

$$\tau = \nu^2/Q^2 \qquad W_{th}^2 = (M+m_\pi)^2$$

$$W^2 = M^2 + 2M\nu - Q^2$$

Electromagnetic Self Energy: Cottingham Formula

renormalization: [J.C. Collins Nucl. Phys. B149 (1979)] one can show the contribution from the operator is numerically second order in isospin breaking with Naive Dimensional Analysis and suitable renormalization (dim. reg.)

quark mass operator renormalizes EM self-energy: can not cleanly separate these two contributions (but mixing is higher order in isospin breaking)

Electromagnetic Self Energy: Cottingham Formula

renormalization: [J.C. Collins Nucl. Phys. B149 (1979)]

$$\delta M^{\gamma}_{UV} \sim \frac{3\alpha_{f.s.}}{16\pi M} \int_{\Lambda^2}^{\infty} \left[\frac{M^2}{Q^2} \int_0^1 dx \Big(2xF_1(x) + F_2(x) \Big) - T_1(0, Q^2) \right]$$

subtraction function

- use OPE to connect to perturbative QCD
- log divergence arising from $2xF_1(x) + F_2(x)$ exactly cancels against log divergence from $T_1(0, Q^2)$
- counter term comes entirely from subtraction function

Electromagnetic Self Energy: Cottingham Formula

renormalization: [J.C. Collins Nucl. Phys. B149 (1979)]

$$\delta M^\gamma = \frac{3\alpha_{f.s.}}{16\pi M}\left\{ \int_0^{\mu^2} \frac{dQ^2}{Q^2} f(Q^2) + \lim_{\Lambda^2\to\infty}\left[\int_{\mu^2}^{\Lambda^2}\frac{dQ^2}{Q^2}\left(f(Q^2)+\sum_i C^0_{1,i}\langle\mathcal{O}^{i,0}\rangle\right)\right]\right\}$$

$$\langle N|\sum_i C^0_{1,i}\mathcal{O}^{i,0}|N\rangle_{p-n} = \frac{2}{Q^2}(e_u^2 m_u - e_d^2 m_d)\langle p|\bar{u}u - \bar{d}d|p\rangle$$

- $\ln(\Lambda^2)$ divergence exactly cancels
- residual dependence on scale μ
- use Naive Dimensional Analysis to estimate size

Electromagnetic Self Energy: Cottingham Formula

renormalization: [J.C. Collins Nucl. Phys. B149 (1979)]

$$\delta \tilde{M}^{ct} = -\frac{3\alpha}{4\pi}\sigma_{\pi N} \ln\left(\frac{\Lambda_1^2}{\Lambda_0^2}\right) \frac{3\hat{m} - 5\delta}{9\hat{m}} \frac{\langle p|\bar{u}u - \bar{d}d|p\rangle}{\langle p|\bar{u}u + \bar{d}d|p\rangle}$$

$$\sigma_{\pi N} = \frac{1}{2M}\langle p|\hat{m}(\bar{u}u + \bar{d}d)|p\rangle \simeq 45 \text{ MeV}$$

- saturate matrix elements in valence limit $$\frac{\langle p|\bar{u}u - \bar{d}d|p\rangle}{\langle p|\bar{u}u + \bar{d}d|p\rangle} \leq \frac{1}{3}$$
- vary arbitrary scales in scaling region $$\Lambda_0^2 = 2 \text{ GeV}^2 , \quad \Lambda_1^2 = 100 \text{ GeV}^2$$

$$|\delta \tilde{M}^{ct}| \lesssim 0.02 \text{ MeV}$$

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

subtraction term: most challenging part - dealing with unknown subtraction function

$$\delta M^{sub} = -\frac{3\alpha}{16\pi M}\int_0^{\Lambda_0^2} dQ^2\; T_1(0,Q^2)\,,$$

- low energy: constrained by effective field theory

$$T_1(0,Q^2) = 2\kappa(2+\kappa) - Q^2\left\{\frac{2}{3}\left[(1+\kappa)^2 r_M^2 - r_E^2\right] + \frac{\kappa}{M^2} - 2M\frac{\beta_M}{\alpha}\right\} + \mathcal{O}(Q^4)\,,$$

most of these contributions come from Low Energy Theorems and are "elastic" (arising from a photon striking an on-shell nucleon)

intimately related to the proton size puzzle which suffers from the same subtracted dispersive problem

K. Pachucki: Phys. Rev. A53 (1996); A. Pineda: Phys. Rev. C67 (2003); Phys. Rev. C71 (2005); R.J. Hill, G. Paz: PRL 107 (2011); C. Carlson, M. Vanderhaeghen: Phys.Rev.A84 (2011); arXiv1109.3779; M.. Birse, J. McGovern: arXiv:1206.3030

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

subtraction term: most challenging part - dealing with unknown subtraction function

$$\delta M^{sub} = -\frac{3\alpha}{16\pi M}\int_0^{\Lambda_0^2} dQ^2\ T_1(0,Q^2)\,,$$

- high energy: OPE (perturbative QCD) constrains

$$\lim_{Q^2\to\infty} T_1(0,Q^2) \propto \frac{1}{Q^2}$$

$$T_1(0,Q^2) \simeq 2G_M^2(Q^2) - 2F_1^2(Q^2) + Q^2 2M\frac{\beta_M}{\alpha}\left(\frac{m_0^2}{m_0^2+Q^2}\right)^2$$

$\mathcal{O}(Q^4)$ inelastic terms known

Birse and McGovern Eur.Phys.J A48 (2012) [arXiv:1206.3030]

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

subtraction term: most challenging part - dealing with unknown subtraction function

$$\delta M_{el}^{sub} = -\frac{3\alpha}{16\pi M}\int_0^{\Lambda_0^2} dQ^2 \left[2G_M^2 - 2F_1^2\right], \qquad \delta M_{el}^{sub}\Big|_{p-n} = -0.62\ \mathrm{MeV}$$

$$\delta M_{inel}^{sub} = -\frac{3\beta_M}{8\pi}\int_0^{\Lambda_0^2} dQ^2 Q^2 \left(\frac{m_0^2}{m_0^2 + Q^2}\right)^2$$

$$\beta_M^{p-n} = -1.0 \pm 1.0 \times 10^{-4}\ \mathrm{fm}^3$$

H.W. Griesshammer, J.A. McGovern, D.R. Phillips, G. Feldman: Prog.Nucl.Part.Phys. (2012)

taking $m_0^2 = 0.71\ \mathrm{GeV}^2$

$$\delta M_{inel}^{sub}\Big|_{p-n} = 0.47 \pm 0.47\ \mathrm{MeV}$$

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

adding it all up:

$$\delta M^{\gamma}|_{p-n} = +1.39(02) - 0.62(02) + 0.057(16) + 0.47(47)\ \text{MeV} = 1.30(.03)(.47)\ \text{MeV}$$

$+1.39(02) - 0.62(02) = 0.77(03)$ MeV — elastic terms

$+0.057(16)$ — inelastic terms

$+0.47(47)$ MeV — unknown subtraction term

recall the fixed pole in the elastic contribution makes a negligible contribtion

Cottingham's Formula

adding it all up:

$$\delta M^{\gamma}\Big|_{p-n} = 1.30(03)(47)\ \mathrm{MeV}$$

AWL, C.Carlson, G.Miller: PRL 108 (2012)

$$= 0.76(30)\ \mathrm{MeV}$$

J. Gasser and H. Leutwyler: Nucl Phys B94 (1975)

We reduced the uncertainty from structure by an order of magnitude! But we uncovered an oversight that dominates the uncertainty :(

Cottingham's Formula

adding it all up:

$$\delta M^\gamma\Big|_{p-n} = 1.30(03)(47)\ \mathrm{MeV}$$

AWL, C.Carlson, G.Miller:
PRL 108 (2012)

$$= 0.76(30)\ \mathrm{MeV}$$

J. Gasser and H. Leutwyler:
Nucl Phys B94 (1975)

expectation from experiment + lattice QCD

$$\delta M^\gamma_{p-n} = -1.29333217(42) + \underline{2.71(15)}\ \mathrm{MeV}$$
$$= \quad 1.42(15)\ \mathrm{MeV}$$

average of 4 independent lattice results

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

subtraction term: most challenging part - dealing with unknown subtraction function

$$\delta M^{sub}_{inel} = -\frac{3\beta_M}{8\pi} \int_0^{\Lambda_0^2} dQ^2 Q^2 \left(\frac{m_0^2}{m_0^2 + Q^2} \right)^2$$

$$\beta_M^{p-n} = -1.0 \pm 1.0 \times 10^{-4} \text{ fm}^3$$

H.W. Griesshammer, J.A. McGovern, D.R. Phillips, G. Feldman: Prog.Nucl.Part.Phys. (2012)

compute $\beta_M^{p,n}$ from lattice QCD

W. Detmold
B. Tiburzi
AWL

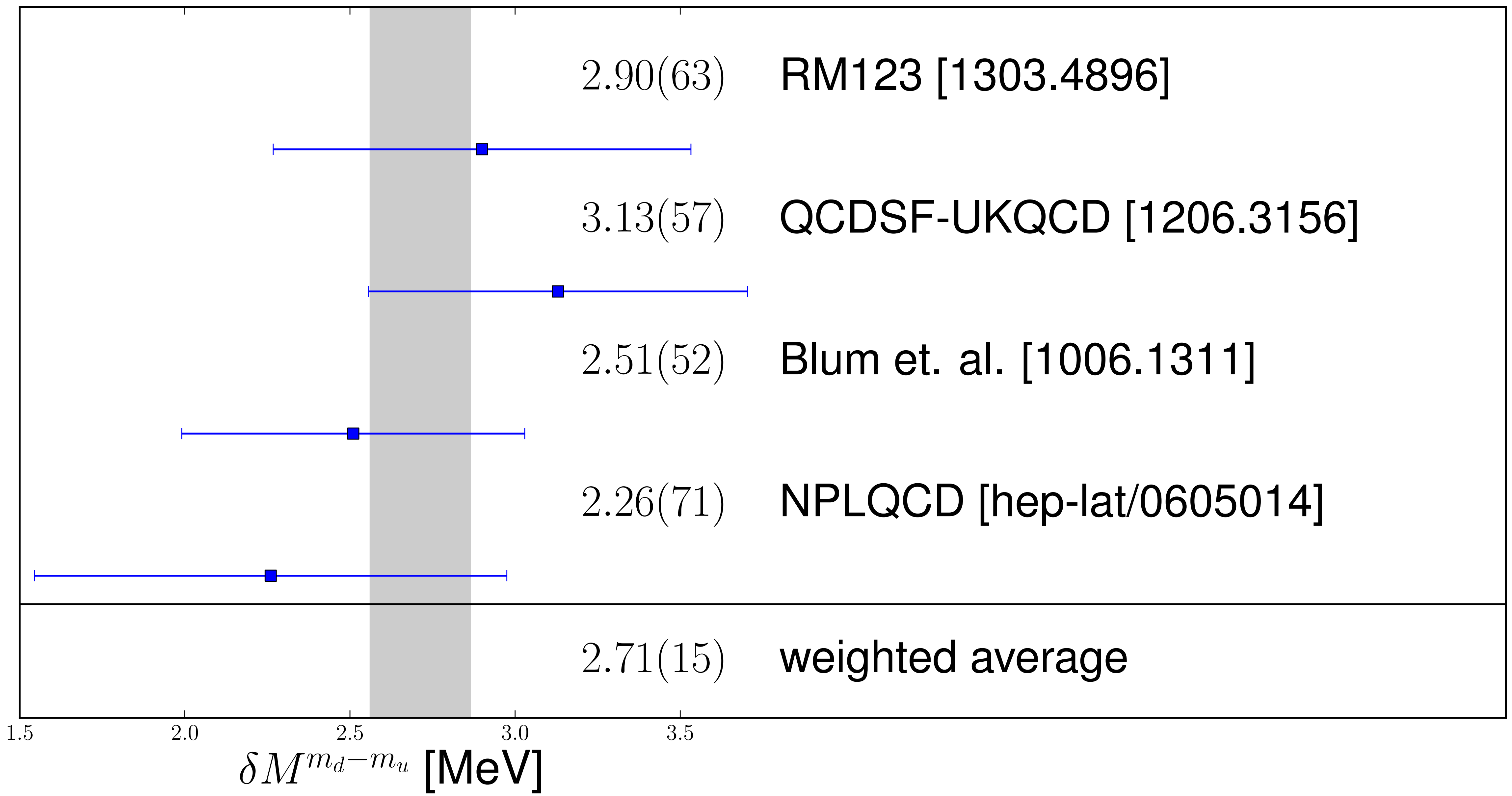
Then, one must improve the modeling of intermediate Q^2

Strong Isospin Breaking: m_d - m_u

$$\delta M_{n-p}^{m_d - m_u} = \alpha(m_d - m_u)$$

M_n - M_p

- $\delta M^{m_d - m_u}_{LQCD} = 2.71(15)$ MeV



Strong Isospin Breaking: $m_d - m_u$

strong isospin breaking correction

$$\delta M_{n-p}^{m_d - m_u} = \alpha(m_d - m_u)$$

ideal problem for lattice QCD

$$\delta M_{n-p}^{m_d - m_u} = 2.71(15)\ \mathrm{MeV}$$

lattice average

B. Tiburzi, AWL	Nucl. Phys. A764 (2006)
Beane, Orginos, Savage	Nucl. Phys. B768 (2007)
AWL	arXiv:0904.2404
Blum, Izubuchi, etal	Phys. Rev. D82 (2010)
AWL	PoS Lattice2010 (2010)
de Divitiis etal	JHEP 1204 (2012)
Horsley etal	Phys. Rev. D86 (2012)
de Divitiis etal	arXiv:1303.4896

But in lattice calculation $m_u = m_d = m_l$?

Strong Isospin Breaking: m_d - m_u

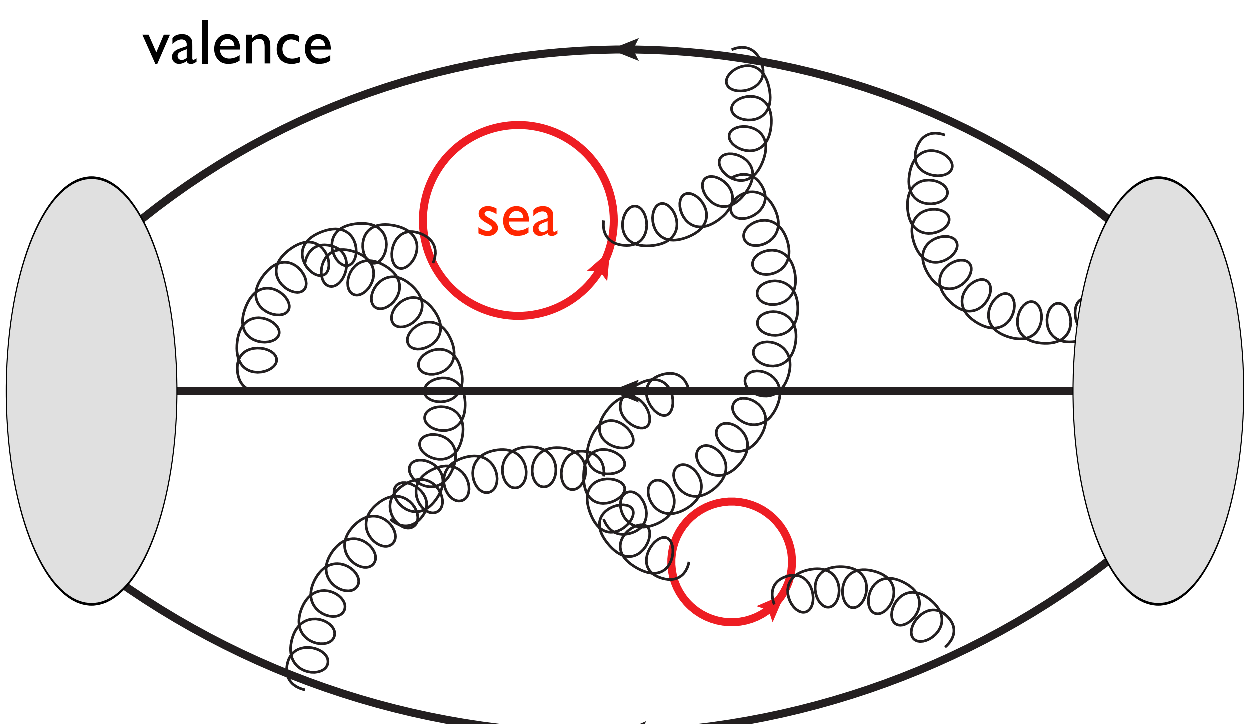
strong isospin breaking correction

$$\delta M_{n-p}^{m_d - m_u} = \alpha(m_d - m_u)$$

ideal problem for lattice QCD

$$\delta M_{n-p}^{m_d - m_u} = 2.71(15)\ \text{MeV}$$

lattice average



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de Divitiis etal	arXiv:1303.4896

$$m_{u,d}^{valence} \neq m_l^{sea}$$

"partially quenched" lattice QCD trick that works on the computer but introduces error which must be corrected

Strong Isospin Breaking: $m_d - m_u$

strong isospin breaking correction

$$\delta M_{n-p}^{m_d - m_u} = \alpha(m_d - m_u)$$

ideal problem for lattice QCD

$$\delta M_{n-p}^{m_d - m_u} = 2.71(15)\ \mathrm{MeV}$$

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Horsley etal	Phys. Rev. D86 (2012)
de Divitiis etal	arXiv:1303.4896

can we improve this method?

of course!

"Symmetric breaking of isospin symmetry" AWL arXiv:0904.2404

Strong Isospin Breaking: m_d - m_u

AWL arXiv:0904.2404

"Symmetric breaking of isospin symmetry"

$$m_{u,d}^{sea} = m_l, \quad m_u^{valence} = m_l - \delta, \quad m_d^{valence} = m_l + \delta$$

$$\mathcal{Z}_{u,d} = \int DU_\mu \, \mathrm{Det}(D + m_l - \delta\tau_3) \, e^{-S[U_\mu]}$$

$$= \int DU_\mu \, \mathrm{Det}(D + m_l) \, \det\left(1 - \frac{\delta^2}{(D + m_l)^2}\right) \, e^{-S[U_\mu]}$$

Isospin symmetric quantities: error $\mathcal{O}(\delta^2)$

Isospin violating quantities: error $\mathcal{O}(\delta^3)$

Strong Isospin Breaking: m_d - m_u

AWL arXiv:0904.2404

"Symmetric breaking of isospin symmetry"

$$m_{u,d}^{sea} = m_l, \quad m_u^{valence} = m_l - \delta, \quad m_d^{valence} = m_l + \delta$$

Pion Chiral Lagrangian

$$\mathcal{L} = \frac{f^2}{8}\,\mathrm{tr}\left(\partial_\mu \Sigma \partial_\mu \Sigma^\dagger\right) - \frac{f^2}{8}\,\mathrm{tr}\left(\chi'^\dagger \Sigma + \Sigma^\dagger \chi'\right) - \frac{l_1}{4}\left[\mathrm{tr}\left(\partial_\mu \Sigma \partial_\mu \Sigma^\dagger\right)\right]^2 - \frac{l_2}{4}\,\mathrm{tr}\left(\partial_\mu \Sigma \partial_\nu \Sigma^\dagger\right)\mathrm{tr}\left(\partial_\mu \Sigma \partial_\nu \Sigma^\dagger\right)$$
$$- \frac{l_3 + l_4}{16}\left[\mathrm{tr}\left(\chi'^\dagger \Sigma + \Sigma^\dagger \chi'\right)\right]^2 + \frac{l_4}{8}\,\mathrm{tr}\left(\partial_\mu \Sigma \partial_\mu \Sigma^\dagger\right)\mathrm{tr}\left(\chi'^\dagger \Sigma + \Sigma^\dagger \chi'\right) - \frac{l_7}{16}\left[\mathrm{tr}\left(\chi'^\dagger \Sigma - \Sigma^\dagger \chi'\right)\right]^2$$

$$m_{\pi^\pm}^2 = 2Bm_l\left\{1 + \frac{m_\pi^2}{(4\pi f_\pi)^2}\ln\left(\frac{m_\pi^2}{\mu^2}\right) + \frac{4m_\pi^2}{f_\pi^2} l_4^r(\mu)\right\} - \frac{\Delta_{PQ}^4}{2(4\pi f_\pi)^2}$$

$$m_{\pi^0}^2 = m_{\pi^\pm}^2 + \frac{16B^2\delta^2}{f_\pi^2} l_7$$

$$\Delta_{PQ}^2 = 2B\delta$$

Strong Isospin Breaking: m_d - m_u

AWL arXiv:0904.2404

"Symmetric breaking of isospin symmetry"

$$m_{u,d}^{sea} = m_l, \quad m_u^{valence} = m_l - \delta, \quad m_d^{valence} = m_l + \delta$$

Can also construct the partially quenched baryon chiral Lagrangian

$$m_p = M_0 - \alpha\delta + m_l(\alpha + \sigma_N) - \frac{3\pi g_A^2}{(4\pi f_\pi)^2} m_\pi^3 - \frac{8 g_{\pi N\Delta}^2}{3(4\pi f_\pi)^2} \mathcal{F}(m_\pi, \Delta, \mu) + \frac{3\pi \Delta_{PQ}^4 (g_A + g_1)^2}{8 m_\pi (4\pi f_\pi)^2}$$

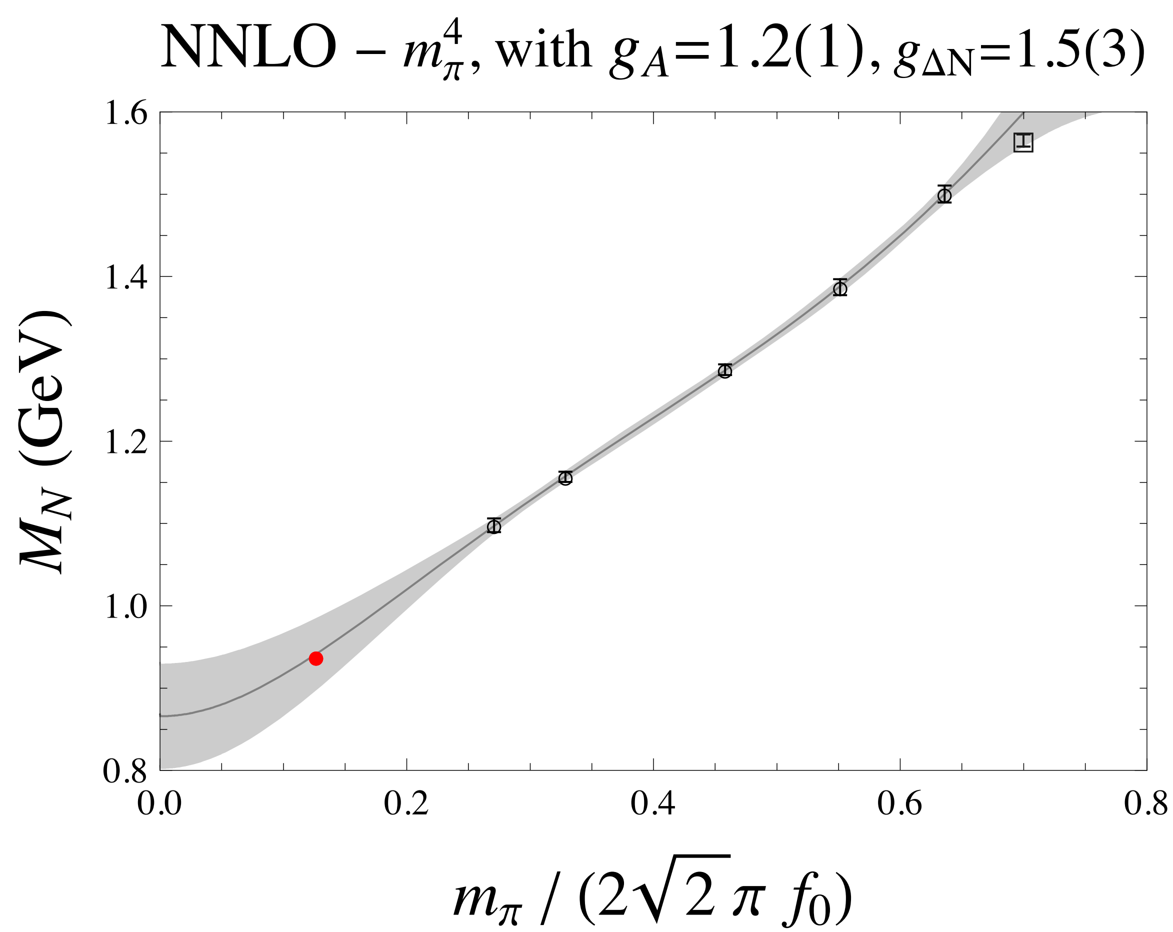
$$m_n = M_0 + \alpha\delta + m_l(\alpha + \sigma_N) - \frac{3\pi g_A^2}{(4\pi f_\pi)^2} m_\pi^3 - \frac{8 g_{\pi N\Delta}^2}{3(4\pi f_\pi)^2} \mathcal{F}(m_\pi, \Delta, \mu) + \frac{3\pi \Delta_{PQ}^4 (g_A + g_1)^2}{8 m_\pi (4\pi f_\pi)^2}$$

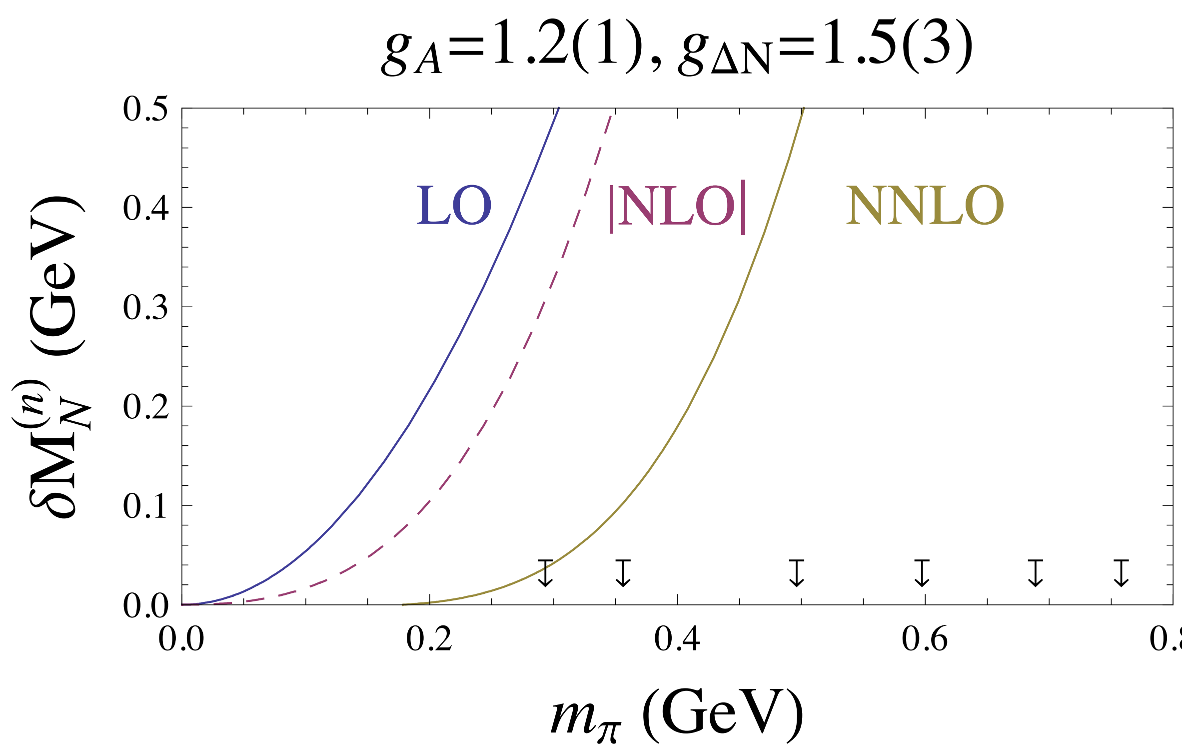
$$M_n - M_p = \alpha(m_d - m_u) + \mathcal{O}(\delta^2, m_\pi^2 \delta)$$

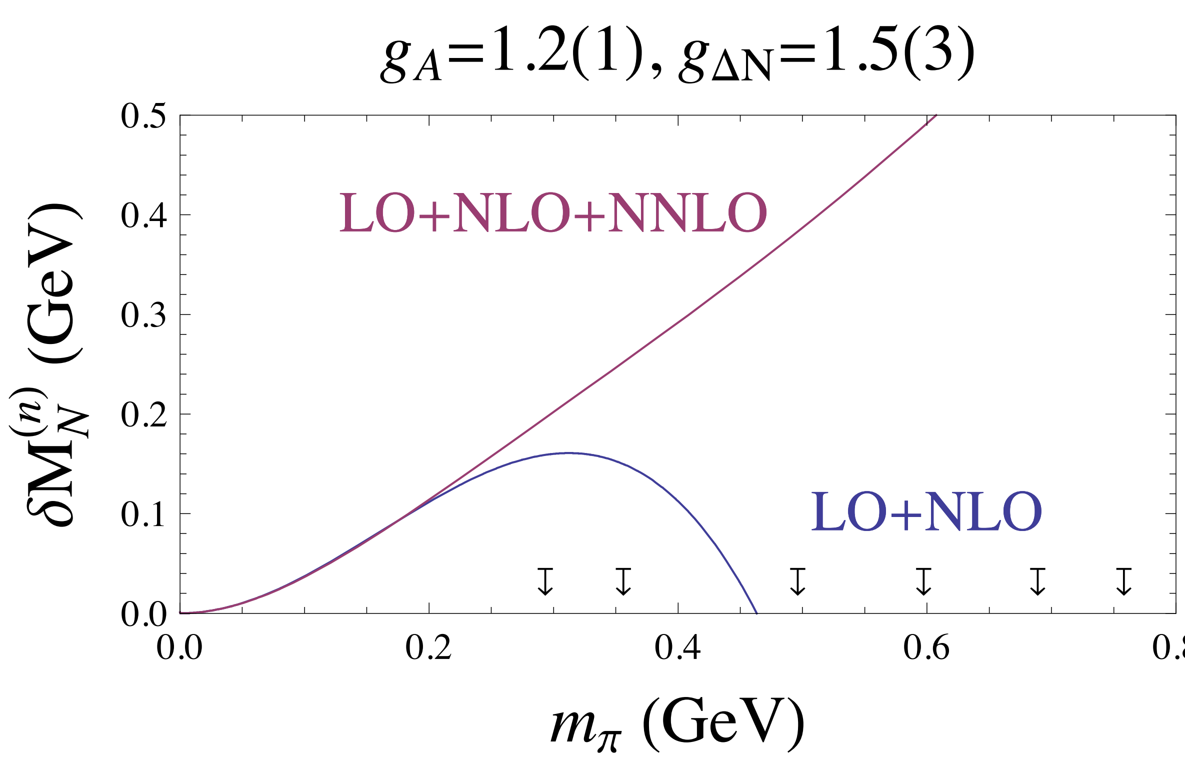
$$(2\delta = m_d - m_u)$$

Problematic terms exactly drop out of expansion for mass difference!

Baryons in lattice QCD Light quark mass dependence of M_N







NNLO Heavy Baryon Fit

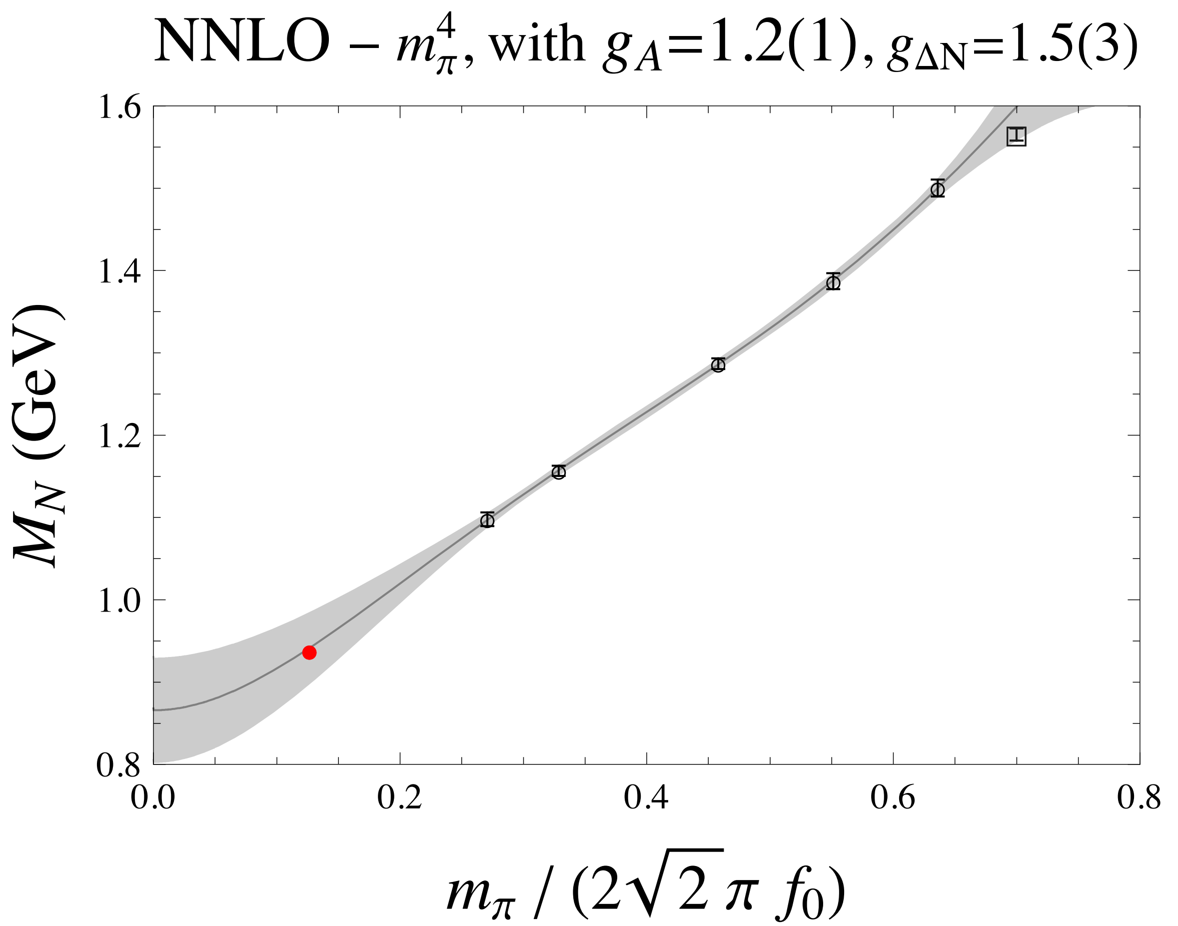
$$M_N = 954 \pm 42 \pm 20 \text{ MeV}$$

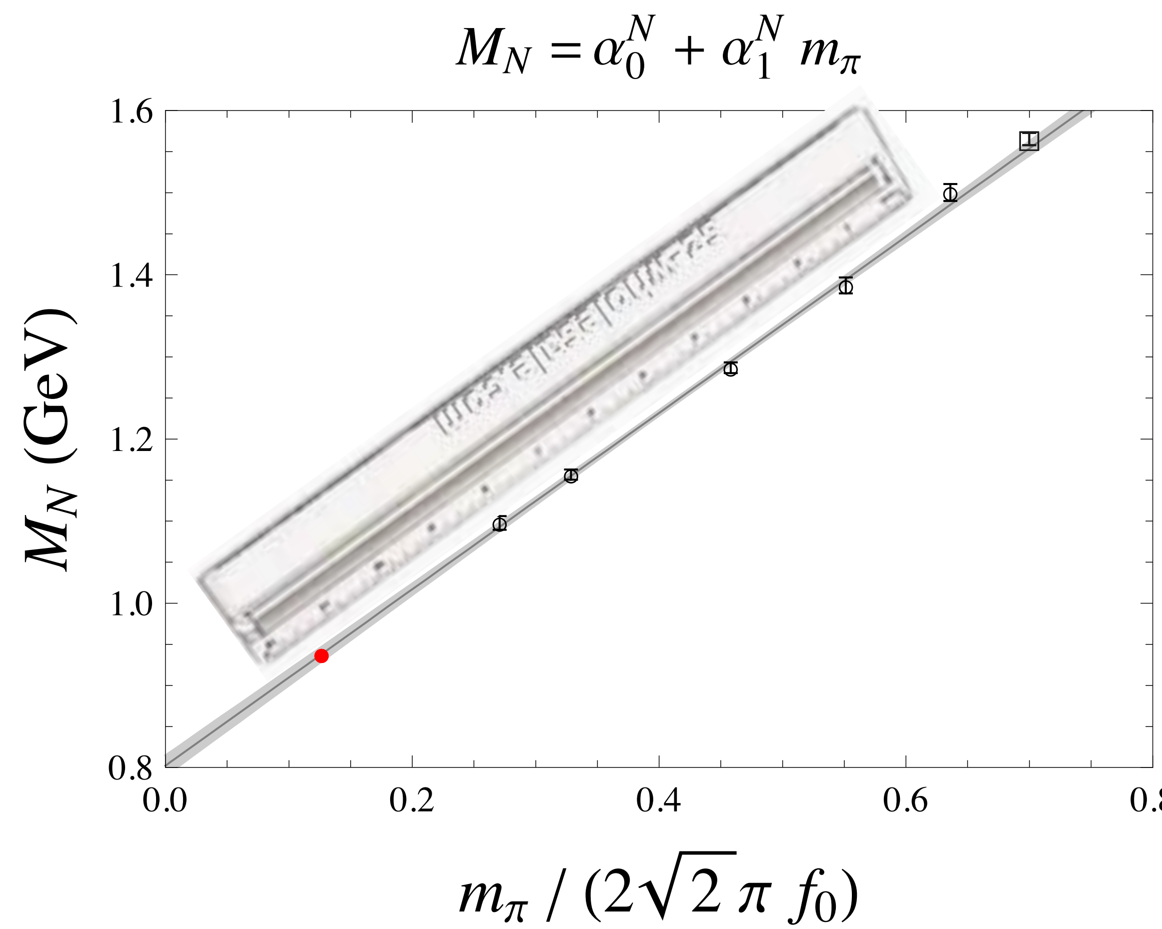
statistical

varying inputs

LHP Collaboration arXiv:0806.4549

Baryons in lattice QCD   Light quark mass dependence of M_N





NNLO Heavy Baryon Fit

$$M_N = 954 \pm 42 \pm 20 \text{ MeV}$$

Ruler Approximation

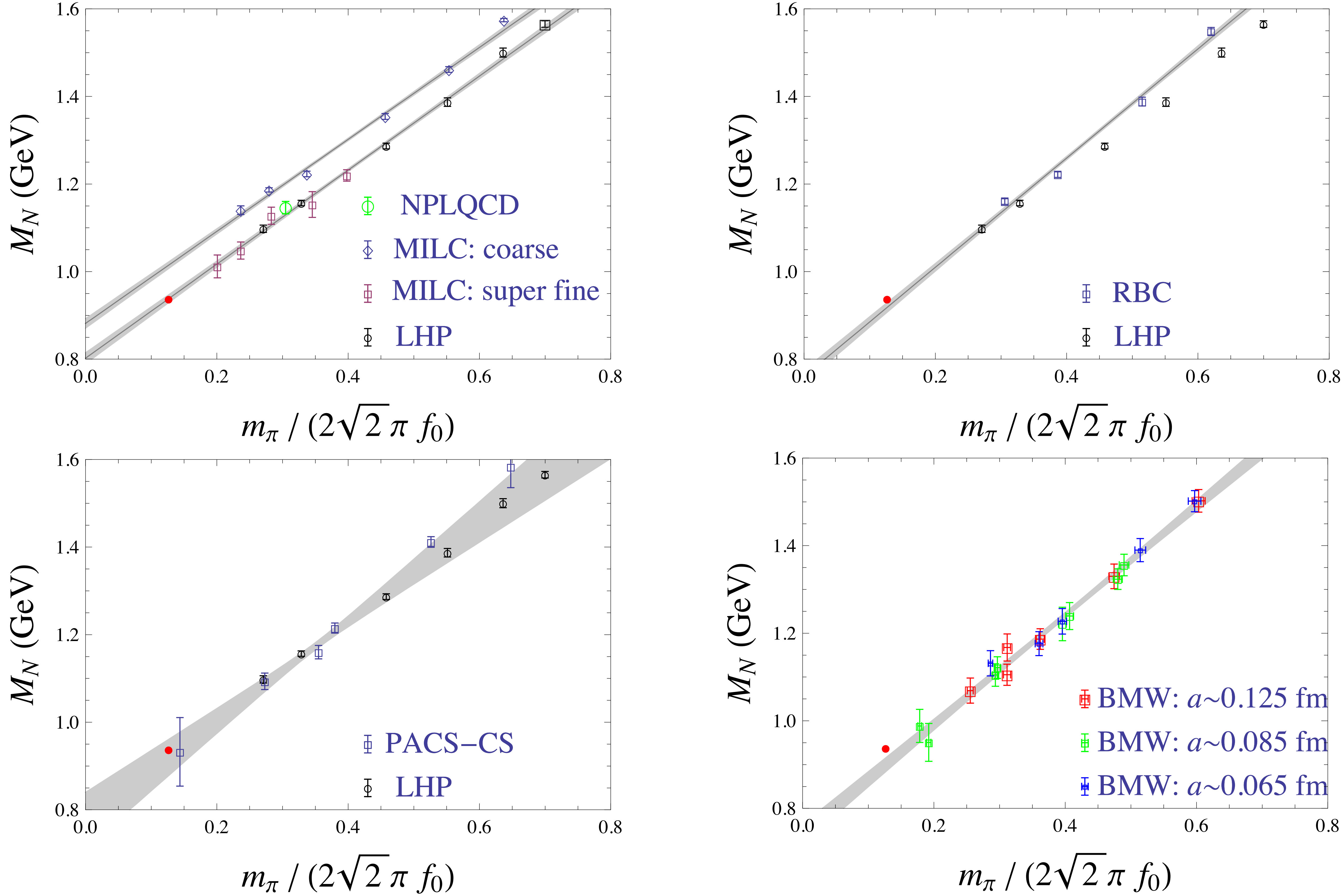
$$M_N = \alpha_0^N + \alpha_1^N m_\pi$$
$$= 938 \pm 9 \text{ MeV}$$

I am not advocating this as a good model for QCD!

LHP Collaboration arXiv:0806.4549

Baryons in lattice QCD   Light quark mass dependence of M_N

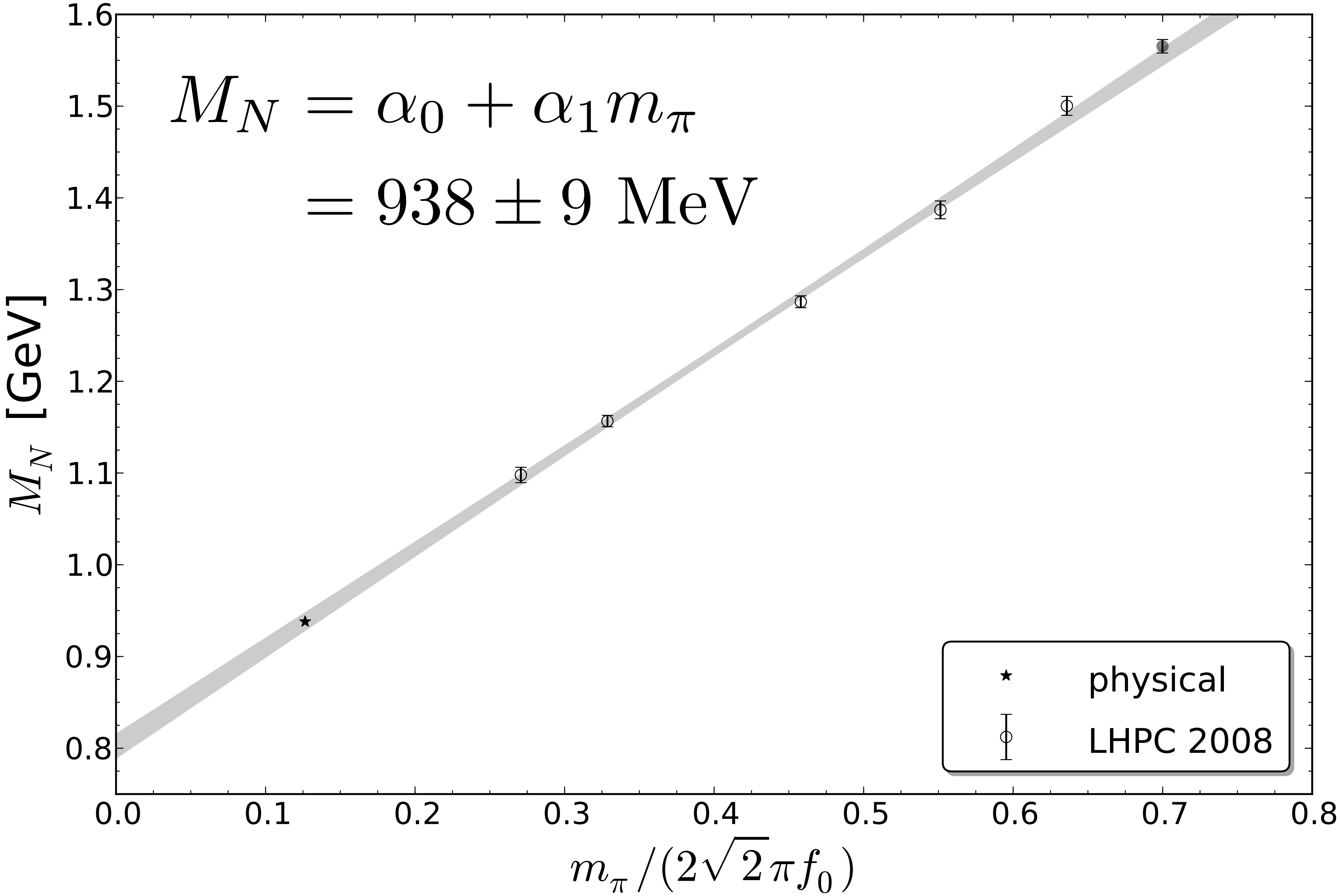
MILC: $M_N = \alpha_0^N + \alpha_1^N\, m_\pi$ Latt 2008, arXiv:0810.0663 $M_N = \alpha_0^N + \alpha_1^N\, m_\pi$



Light quark mass dependence of M_N

What is the status now (2012)?

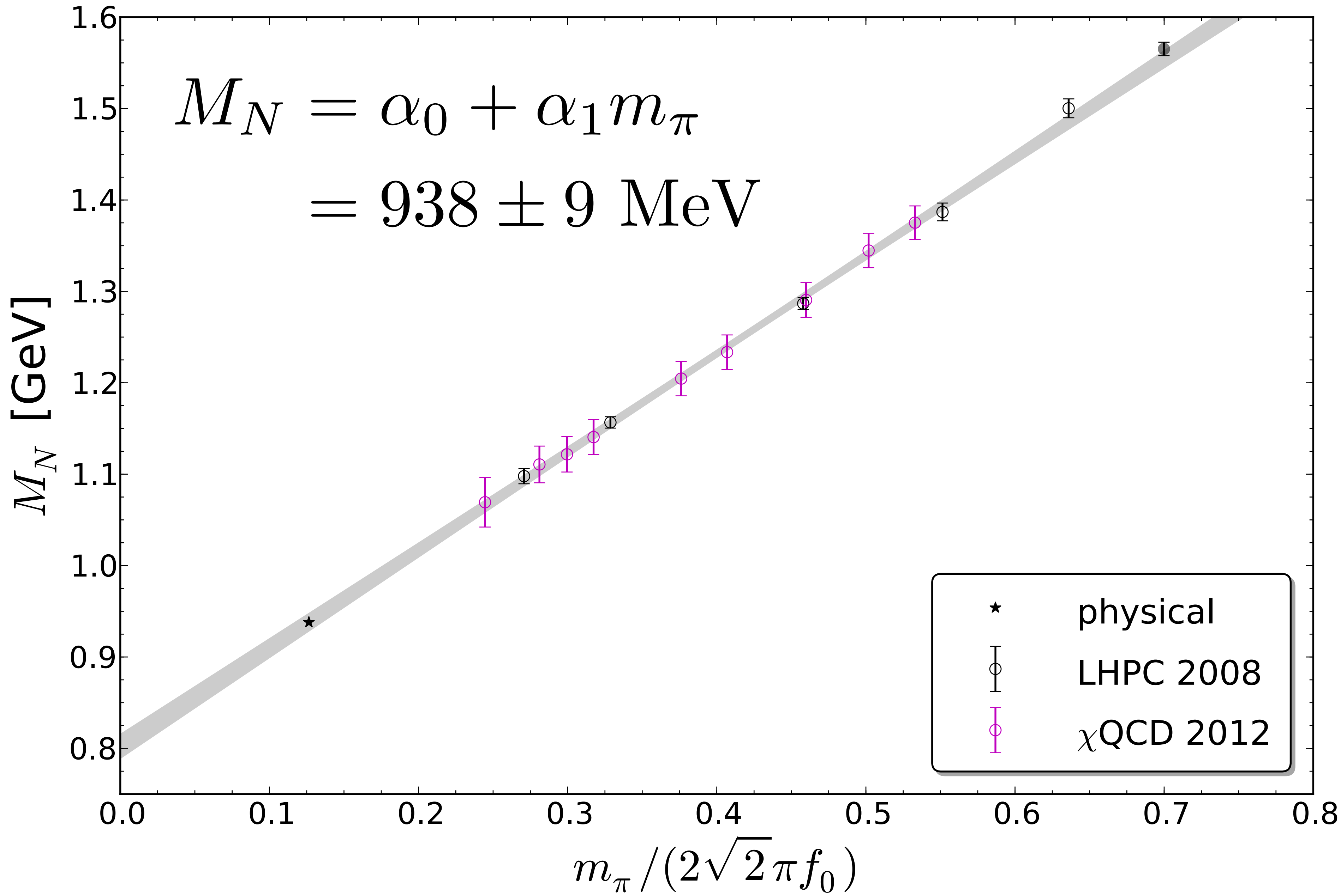
Chiral Dynamics 2013
AWL



Physical point NOT included in fit

What is the status now (2012)?

Chiral Dynamics 2013
AWL

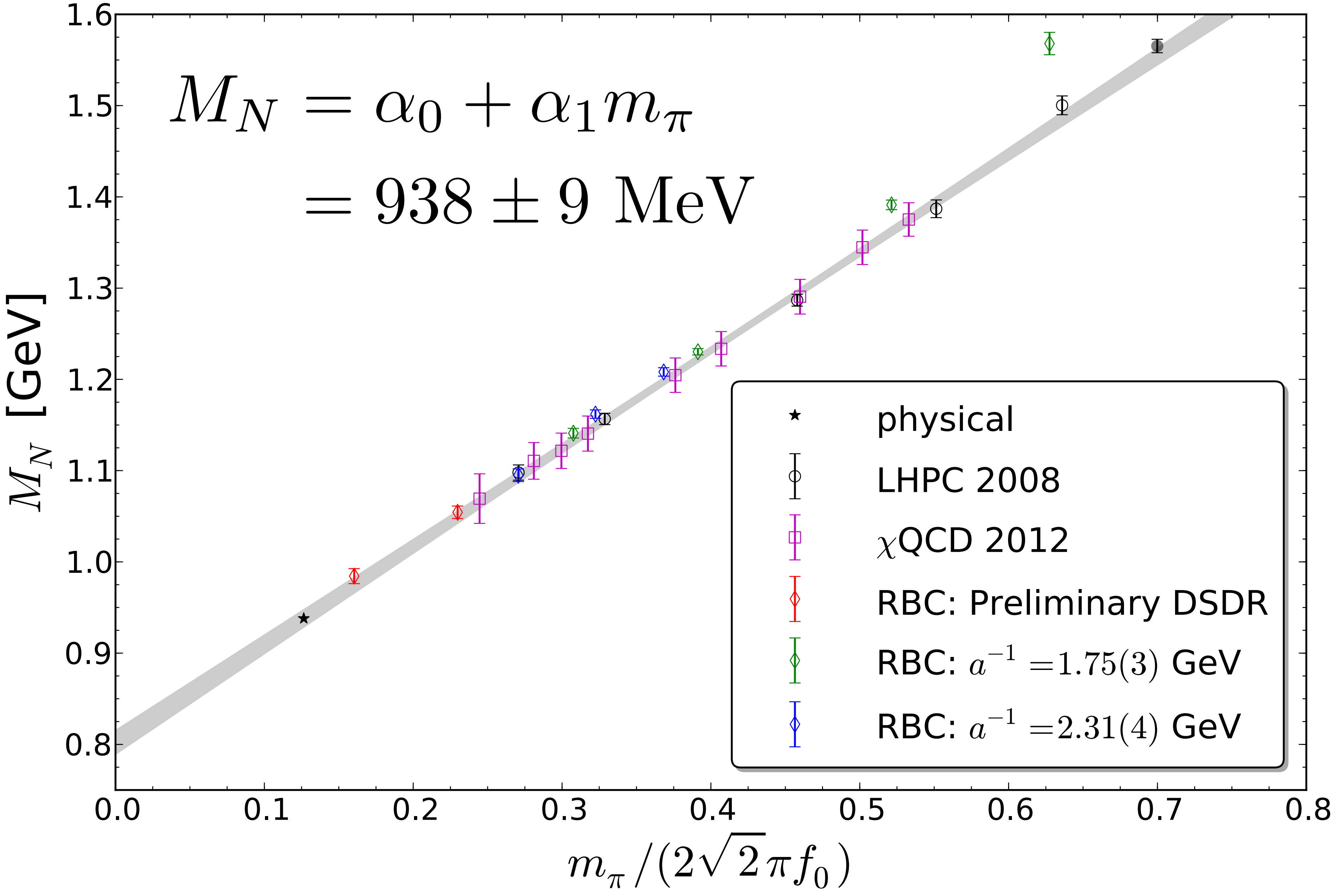


χQCD Collaboration uses Overlap Valence fermions on Domain-Wall (RBC-UKQCD) sea fermions

Light quark mass dependence of M_N

What is the status now (2012)?

Chiral Dynamics 2013
AWL

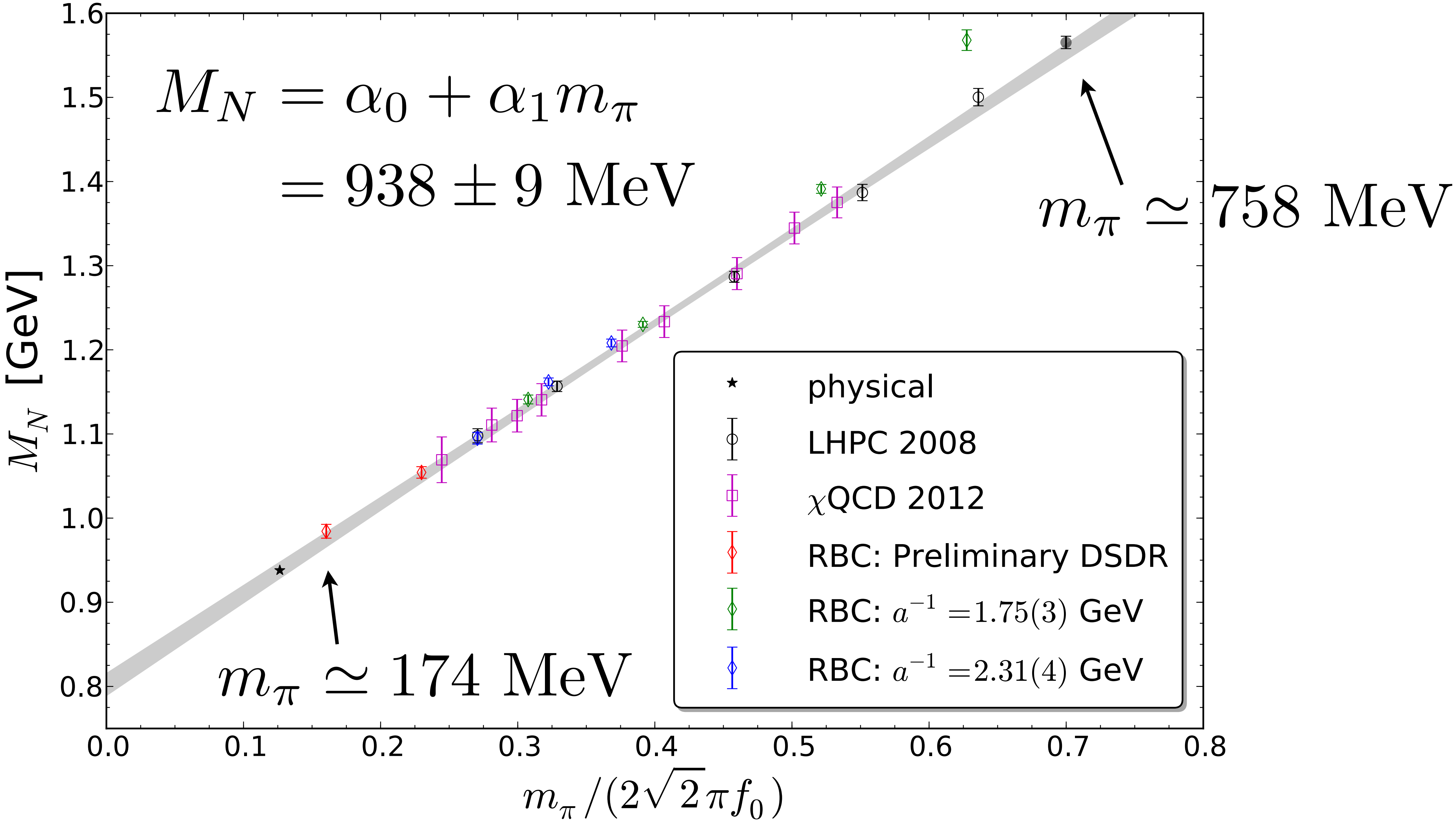


RBC-UKQCD Collaboration uses Domain-Wall valence and sea fermions

Light quark mass dependence of M_N

What is the status now (2012)?

Chiral Dynamics 2013
AWL



Taking this seriously yields

$$\sigma_{\pi N} = 67 \pm 4 \text{ MeV}$$

I am not advocating this as a good model for QCD!

Strong Isospin Breaking: m_d - m_u

AWL arXiv:0904.2404

"Symmetric breaking of isospin symmetry"

$$m_{u,d}^{sea} = m_l, \quad m_u^{valence} = m_l - \delta, \quad m_d^{valence} = m_l + \delta$$

Can also construct the partially quenched baryon chiral Lagrangian

$$m_p = M_0 - \alpha\delta + m_l(\alpha + \sigma_N) - \frac{3\pi g_A^2}{(4\pi f_\pi)^2} m_\pi^3 - \frac{8 g_{\pi N\Delta}^2}{3(4\pi f_\pi)^2} \mathcal{F}(m_\pi, \Delta, \mu) + \frac{3\pi\Delta_{PQ}^4 (g_A + g_1)^2}{8 m_\pi (4\pi f_\pi)^2}$$

$$m_n = M_0 + \alpha\delta + m_l(\alpha + \sigma_N) - \frac{3\pi g_A^2}{(4\pi f_\pi)^2} m_\pi^3 - \frac{8 g_{\pi N\Delta}^2}{3(4\pi f_\pi)^2} \mathcal{F}(m_\pi, \Delta, \mu) + \frac{3\pi\Delta_{PQ}^4 (g_A + g_1)^2}{8 m_\pi (4\pi f_\pi)^2}$$

$$M_n - M_p = \alpha(m_d - m_u) + \mathcal{O}(\delta^2, m_\pi^2 \delta)$$

$$(2\delta = m_d - m_u)$$

Problematic terms exactly drop out of expansion for mass difference!

Strong Isospin Breaking: m_d - m_u

PRELIMINARY

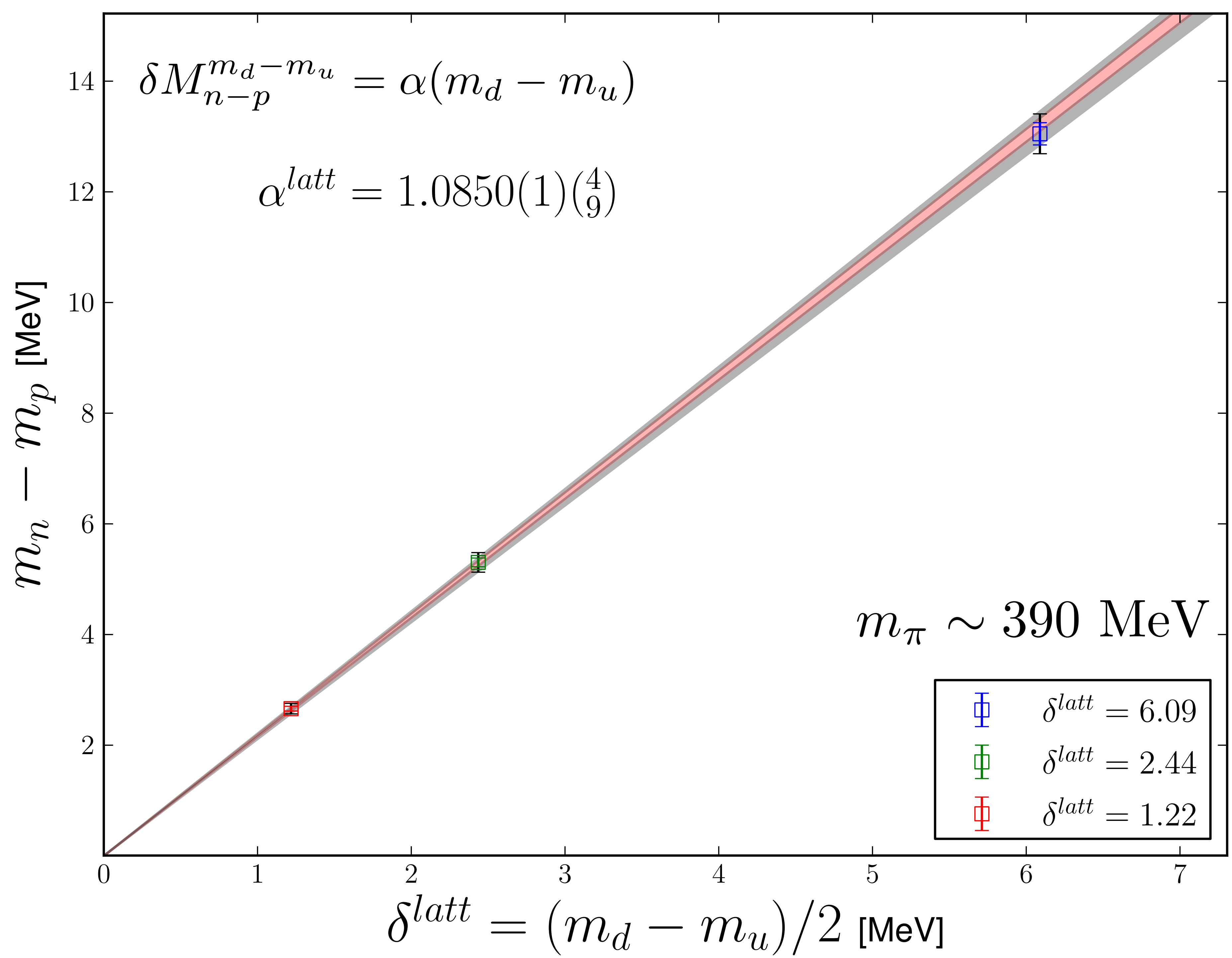
performed the lattice calculation using the Spectrum Collaboration anisotropic clover-Wilson gauge ensembles (developed @ JLAB)

ensemble				m_π	m_K	$a_t\delta$ $[N_{cfg} \times N_{src}]$			
L	T	$a_t m_l$	$a_t m_s$	[MeV]	[MeV]	0.0002	0.0004	0.0010	0.0020
16	128	-0.0830	-0.0743	449	581	207×16	207×16	207×16	207×16
16	128	-0.0840	-0.0743	390	546	166×25	166×25	166×25	166×50
20	128	-0.0840	-0.0743	390	546	120×25	–	–	–
24	128	-0.0840	-0.0743	390	546	97×25	–	193×25	–
32	256	-0.0840	-0.0743	390	546	291×10	291×10	291×10	–
24	128	-0.0860	-0.0743	225	467	118×26	–	–	–
32	256	-0.0860	-0.0743	225	467	842×11	–	–	–

C.Aubin, W.Detmold,
E.Mereghetti,
K.Orginos, S.Syritsyn,
B.Tiburzi,
AWL

Strong Isospin Breaking: m_d - m_u

PRELIMINARY



C.Aubin, W.Detmold, E.Mereghetti, K.Orginos, S.Syritsyn, B.Tiburzi, AWL

This technique orders of magnitude less numerically expensive than using lattice QCD with full $m_u^{sea} \neq m_d^{sea}$

Big Bang Nucleosynthesis and $M_n - M_p$

calculations performed at

Jefferson Lab HPC Center

Sporades Cluster

Big Bang Nucleosynthesis and $M_n - M_p$

$$M_n - M_p = \delta M^{\gamma}_{n-p} + \delta M^{m_d - m_u}_{n-p}$$
$$= -178(04)(64)\ \mathrm{MeV} \times \alpha_{f.s.} + 1.08(6)(9) \times (m_d - m_u)$$

(lattice average)

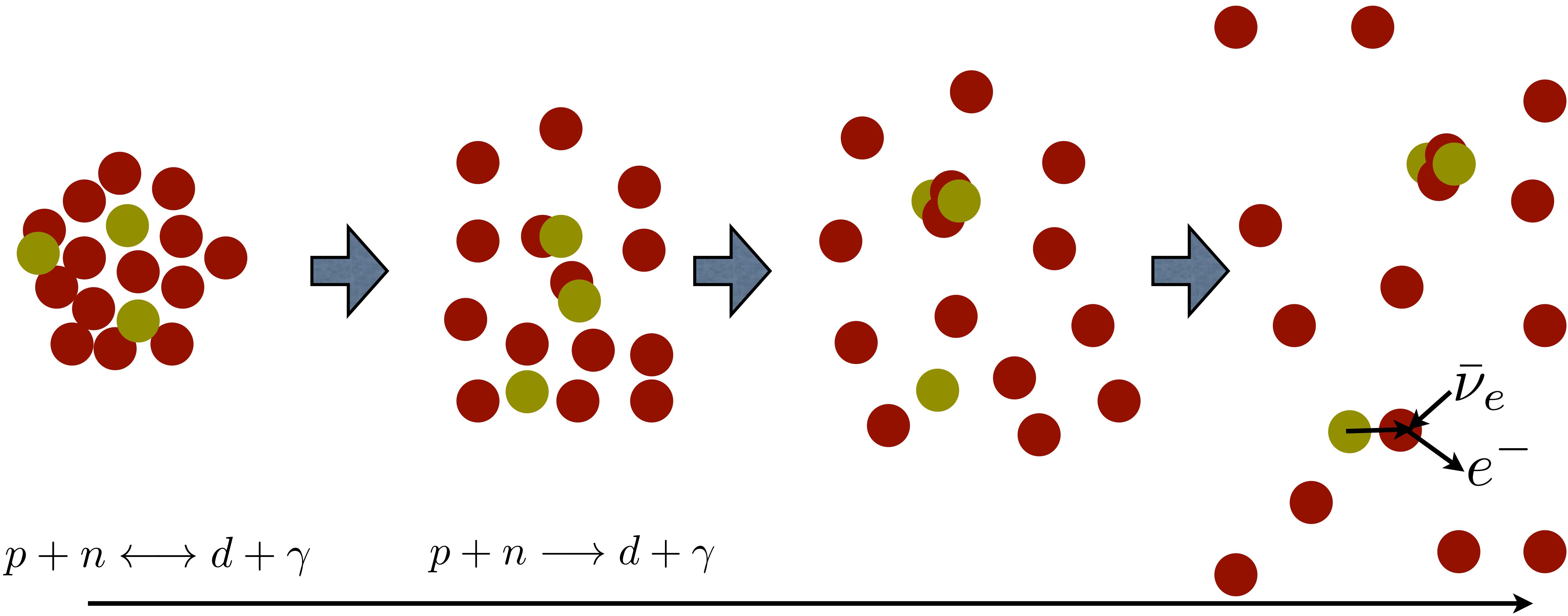
my value an order of magnitude more precise

Big Bang Nucleosynthesis highly constrains variation of $M_n - M_p$ and hence variation of fundamental constants

considering $\alpha_{f.s.}$ and $m_d - m_u$ simultaneously relaxes constraints (not yet simultaneously considered)

for now - freeze electromagnetic coupling and just look at effects of quark mass splitting

Big Bang Nucleosynthesis and $M_n - M_p$



$t \sim 1$ sec	$t \sim 3$ min	$t \sim 3^+$ min	$t \sim 15$min
$T \sim 1$ MeV	$T \sim 0.1$ MeV	$T \sim 0.1^-$ MeV	$T \sim 0.01$ MeV

$$\frac{X_n}{X_p} = e^{-\frac{M_n - M_p}{T}}$$

Initial conditions

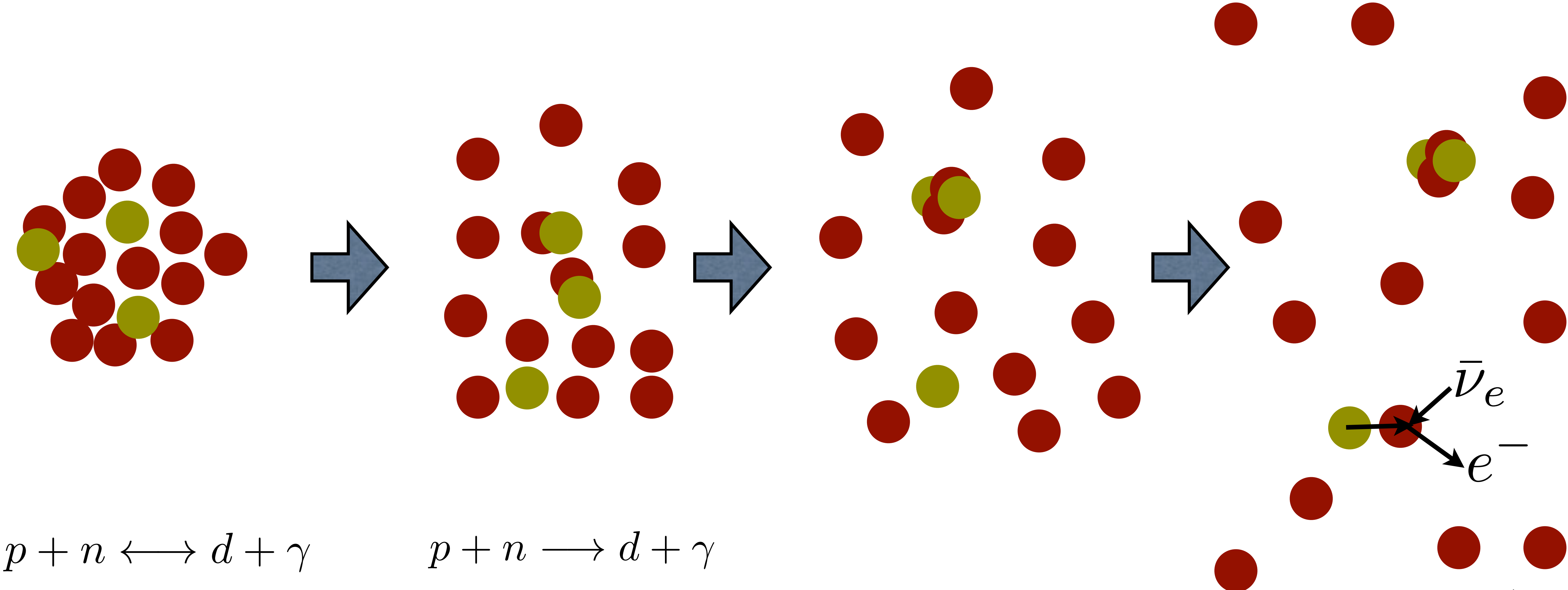
B_d

deuterium binding energy

τ_n

neutron lifetime

Big Bang Nucleosynthesis and $M_n - M_p$



$p + n \longleftrightarrow d + \gamma$ $\quad$ $p + n \longrightarrow d + \gamma$

$t \sim 1$ sec	$t \sim 3$ min	$t \sim 3^+$ min	$t \sim 15$min
$T \sim 1$ MeV	$T \sim 0.1$ MeV	$T \sim 0.1^-$ MeV	$T \sim 0.01$ MeV

$$\frac{X_n}{X_p} = e^{-\frac{M_n - M_p}{T}}$$

Initial conditions

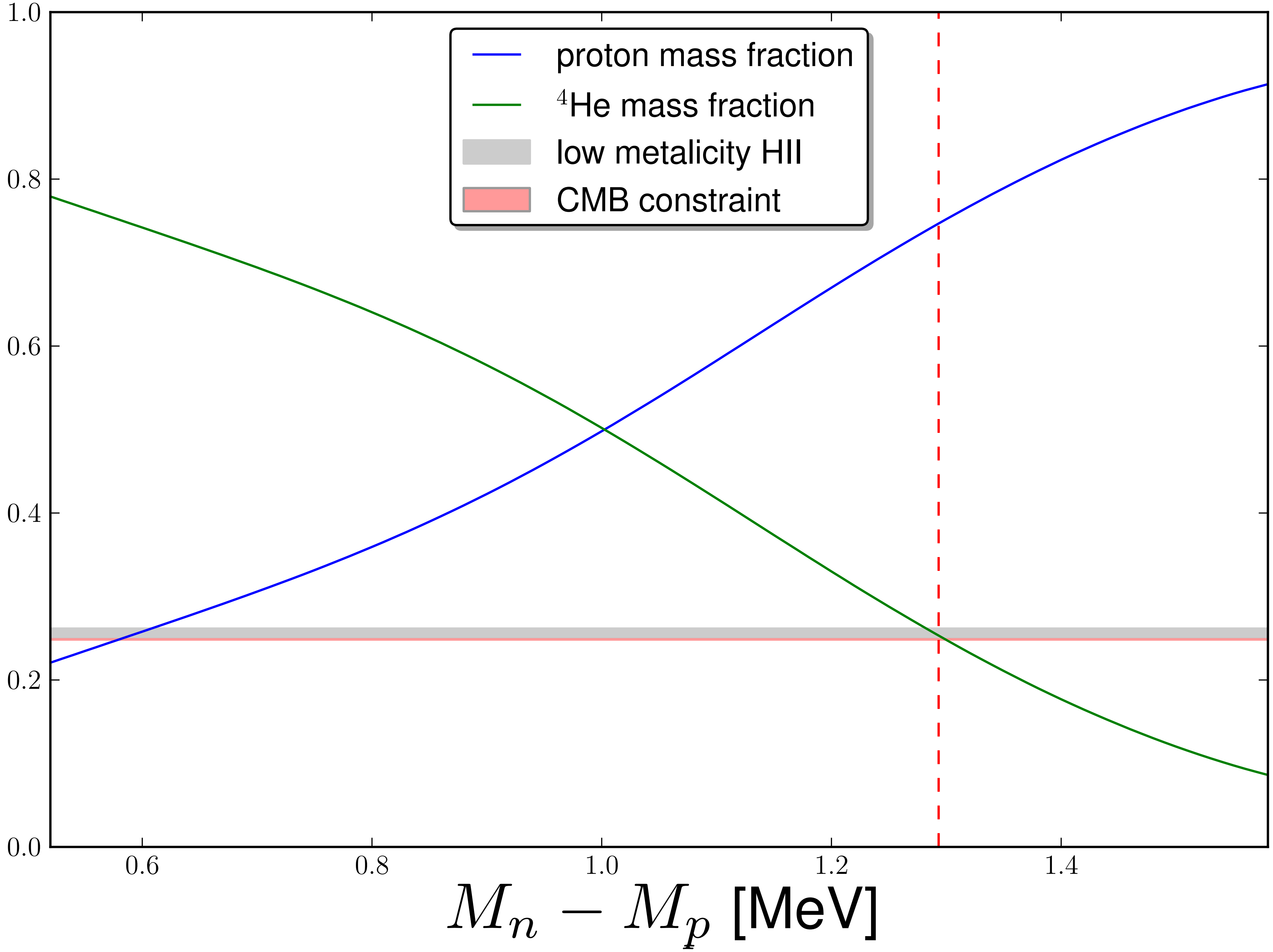
focus on leading isospin breaking

τ_n

neutron lifetime

Big Bang Nucleosynthesis and $M_n - M_p$ P. Banerjee, T. Luu, AWL

PRELIMINARY



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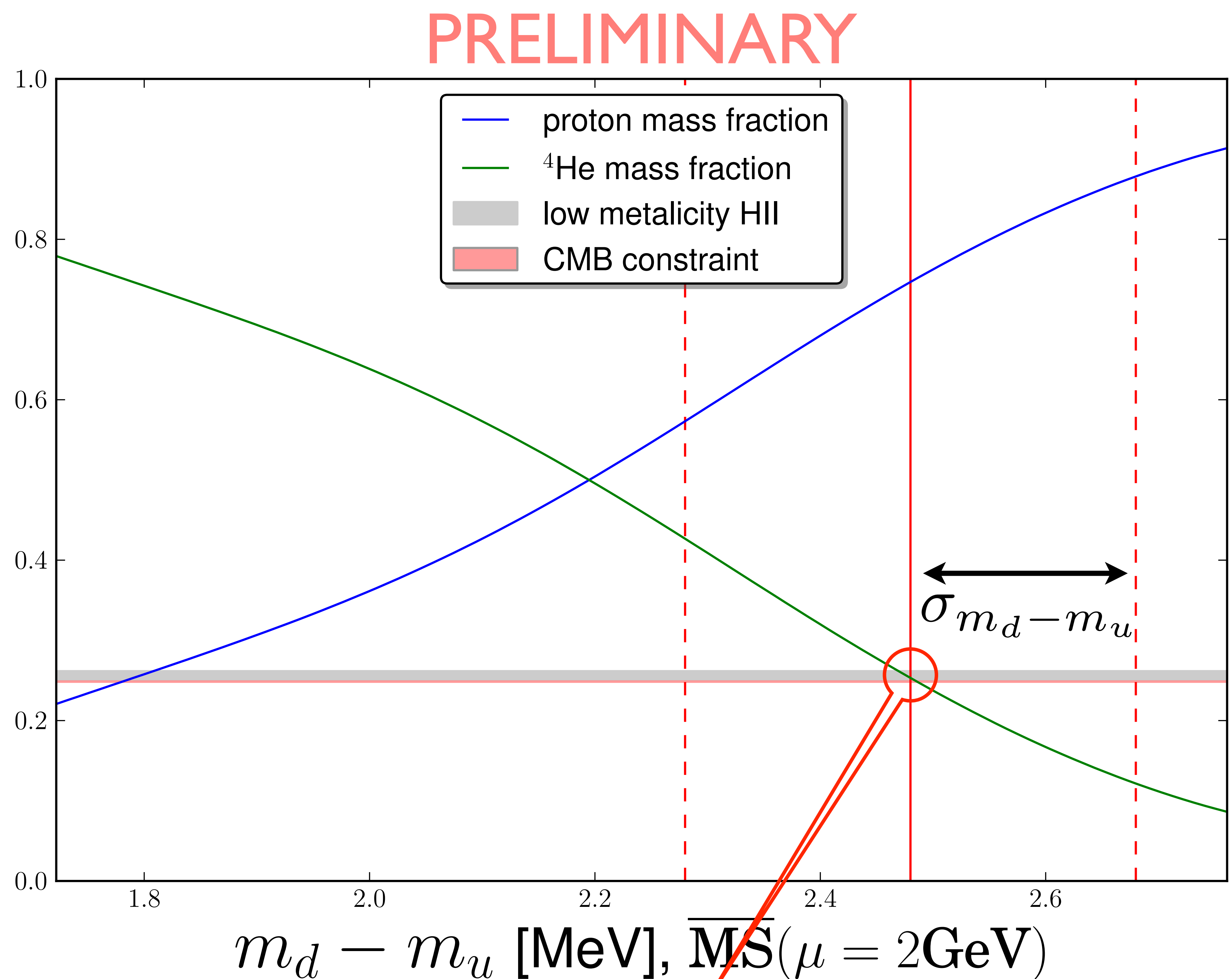


Lattice QCD

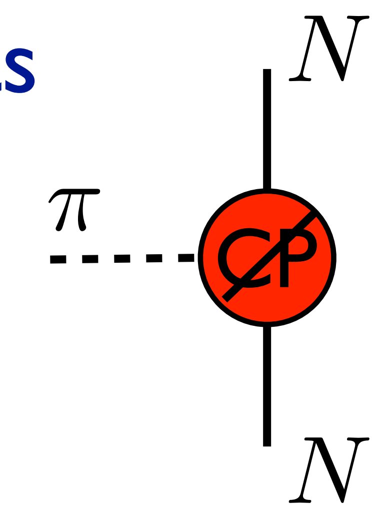
A precise determination of α + BBN can constrain $m_d - m_u$

$$\delta M_{n-p}^{m_d - m_u} \equiv \alpha(m_d - m_u)$$

Yet More Connections

Electric Dipole Moments

long range $\not{CP}$ interaction
dominates nuclear EDMs



$g_0(\bar\theta)$

$$g_0(\bar\theta) = \delta M_{n-p}^{m_d-m_u} \frac{2 m_d m_u \sin(\bar\theta)}{(m_d+m_u)(m_d-m_u)} \qquad \delta M_{n-p}^{m_d-m_u} \equiv \alpha (m_d - m_u)$$

$$= \sin(\bar\theta)\, \alpha \frac{2 m_d m_u}{(m_d+m_u)}$$

I am computing this
with lattice QCD

The world's most stringent constraint on an EDM from Atomic measurement
competitive constraint on $\bar\theta$

Griffith, Swallows, Loftus, Romalis, Heckel, Fortson
PRL 102 101601 (2009)

Conclusions and New Horizons

- goal was to show you with a little cleverness, you can relate simple quantities, computable (cheaply) with lattice QCD to other interesting and complicated physics encourage you to be clever and adventurous

- related a simple quantity $M_n - M_p$ to the primordial abundance of light nuclear elements, formed in the first few minutes after the Big Bang
also related to strong CP violation

- showed how modern knowledge of nucleon structure can be used to determine the electromagnetic self-energy contribution
    - improvements will come with a determination of the iso-vector nucleon magnetic polarizability - either experimentally of from lattice QCD

- the strong contribution $(m_d - m_u)$ can only be determined with lattice QCD

- this was just a simple example of exciting connections we can now make between the universe and QCD because of the tremendous growth of lattice QCD as a tool for non-perturbative QCD phenomena

Thank You