Bottomonium at finite temperature

A signal for the quark-gluon plasma from lattice NRQCD

Tim Harris + FASTSUM

TRINITY
COLLEGE
DUBLIN

Anisotropic Symanzik gauge/2+1 Wilson clover

a_s^{-1} (GeV)	ξ	m_π/m_ρ	$m_\pi L$	N_s	N_τ
1.5	3.5	0.446	3.9	16	128

- But $m_b \sim 5$ GeV...

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Effective field theory for heavy quarks

- Omit modes $\gtrsim m_b \sim 5$ GeV.
- Power counting in $(p/m_b)^2 \sim v^2 \approx 0.1$.
- Heavy quark phase symmetry.

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Eucl. Cont. NRQCD Lagrangian

$$\mathcal{L}_0 = \psi^\dagger(x)\left[+D_\tau - \frac{\boldsymbol{D}^2}{2m_b}\right]\psi(x),$$

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$$\delta\mathcal{L}_{v^2} = \psi^\dagger(x)\left[-\frac{(\boldsymbol{D}^2)^2}{8m_b^3} + \frac{ig_0}{8m_b^2}(\boldsymbol{D}\cdot\boldsymbol{E} - \boldsymbol{E}\cdot\boldsymbol{D})\right]\psi(x),$$

$$\delta\mathcal{L}_{\boldsymbol{\sigma},v^4} = \psi^\dagger(x)\left[-\frac{g_0}{8m_b^2}\boldsymbol{\sigma}\cdot(\boldsymbol{D}\times\boldsymbol{E} - \boldsymbol{E}\times\boldsymbol{D}) - \frac{g_0}{2m_b}\boldsymbol{\sigma}\cdot\boldsymbol{B}\right]\psi(x).$$

$$G(n + a_\tau \boldsymbol{e}_\tau) = \left(1 - \frac{a_\tau H_0|_{n_\tau + a_\tau}}{2}\right) U_\tau^\dagger(n) \left(1 - \frac{a_\tau H_0|_{n_\tau}}{2}\right) (1 - a_\tau \delta H)\, G(n)$$

Lattice NRQCD

- Heavy quark propagators solve initial value problem $\implies$ cheap.
- ☛ 'Energy shift' undefined.
- No continuum limit! Must keep $a^{-1} \lesssim m_b$.

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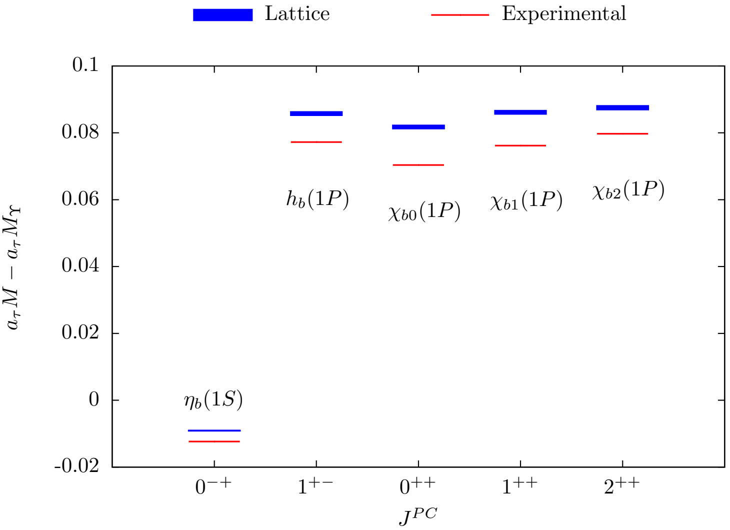
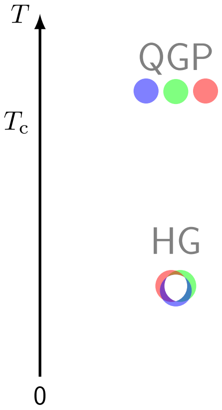
Tuning bare parameters

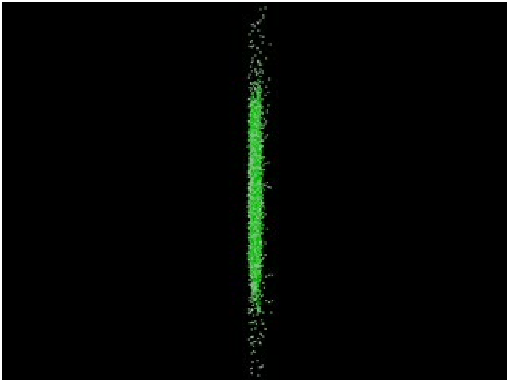
- Heavy quark rest mass plays no role.
- Tune $\hat{m}_b$ via meson dispersion relation.
- Use $1S$ spin-averaged 'kinetic mass'.



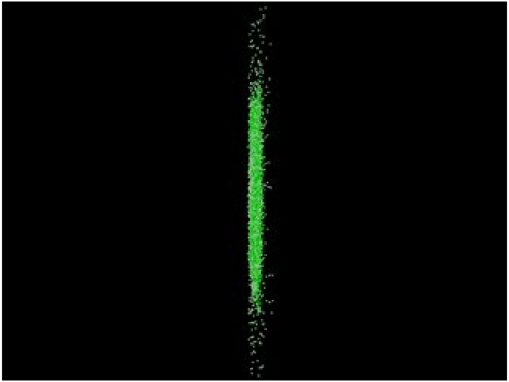
Lattice
Experimental
0.1
0.08
0.06
0.04
0.02
0
-0.02
$a_\tau M - a_\tau M_\Upsilon$
$h_b(1P)$
$\chi_{b0}(1P)$
$\chi_{b1}(1P)$
$\chi_{b2}(1P)$
$\eta_b(1S)$
0^{-+}
1^{+-}
0^{++}
1^{++}
2^{++}
J^{PC}


T
0


T
QGP
T_c
HG
0



Credit: *Jeffery Mitchell. VNI model by Klaus Kinder-Geiger and Ron Longacre, Brookhaven National Laboratory*



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A signal for the QGP

- Suppression of J/ψ yield in RHICs [Matsui & Satz].
- b-quark ‘cleaner’ probe.
- Sequential Υ suppression observed at LHC `arxiv:1208.2826`.

Finite temperature

- Simulate with $L_\tau = N_\tau a_\tau = \beta$ and appropriate b.c.s.
- Fixed-scale approach: vary temperature by changing N_τ.

N_s	N_τ	T/T_c	N_{cfg}
24	{16,...,40}	{1.75,...,0.70}	≥500

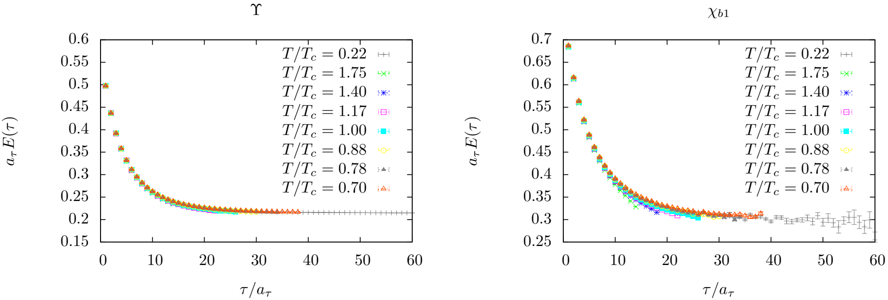

Figure: Temperature dependence of the effective energies

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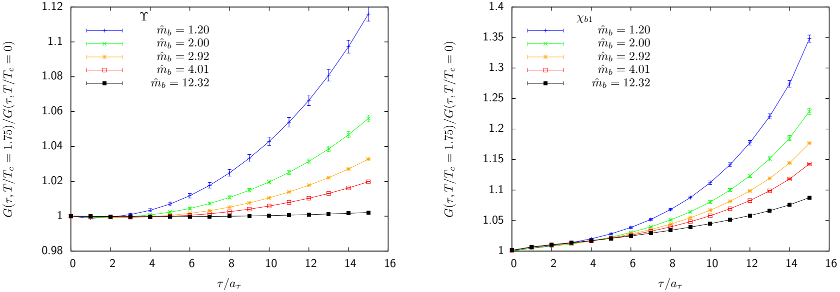

Figure: $\hat{m}_b$-dependence of the correlators

Cont. nearly-free dynamics in NRQCD

$$G(\tau) \sim \int_{-\omega_0}^{\infty} \frac{\mathrm{d}\omega}{\pi} e^{-(\omega+\omega_0)\tau} \rho(\omega)$$

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☛ Temperature dependence enters through interaction with the hot medium and not kinematically via the boundary conditions.

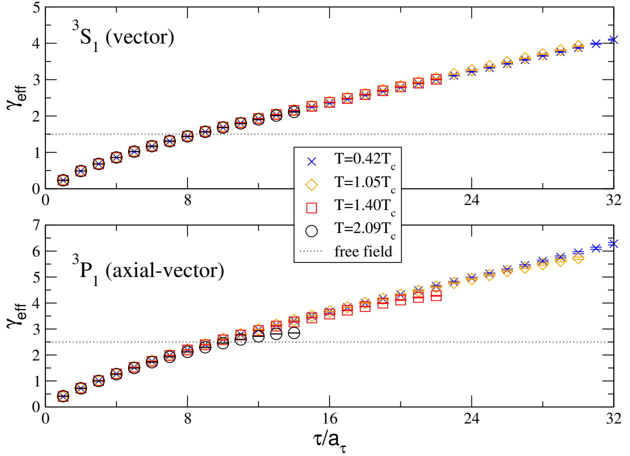
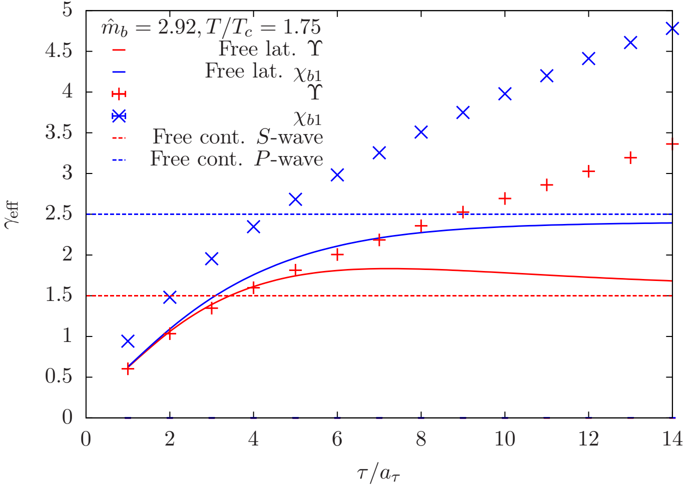
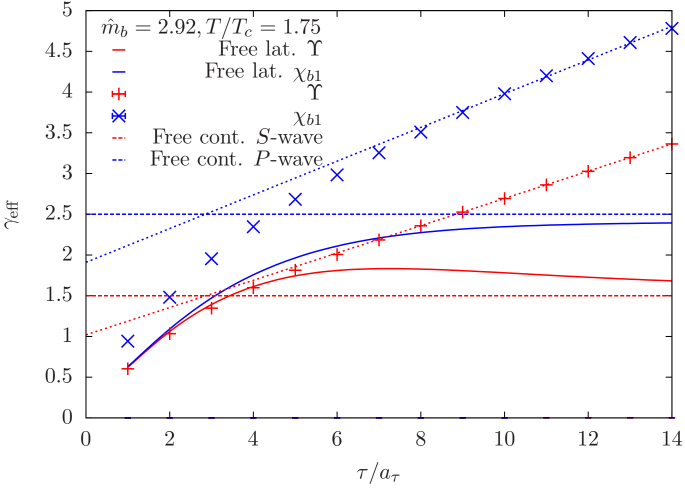


Figure: Previous study suggested melting P-wave at $T/T_c \sim 2$
`arXiv:1010.3725`



$\hat{m}_b = 2.92, T/T_c = 1.75$
Free lat. Υ
Free lat. χ_{b1}
Υ
χ_{b1}
Free cont. S-wave
Free cont. P-wave
$\gamma_{\rm eff}$
τ/a_τ
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
0
2
4
6
8
10
12
14



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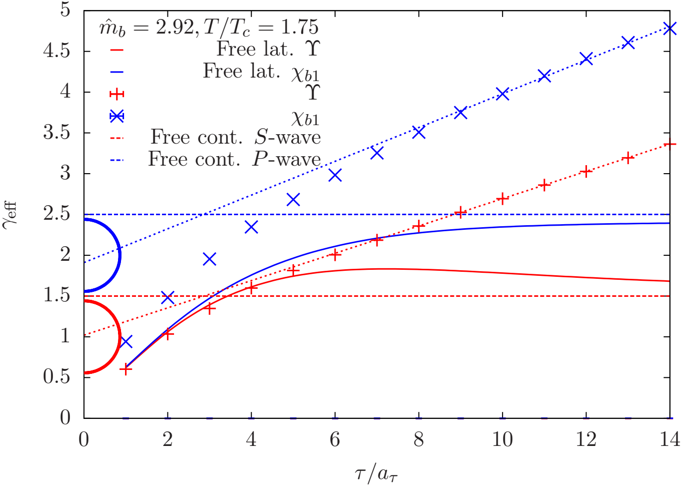
Free lat. Υ
Free lat. χ_{b1}
Υ
χ_{b1}
Free cont. S-wave
Free cont. P-wave
$\gamma_{\rm eff}$
τ/a_τ
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
0
2
4
6
8
10
12
14



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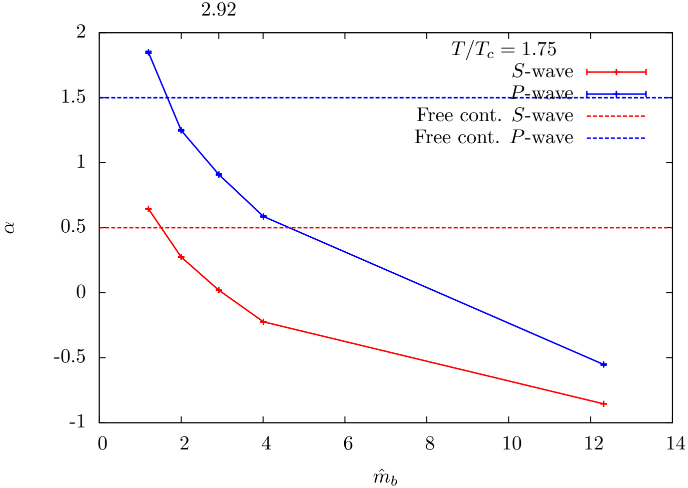
Free lat. Υ
Free lat. χ_{b1}
Υ
χ_{b1}
Free cont. S-wave
Free cont. P-wave
$\gamma_{\rm eff}$
τ/a_τ
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
0
2
4
6
8
10
12
14



2.92
$T/T_c = 1.75$
S-wave
P-wave
Free cont. S-wave
Free cont. P-wave
2
1.5
1
0.5
0
-0.5
-1
α
0
2
4
6
8
10
12
14
$\hat{m}_b$

NRQCD

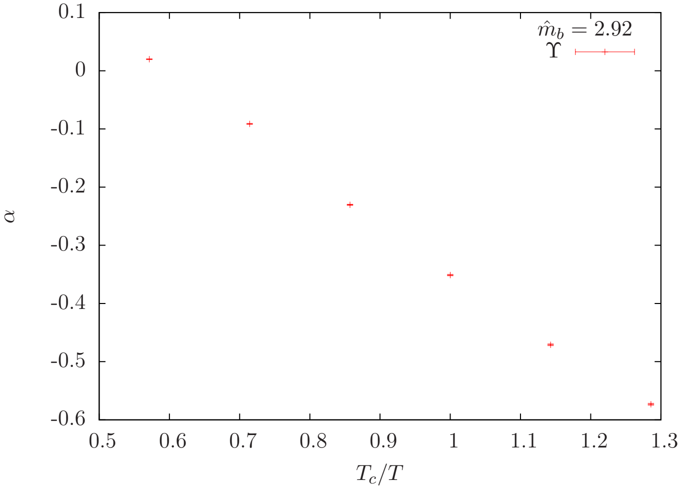
- Radiatively improve NRQCD action with automated LPT.

Finite temperature

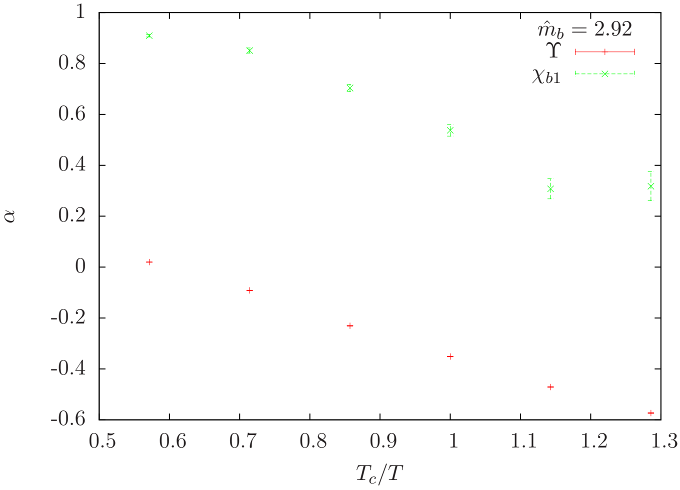
- Compare analysis of correlators with spectral functions from MaxEnt.

New ensembles with $\xi = 7$

- Anisotropy tuning with Wilson flow.



$\hat{m}_b = 2.92$
Υ
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
α
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
T_c/T



$\hat{m}_b = 2.92$
Υ
χ_{b1}
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
α
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
T_c/T

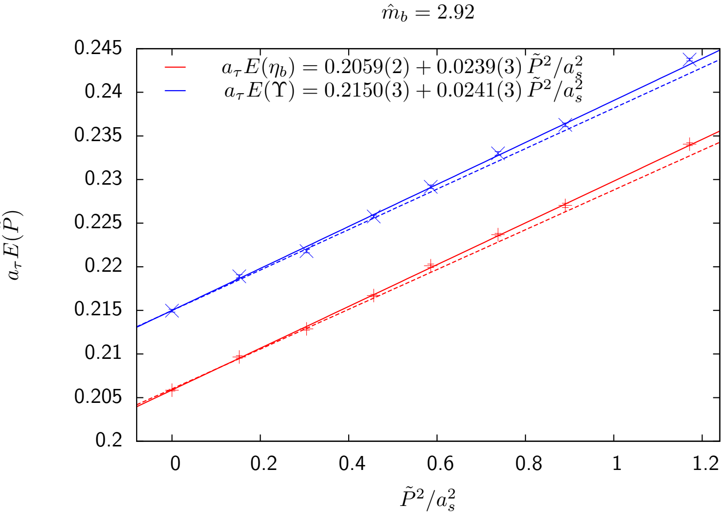
$$S_\psi = a_s^3 \sum_{n\in\Lambda} \psi^\dagger(n)\left[\psi(n) - K(n_\tau)\psi(n - a_\tau \boldsymbol{e}_\tau)\right]$$

$$\begin{aligned}
H_0 &= -\frac{\Delta^{(2)}}{2m_b} \\
\delta H_{v^2} &= -\frac{(\Delta^{(2)})^2}{8m_b^3} + \frac{ig_0}{8m_b^2}(\boldsymbol{\nabla}^\pm \cdot \boldsymbol{E} - \boldsymbol{E} \cdot \boldsymbol{\nabla}^\pm) \\
\delta H_{\boldsymbol{\sigma}} &= -\frac{g_0}{8m_b^2}\boldsymbol{\sigma} \cdot (\boldsymbol{\nabla}^\pm \times \boldsymbol{E} - \boldsymbol{E} \times \boldsymbol{\nabla}^\pm) - \frac{g_0}{2m_b}\boldsymbol{\sigma} \cdot \boldsymbol{B} \\
\delta H_{\text{imp}} &= \frac{a_s^2 \Delta^{(4)}}{24m_b} - \frac{a_\tau (\Delta^{(2)})^2}{16km_b^2} \\
\text{where} \quad \Delta^{(2)} &= \sum_i \nabla_i^+ \nabla_i^-, \quad \text{and} \quad \Delta^{(4)} = \sum_i (\nabla_i^+ \nabla_i^-)^2
\end{aligned}$$

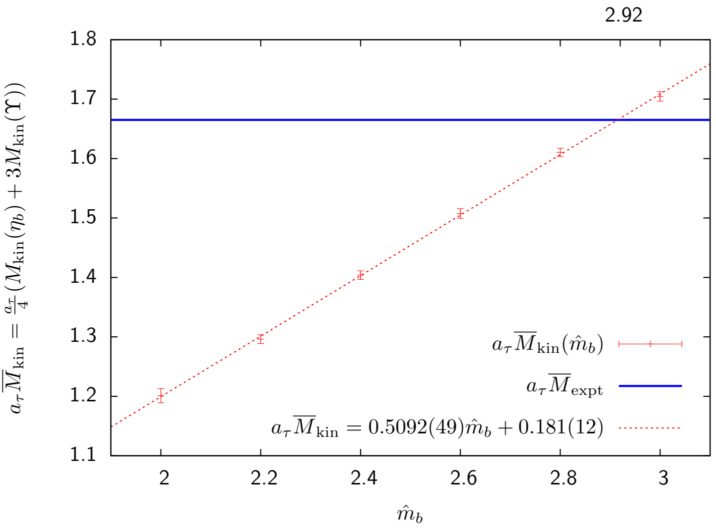
[1] CMS Collaboration. Observation of sequential Υ suppression in PbPb collisions. *Physical Review Letters*, 109:222301, 2012, arXiv:1208.2826.
[2] R. Rapp, D. Blaschke, and P. Crochet. Charmonium and bottomonium in heavy-ion collisions. *Progress in Particle and Nuclear Physics*, 65:209, 2010, arXiv:0807.2470.
[3] Y. Burnier, M. Laine, and M. Vepsäläinen. Heavy quarkonium in any channel in resummed hot QCD. *Journal of High Energy Physics*, 0801:043, 2008, arXiv:0711.1743.
[4] G. Aarts, S. Kim, M. P. Lombardo, M. B. Oktay, S. M. Ryan, D. K. Sinclair, and J.-I. Skullerud. Bottomonium above deconfinement in lattice nonrelativistic QCD. *Physical Review Letters*, 106:061602, 2011, arXiv:1010.3725.

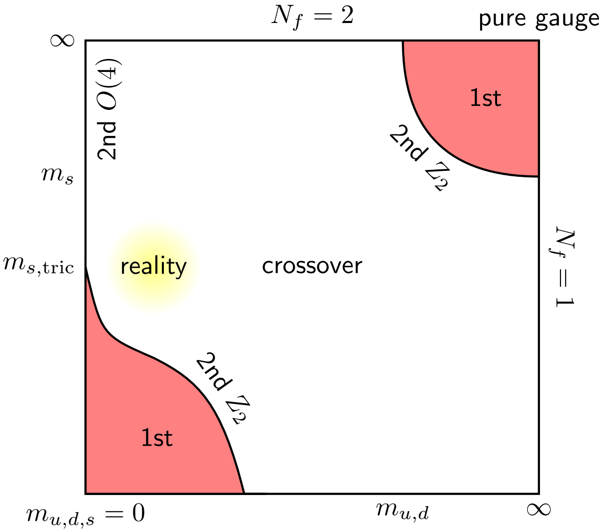
Latticed Window
(with the Camera Obscura)
August 1835

When first made, the squares
of glass about 200 in number
could be counted, with help
of a lens.



$\hat{m}_b = 2.92$
$a_\tau E(\eta_b) = 0.2059(2) + 0.0239(3)\,\tilde{P}^2/a_s^2$
$a_\tau E(\Upsilon) = 0.2150(3) + 0.0241(3)\,\tilde{P}^2/a_s^2$
$a_\tau E(\tilde{P})$
$\tilde{P}^2/a_s^2$
0.245
0.24
0.235
0.23
0.225
0.22
0.215
0.21
0.205
0.2
0
0.2
0.4
0.6
0.8
1
1.2


2.92
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
2
2.2
2.4
2.6
2.8
3
$\hat{m}_b$
$a_\tau \overline{M}_{\rm kin} = \frac{a_\tau}{4}(M_{\rm kin}(\eta_b) + 3M_{\rm kin}(\Upsilon))$
$a_\tau \overline{M}_{\rm kin}(\hat{m}_b)$
$a_\tau \overline{M}_{\rm expt}$
$a_\tau \overline{M}_{\rm kin} = 0.5092(49)\hat{m}_b + 0.181(12)$


$N_f = 2$
pure gauge
∞
2nd $O(4)$
1st
2nd Z_2
m_s
$N_f = 1$
$m_{s,\mathrm{tric}}$
reality
crossover
2nd Z_2
1st
$m_{u,d,s} = 0$
$m_{u,d}$
∞