Lattice calculations of the leading hadronic contribution to $(g-2)_\mu$

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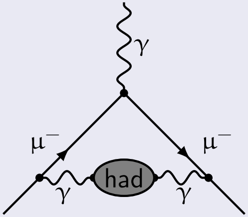
based on arXiv:1211.1159

Motivation

Motivation

The anomalous magnetic moment of the muon

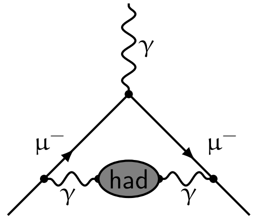
- $a_\mu = (g_\mu - 2)/2$ shows a discrepancy of $\sim 3\sigma$ between experiment and theory
- Strong interaction dominates the theoretical uncertainty of a_μ:
  - **QCD** (α_s^2): $4.1 \cdot 10^{-10}$
  - QCD (α_s^3,LbL): $2.6 \cdot 10^{-10}$
  - Weak (up to $\mathcal{O}(\alpha_W^2)$): $0.2 \cdot 10^{-10}$
  - QED (up to $\mathcal{O}(\alpha^5)$): $0.02 \cdot 10^{-10}$ [PDG, 2010]



Methods of determine a_μ^{had}

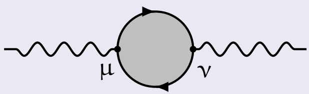
- Optical Theorem using $e^+ e^- \to$ hadrons data (Rogerio Rosenfeld's talk)
- Lattice QCD allows an ab initio calculation
- ChPT, ...

a_μ^{had} in Lattice QCD



In the continuum

- Vacuum polarization tensor defined as current-current correlator



$$\Pi_{\mu\nu}(q) = i \int d^4x\, e^{iqx} \langle J_\mu(x) J_\nu(0) \rangle = \left(q_\mu q_\nu - q^2 g_{\mu\nu}\right) \Pi(q^2)$$

- Currently only connected diagram considered
  - Two flavour χPT $\Rightarrow$ Disconnected diagram $\approx -10\%$ [Jüttner, Della Morte, 2010]
  - Lattice study $\Rightarrow$ Disconnected diagram compatible with 0 (large error bars) [ETMC, 2011]

On the lattice

- $\Pi_{\mu\nu}$ can be expressed in terms of gauge links $U_\mu(n)$ and propagators D_{lat}^{-1}

$$\Pi_{\mu\nu}(q) = a^4 \sum_{n\in\Lambda} e^{iq(n+a\hat{\mu}/2)} \left\langle J_\mu^c(n) J_\nu^l(0) \right\rangle = \left(q_\mu q_\nu - q^2 \delta_{\mu\nu}\right) \Pi(q^2)$$

- Use local current J^l at the source and conserved point-split current J^c at sink:
  $\rightarrow$ Only 1 inversion needed, but $\Pi_{\mu\nu}$ needs to renormalized. [Boyle, et al, 2011]
- Twisted boundary conditions applied to valence quarks

$$\psi(x+L) = \exp\left(i\frac{\Theta_i}{L}x_i\right)\psi(x)$$

  $\Rightarrow$ Momentum becomes tunable by Θ_i: $q_i = \frac{2\pi n_i}{L} - \frac{\Theta_i}{L}$ [Sachrajda, Villadoro, 2005]
- Determine a_μ^{had} by convolution integral: $4\alpha^2 \int_0^\infty F\left(\frac{q^2}{m_\mu^2}\right)\left(\Pi(0) - \Pi(q^2)\right) dq^2$

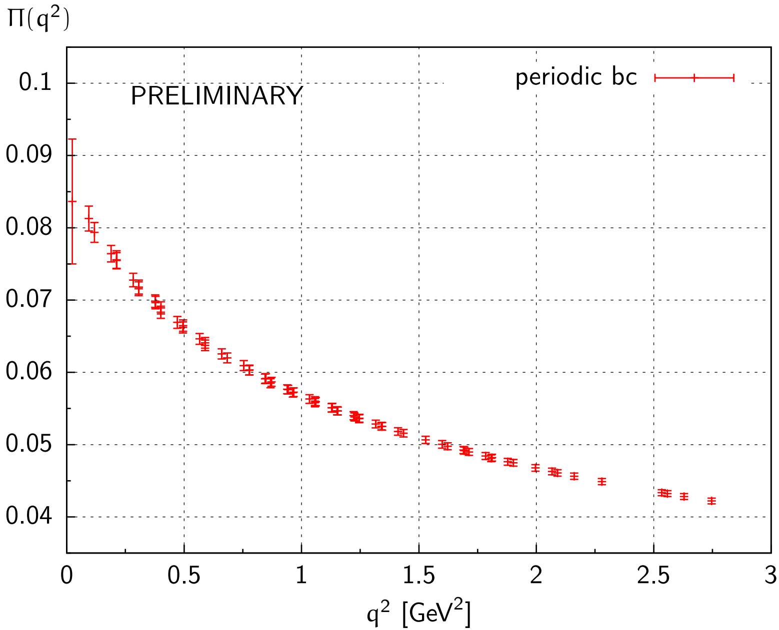
Lattice Simulations

Simulation details

- $\mathcal{O}(a)$ improved Wilson fermions (Wilson clover)
- $N_f = 2$ and $N_f = 2+$ quenched strange
- CLS ensembles:

β	a [fm]	lattice	L [fm]	m_π [MeV]	$m_\pi L$	Labels
5.20	0.079	64×32^3	2.5	473, 363, 312	6.0, 4.7, 4.0	A3, A4, A5
5.30	0.063	64×32^3	2.0	606, 451	6.2 , 4.7	E4, E5
5.30	0.063	96×48^3	3.0	324, 277	5.0, 4.2	F6, F7
5.30	0.063	128×64^3	4.0	190	4.0	G8
5.50	0.050	96×48^3	2.4	536, 430, 330	6.5, 5.2, 4.1	N4, N5, N6
5.50	0.050	128×64^3	3.2	260	4.4	O7

Vacuum Polarization $\Pi(q^2)$



- G8 ensemble: $\beta = 5.3$, $m_\pi = 190\,\text{MeV}$, $L = 4.0\,\text{fm}$

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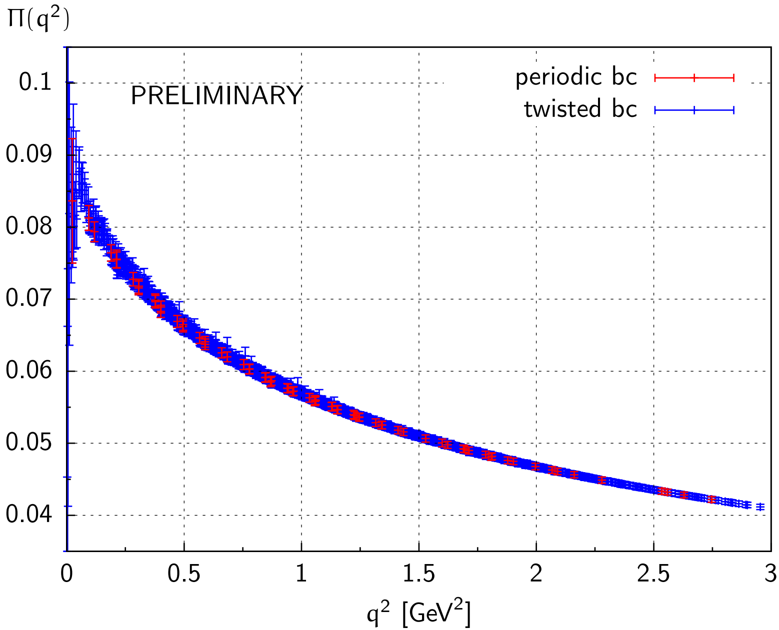

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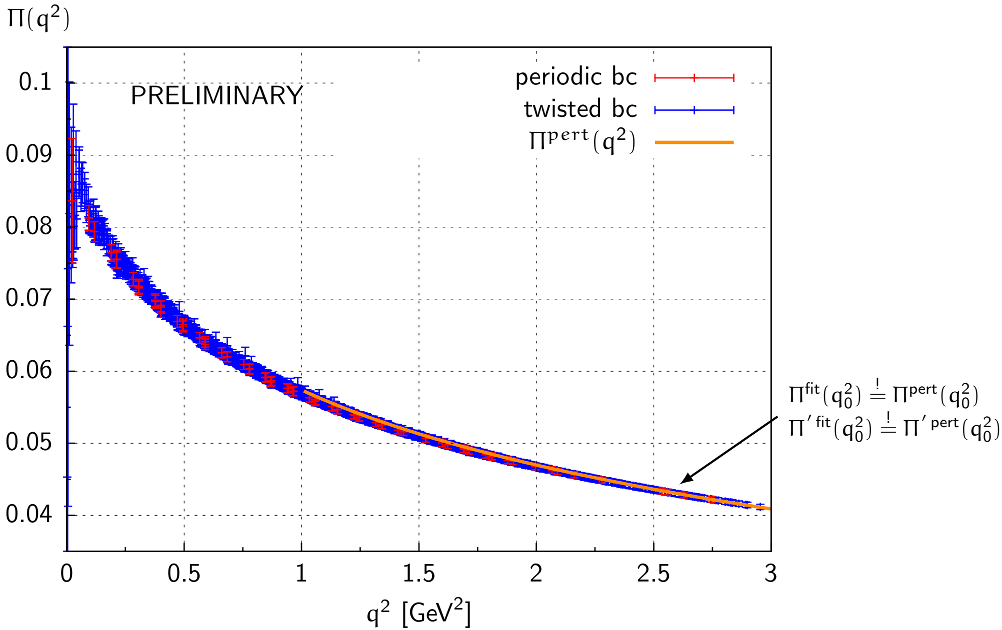

- 2-loop perturbation theory matched to lattice data at $q_0^2 \approx 2.6\,\mathrm{GeV}^2$

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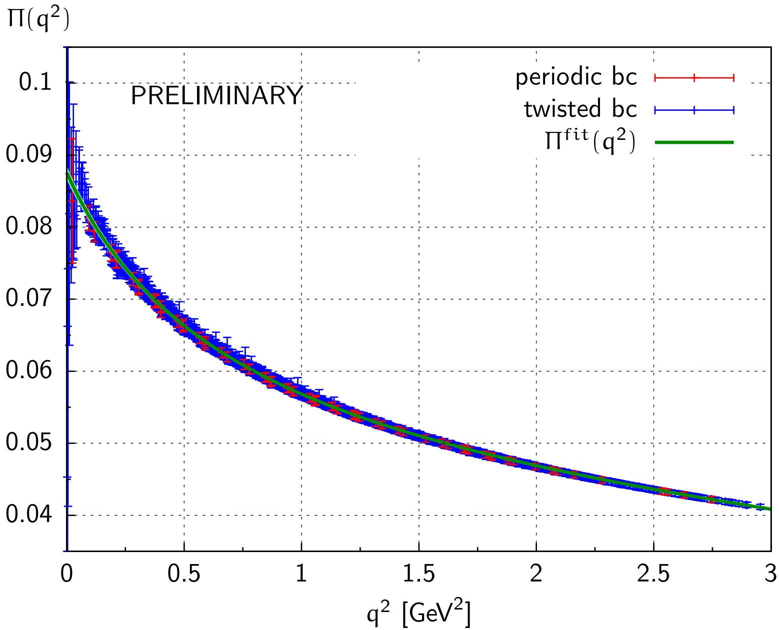
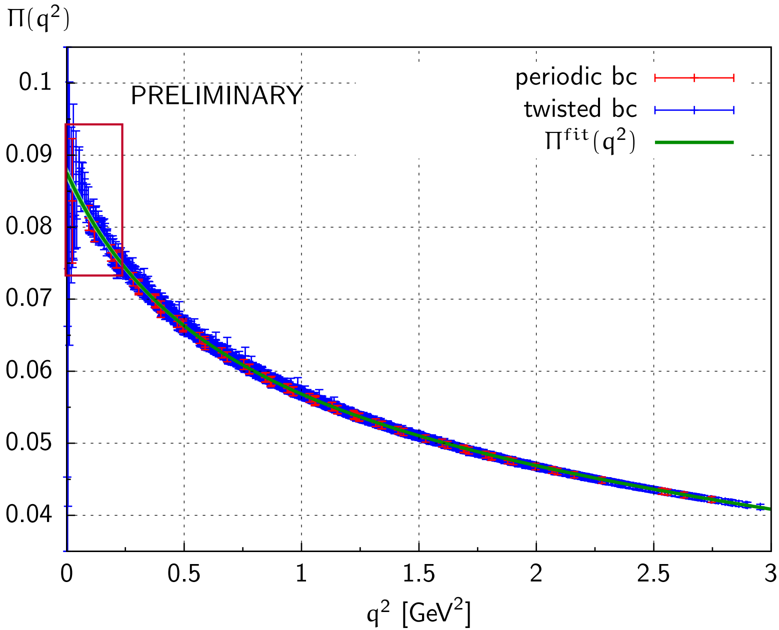


- Fit $\Pi(q^2)$ to well-behaved functions (e.g. Padé) [Blum, et al, 2012]

Vacuum Polarization $\Pi(q^2)$



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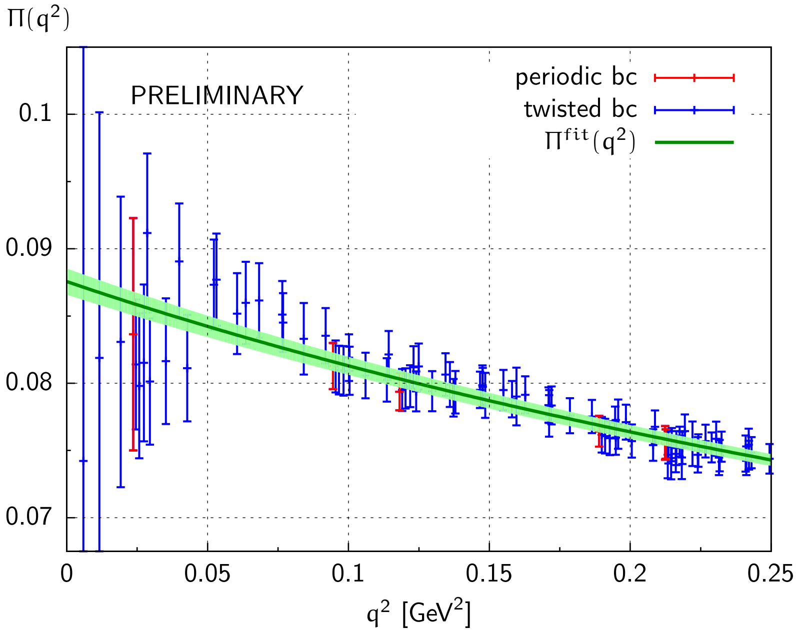
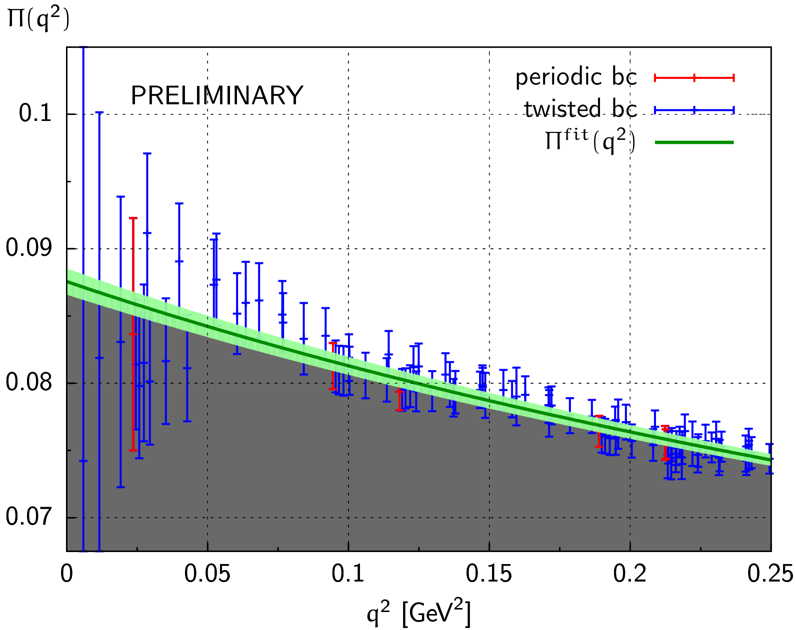


- Data from twisted boundary conditions improve stability of the fit

Vacuum Polarization $\Pi(q^2)$



- Determine a_μ^{had} by convolution integral: $a_\mu^{had} = 4\alpha^2 \int_0^\infty F\left(\frac{q^2}{m_\mu^2}\right) \left(\Pi(0) - \Pi(q^2)\right) dq^2$

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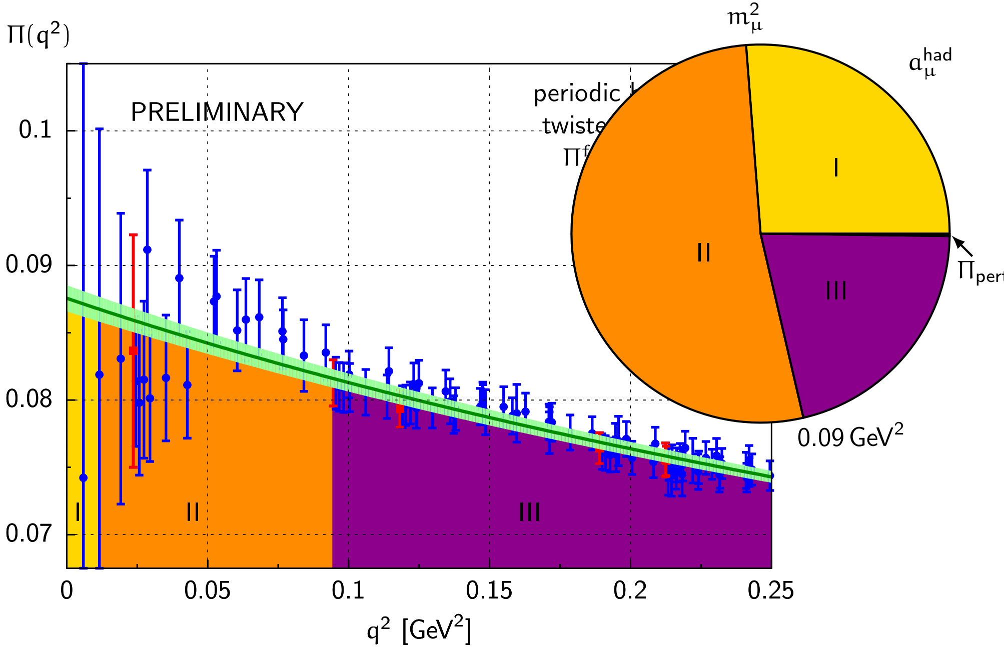
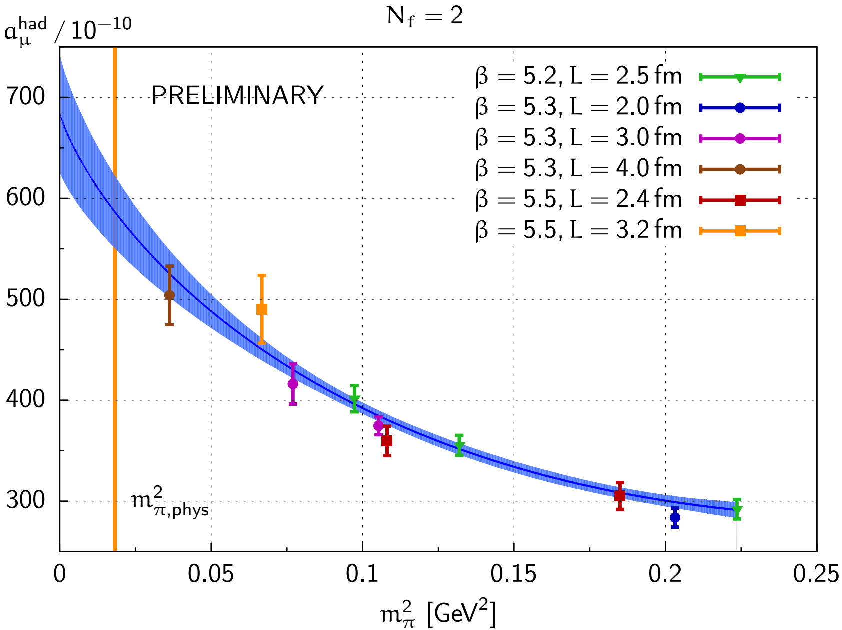


- Twisted boundary conditions improve the crucial low momentum behaviour

Hadronic Contribution to a_μ for $N_f = 2$



- Chiral behavior unknown: χPT inspired fit : $A + Bm_\pi^2 + Cm_\pi^2 \ln\left(m_\pi^2\right)$

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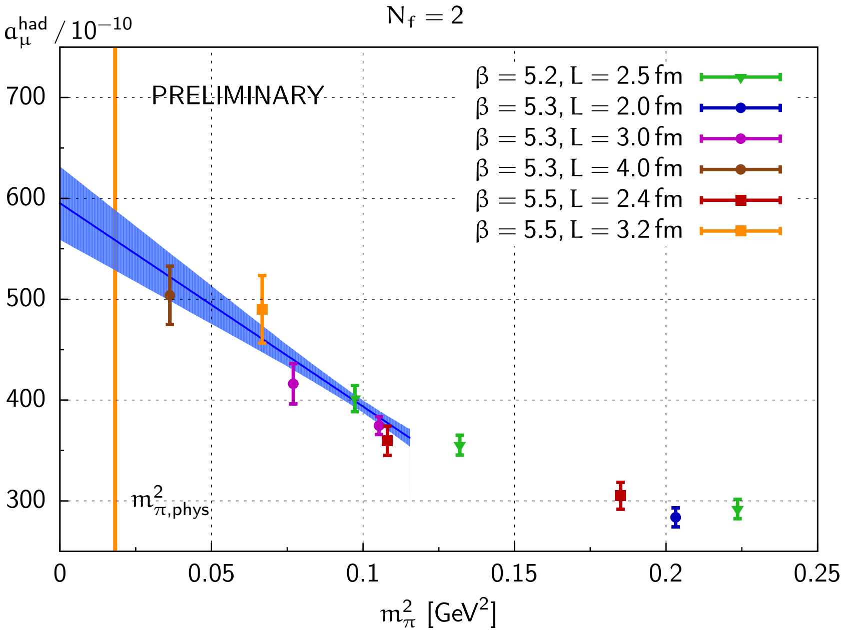
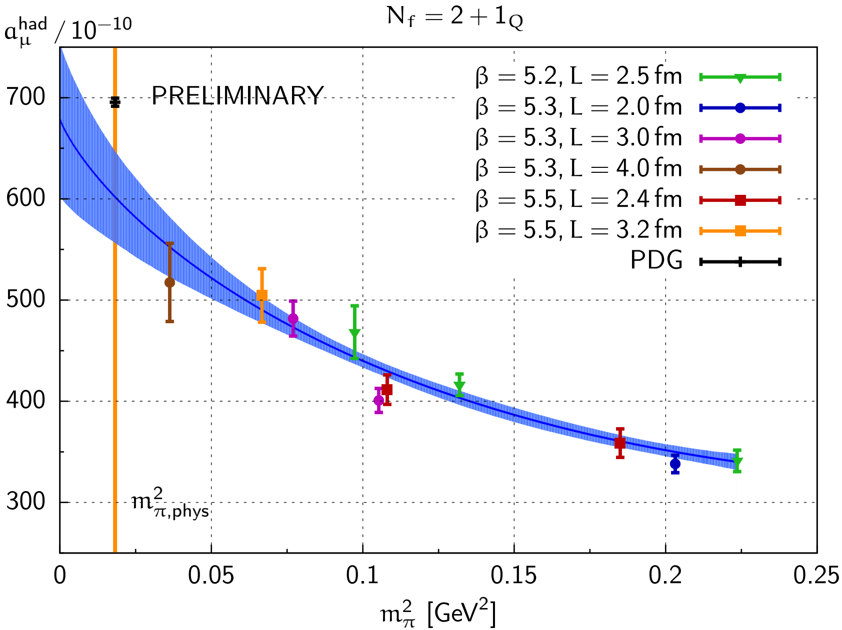


- Chiral behaviour unknown: Try linear extrapolation on most chiral points

Hadronic Contribution to a_μ for $N_f = 2 + 1_Q$



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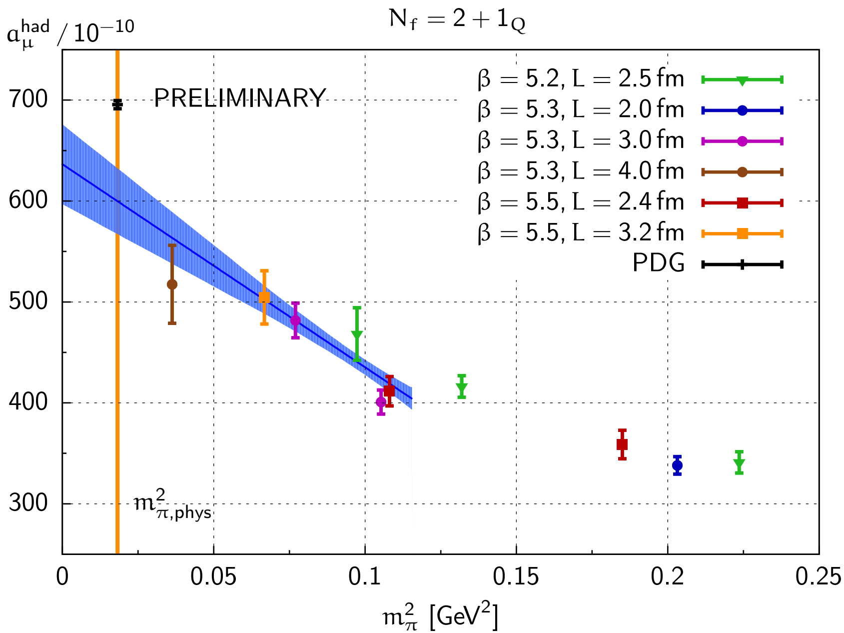


Outlook and Conclusion

Conclusion

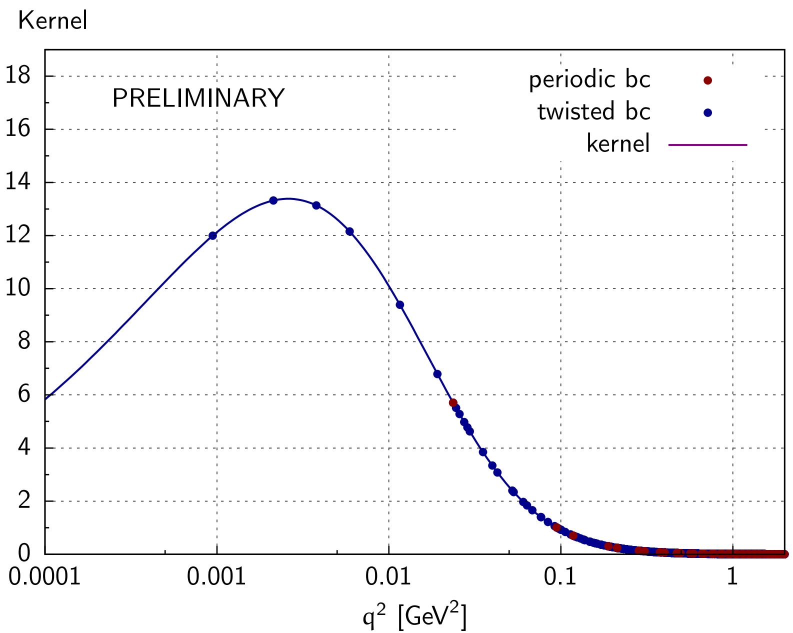
- Lattice QCD can calculate a_μ^{had} from first principles
- Twisted boundary conditions improve momentum dependence of $\Pi(q^2)$ and help to control the systematic uncertainties of a_μ
- New currents (local and conserved) reduce numerical cost by factor 5
- Chiral extrapolation improved by additional ensembles ($m_\pi^2 < 200\,\text{MeV}$)

Outlook

- Further improvements necessary to compete with phenomenological approach
  - Improve statistics (e.g. by multiple sources)
  - Study finite size and volume effects
  - Dynamical strange quark (and charm quark)
  - Disconnected diagrams (e.g. by hopping parameter expansion)
  - Simulations at the physical pion mass
  - Isospin breaking

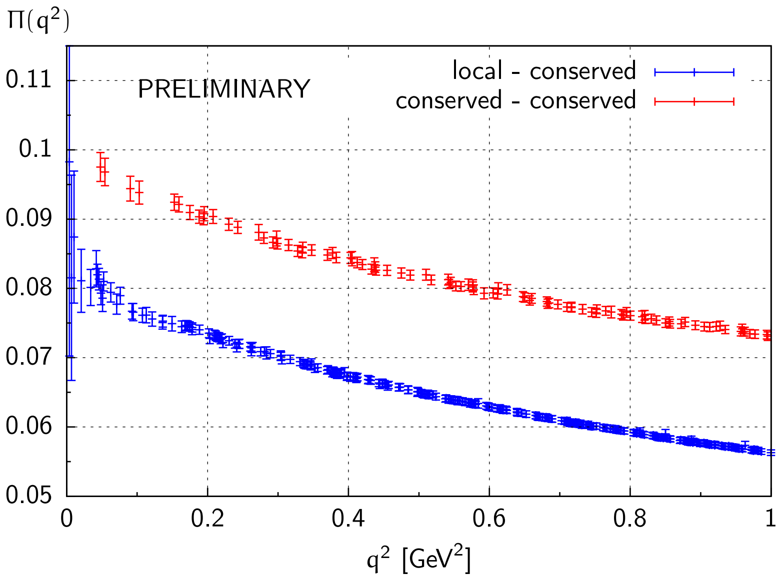
Thank you for your attention!

Vacuum Polarization $\Pi(q^2)$



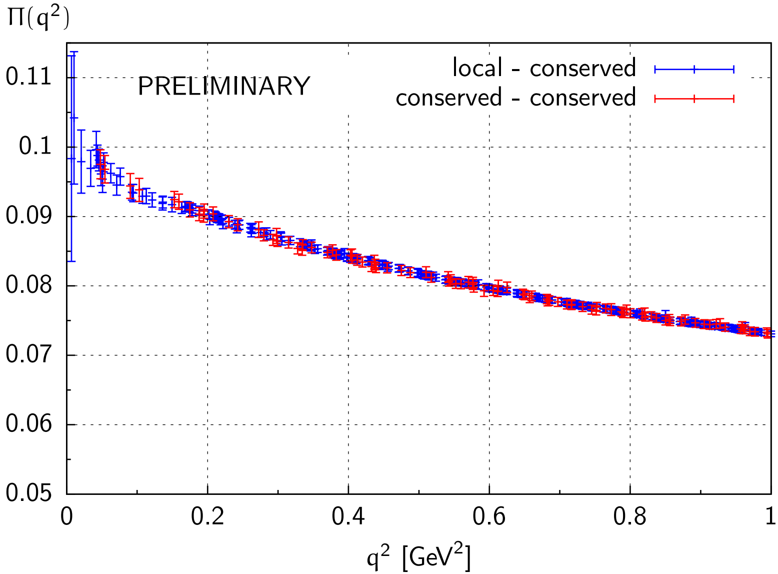
- Small momentum region crucial!

Vacuum Polarization $\Pi(q^2)$



- F6 ensemble: $\beta = 5.3$, $m_\pi = 324$ MeV, $L = 3.0$ fm

Vacuum Polarization $\Pi(q^2)$



- Subtracted vacuum polarisation $\widehat{\Pi}(q^2)$ is unchanged $\rightarrow a_\mu^{had}$ remains unchanged