- Let $f(p^2)$ be a smooth function. For a sufficiently large L:

$$\frac{1}{L}\sum_n f(p_n^2) = \int \frac{dp}{2\pi}\, f(p^2)\,,$$

where $p_n = (2\pi/L)\,n$ and the relation holds "locally".

- In actual lattice calculations the spacing between momenta are $O(\text{few}\,100\,\text{MeV})$ so we would not expect such a local relation to be sufficiently accurate.
- However using the Poisson summation formula:

$$\sum_{n=-\infty}^{\infty} \delta(x-n) = \sum_{n=-\infty}^{\infty} \exp(2\pi i n x)$$

we obtain the powerful exact relation

$$\frac{1}{L}\sum_{n=-\infty}^{\infty} f(p_n^2) = \int_{-\infty}^{\infty} \frac{dp}{2\pi} f(p^2) + \sum_{n\neq 0} \int_{-\infty}^{\infty} \frac{dp}{2\pi} f(p^2)\, e^{inpL}\,,$$

which implies that

$$\boxed{\frac{1}{L}\sum_n f(p_n^2) = \int_{-\infty}^{\infty} \frac{dp}{2\pi}\, f(p^2)\,,}$$

up to exponentially small corrections in L.

- In the approach developed with Steve and Changhoan Kim, this is the starting point for all calculations of FV effects.
- Calculate the leading finite-volume effects for

$$f(p^2) = \frac{1}{p^2 + m^2}\,.$$