Flavour Physics – Lecture 1

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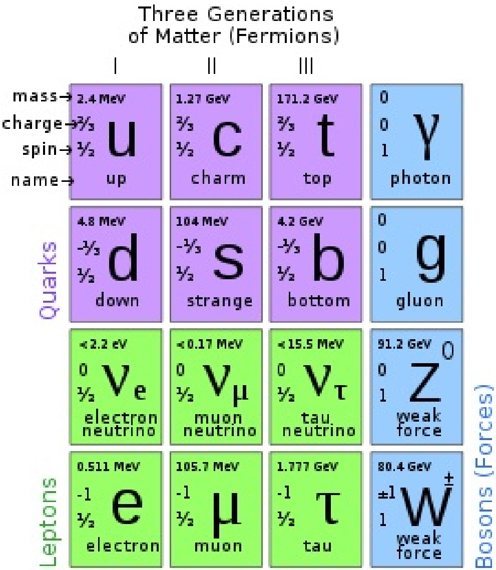
Standard Model of Particle Physics



- "Ordinary Matter" composed of elements from the first column.
- Plus the Higgs Boson.

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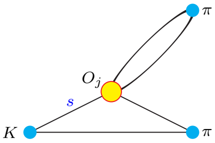
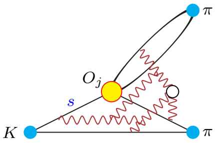
- (Precision) Flavour physics, weak interaction processes in which the flavour (u, d, s, c, b, t) quantum number changes, is a key tool in exploring the limits of the Standard Model of Particle Physics and in searches for new physics.
- It is complementary to high-energy experiments (most notably the LHC).
  - If, as expected/hoped the LHC experiments discover new elementary particles BSM, then precision flavour physics will be necessary to understand the underlying framework.
  - The discovery potential of precision flavour physics should also not be underestimated. (In principle, the reach is about two-orders of magnitude deeper than the LHC!)
  - Precision flavour physics requires control of hadronic effects for which lattice QCD simulations are essential.

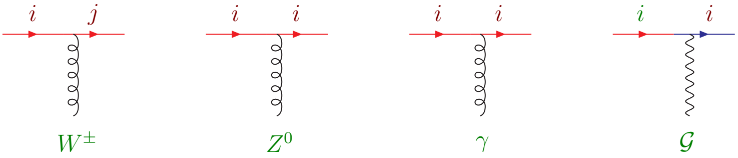


means



- In fact, it is a major surprise to many of us that no unambiguous inconsistencies have arisen up to now.

- In the Standard Model, the interaction of quarks with the gauge-bosons can be illustrated by the following vertices:



i,j represent the quark *flavour* $\{i,j = u,d,c,s,t,b\}$.
Colour is the charge of the strong interactions.

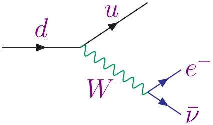
- In these lectures we will be particularly interested in the weak interactions. Feynman rule for W-vertex above is

$$i\frac{g_2}{2\sqrt{2}}V_{ij}\gamma_\mu(1-\gamma_5)\ ,$$

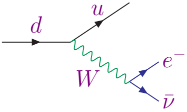
where g_2 is the coupling constant of the $SU(2)_L$ gauge group and V is the (unitary) Cabibbo-Kobayashi-Maskawa (CKM) matrix (see below).

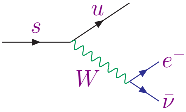
Generalized β-Decays

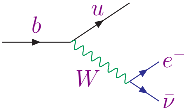
- At the level of quarks we understand nuclear β decay in terms of the fundamental process:



- With the 3 generations of quarks and leptons in the standard model this is generalized to other *charged current* processes, e.g.:







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- Experiment ⇒ only the left-handed components of the fermions participate in charged current weak interactions, i.e. the W's only couple to the left-handed components.
-
$$\psi_L = P_L \psi = \frac{1}{2}(1-\gamma^5)\psi \qquad \psi_R = P_R \psi = \frac{1}{2}(1+\gamma^5)\psi$$
Under parity transformations $\psi_L(x_0,\vec{x}) \to \gamma_0 \psi_R(x_0,-\vec{x})$ and $\psi_R(x_0,\vec{x}) \to \gamma_0 \psi_L(x_0,-\vec{x})$
- P_L and P_R are projection operators
$$P_L^2 = P_L \quad \text{and} \quad P_R^2 = P_R \qquad (P_L P_R = P_R P_L = 0,\; P_L + P_R = I)$$
-
$$\bar{\psi}\gamma^\mu\psi = \bar{\psi}_L\gamma^\mu\psi_L + \bar{\psi}_R\gamma^\mu\psi_R \quad \text{and} \quad \bar{\psi}\psi = \bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L\,.$$
(Thus for QCD with N massless fermions we have a $U(N)\times U(N)$ (global) chiral symmetry ⇒ $SU(N)_L \times SU(N)_R$.) See Steve Sharpe's Lectures
- In order to accommodate the observed nature of the parity violation the left and right-handed fermions are assigned to different representations of $SU(2)\times U(1)$, with the right-handed fields being singlets of $SU(2)$.

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For a general representation of fermions the covariant derivative takes the form:

$$D_\mu = \partial_\mu - igW^a_\mu T^a - ig' Y B_\mu\,,$$

where the T^a are the corresponding generators of $SU(2)$ and the $Y's$ are the weak-hypercharges. The covariant derivative can be rewritten in terms of the mass-eigenstates as:

$$D_\mu = \partial_\mu - \frac{ig}{\sqrt{2}}(W^+_\mu T^+ + W^-_\mu T^-) - i\frac{g^2T^3 - g'^2Y}{\sqrt{g^2+g'^2}}Z_\mu - i\frac{gg'}{\sqrt{g^2+g'^2}}(T^3+Y)A_\mu\,.$$

- Thus the electic charge operator is

$$Q = T_3 + Y \quad \text{and} \quad e = \frac{gg'}{\sqrt{g^2+g'^2}}\,. \qquad (Q=-1 \text{ for the electron}).$$

- The left-handed quarks and leptons are assigned to doublets of $SU(2)$ and the right-handed fermions are singlets.

$$Q = T_3 + Y$$

- The left handed leptons are assigned to the doublet.

$$E_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L .$$

  In order to have the correct charge assignments $Y_{\nu_e} = Y_{e_L} = -1/2$.
- For the right-handed lepton fields $T_3 = 0$ and hence $Y_{e_R} = -1$. In the standard model we do not have a right-handed neutrino!
- For the left-handed quark fields we have the left-handed doublet:

$$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L .$$

  with $Y_{Q_L} = 1/6$.
- The right-handed quark fields therefore have $Y_{u_R} = 2/3$ and $Y_{d_R} = -1/3$.
- Similar assignments are made for the other two generations.

The terms in the Lagrangian involving the first-generation fermions then take the form:

$$\begin{aligned}\mathscr{L} &= \bar{E}_L(i\,\partial\!\!\!/)E_L + \bar{e}_R(i\,\partial\!\!\!/)e_R + \bar{Q}_L(i\,\partial\!\!\!/)Q_L + \bar{u}_R(i\,\partial\!\!\!/)u_R + \bar{d}_R(i\,\partial\!\!\!/)d_R \\ &\quad + g\left(W_\mu^+ J_W^{\mu\,+} + W_\mu^- J_W^{\mu\,-} + Z_\mu^0 J_Z^\mu\right) + eA_\mu J_{\rm EM}^\mu\,,\end{aligned}$$

where

$$\begin{aligned}J_W^{\mu\,+} &= \frac{1}{\sqrt{2}}\left(\bar{\nu}_L\gamma^\mu e_L + \bar{u}_L\gamma^\mu d_L\right); \\ J_W^{\mu\,-} &= \frac{1}{\sqrt{2}}\left(\bar{e}_L\gamma^\mu \nu_L + \bar{d}_L\gamma^\mu u_L\right); \\ J_Z^\mu &= \frac{1}{\cos\theta_W}\left\{\frac{1}{2}\,\bar{\nu}_L\gamma^\mu\nu_L + \left(\sin^2\theta_W - \frac{1}{2}\right)\bar{e}_L\gamma^\mu e_L + \sin^2\theta_W\,\bar{e}_r\gamma^\mu e_R\right. \\ &\quad + \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right)\bar{u}_L\gamma^\mu u_L - \frac{2}{3}\sin^2\theta_W\,\bar{u}_R\gamma^\mu u_R \\ &\quad \left. + \left(\frac{1}{3}\sin^2\theta_W - \frac{1}{2}\right)\bar{d}_L\gamma^\mu d_L + \frac{1}{3}\sin^2\theta_W\,\bar{d}_R\gamma^\mu d_R\right\}; \\ J_{\rm EM}^\mu &= -\bar{e}\gamma^\mu e + \frac{2}{3}\,\bar{u}\gamma^\mu u - \frac{1}{3}\,\bar{d}\gamma^\mu d\,.\end{aligned}$$

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- The *weak mixing angle* θ_W is defined by:

$$\begin{pmatrix} Z^0 \\ A \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W^3 \\ B \end{pmatrix}$$

  so that

$$\cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}\,, \quad \text{and} \quad \sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}}\,.$$

- At tree level

$$m_W = m_Z \cos\theta_W\,.$$

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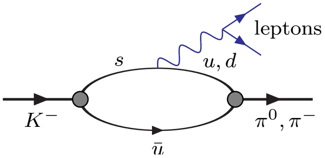
Two Experimental Numbers:

$$B(K^- \to \pi^0 e^- \nu_e) \simeq 5\% \ (K_{e3}\ \text{Decay}) \quad \text{and} \quad B(K^- \to \pi^- e^+ e^-) < 3 \times 10^{-7}\,.$$



- Measurements like this show that $s \to u$ (charged-current) transitions are not very rare, but that *Flavour Changing Neutral Current* (FCNC) transitions, such as $s \to d$ are.
- In the picture that we have developed so far, there are no transitions between fermions of different generations. This has to be modified.
- The picture which has emerged is the Cabibbo-Kobayashi-Maskawa (CKM) theory of quark mixing which we now consider.

In the CKM theory the (quark) mass eigenstates are not the same as the weak-interaction eigenstates which we have been considering up to now.
Let

$$U' = \begin{pmatrix} u' \\ c' \\ t' \end{pmatrix} = U_u \begin{pmatrix} u \\ c \\ t \end{pmatrix} = U_u U \quad \text{and} \quad D' = \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = U_d \begin{pmatrix} d \\ s \\ b \end{pmatrix} = U_d D$$

where the $'$s denote the weak interaction eigenstates and U_u and U_d are unitary matrices.

- For neutral currents:

$$\bar{U}' \cdots U' = \bar{U} \cdots U \quad \text{and} \quad \bar{D}' \cdots D' = \bar{D} \cdots D$$

  and no FCNC are induced. The $\cdots$ represent Dirac Matrices, but the identity in flavour.

- For charged currents:

$$J_W^{\mu\,+} = \frac{1}{\sqrt{2}}\,\bar{U}'_L \gamma^\mu D'_L = \frac{1}{\sqrt{2}}\,\bar{U}_L U_u^\dagger \gamma^\mu U_d D_L = \frac{1}{\sqrt{2}}\,\bar{U}_L \gamma^\mu (U_u^\dagger U_d) D_L \equiv \frac{1}{\sqrt{2}}\,\bar{U}_L \gamma^\mu V_{\text{CKM}} D_L$$

- The charged-current interactions are of the form

$$J^+_\mu = (\bar u, \bar c, \bar t\,)_L \gamma_\mu V_{\rm CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L ,$$

- 2012 Particle Data Group summary for the magnitudes of the entries:

$$\begin{pmatrix} 0.97427 \pm 0.00015 & 0.22534 \pm 0.00065 & 0.00351^{+0.00015}_{-0.00014} \\ 0.22520 \pm 0.00065 & 0.97344 \pm 0.00016 & 0.0412^{+0.0011}_{-0.0005} \\ 0.00867^{+0.00029}_{-0.00031} & 0.0404^{+0.0011}_{-0.0005} & 0.999146^{+0.000021}_{-0.000046} \end{pmatrix} .$$

- How many parameters are there?
  - Let N_g be the number of generations.
  - $N_g \times N_g$ unitary matrix has N_g^2 real parameters.
  - $(2N_g - 1)$ of them can be absorbed into unphysical phases of the quark fields.
  - $(N_g - 1)^2$ physical parameters to be determined.

- For $N_g = 2$ there is only one parameter, which is conventionally chosen to be the Cabibbo angle:

$$V_{\text{CKM}} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}.$$

- For $N_g = 3$, there are 4 real parameters. Three of these can be interpreted as angles of rotation in three dimensions (e.g. the three Euler angles) and the fourth is a phase. The general parametrization recommended by the PDG is

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

  where c_{ij} and s_{ij} represent the cosines and sines respectively of the three angles θ_{ij}, $ij = 12$, 13 and 23. δ_{13} is the phase parameter.

- It is conventional to use approximate parametrizations, based on the hierarchy of values in V_{CKM} $(s_{12} \gg s_{23} \gg s_{13})$.

The Wolfenstein parametrization is

$$V_{\mathrm{CKM}} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}.$$

- $\lambda = s_{12}$ is approximately the Cabibbo angle.
- A, ρ and η are real numbers that a priori were intended to be of order unity.
- Corrections are of $O(\lambda^4)$.

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Unitarity of the CKM-matrix we have a set of relations between the entries. A particularly useful one is:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0\ .$$

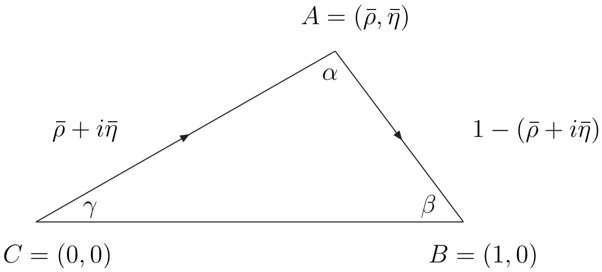
In terms of the Wolfenstein parameters, the components on the left-hand side are given by:

$$\begin{aligned} V_{ud}V_{ub}^* &= A\lambda^3[\bar\rho + i\bar\eta] + O(\lambda^7) \\ V_{cd}V_{cb}^* &= -A\lambda^3 + O(\lambda^7) \\ V_{td}V_{tb}^* &= A\lambda^3[1-(\bar\rho + i\bar\eta)] + O(\lambda^7)\ , \end{aligned}$$

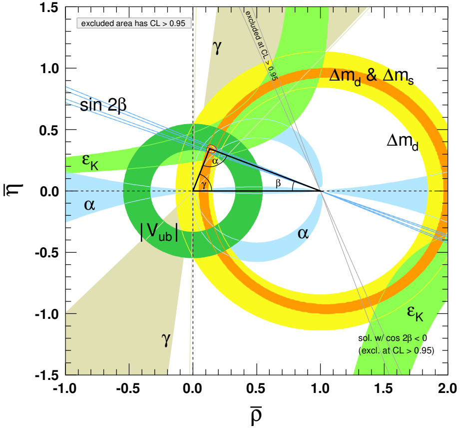
where $\bar\rho = \rho(1-\lambda^2/2)$ and $\bar\eta = \eta(1-\lambda^2/2)$.
The unitarity relation can be represented schematically by the famous “unitarity triangle" (obtained after scaling out a factor of $A\lambda^3$).

The Unitarity Triangle Cont.

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0\ .$$

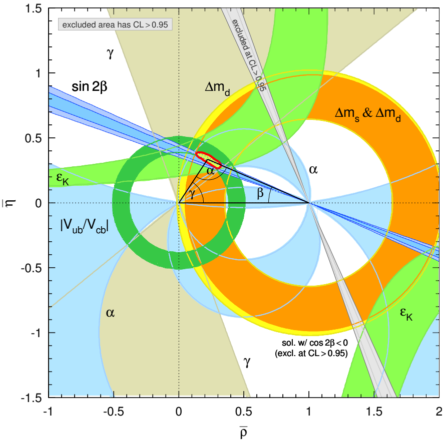


- A particularly important approach to testing the *Limits of the SM* is to over-determine the position of the vertex A to check for consistency.

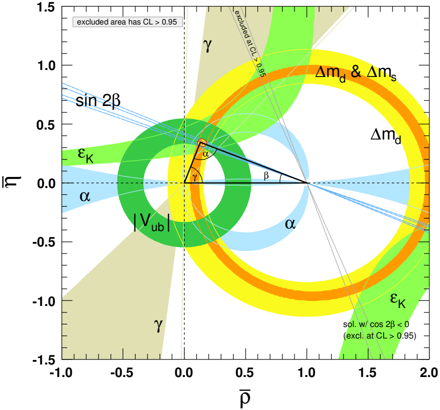


excluded area has CL > 0.95
excluded at CL > 0.95
γ
Δm_d & Δm_s
sin 2β
Δm_d
ε_K
α
β
|V_ub|
sol. w/ cos 2β < 0
(excl. at CL > 0.95)
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0
η̄
ρ̄

PDG2006 & 2012 Unitarity Triangle Comparison



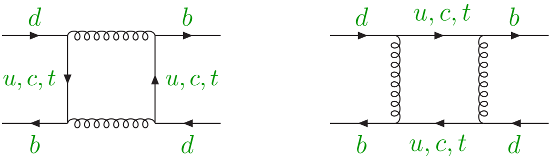
2006



2012

We have seen that in the SM, unitarity implies that there are no FCNC reactions at tree level, i.e. there are no vertices of the type:

Quantum loops, however, can generate FCNC reactions, through *box* diagrams or *penguin* diagrams.
Example relevant for $\bar{B}^0 - B^0$ mixing:



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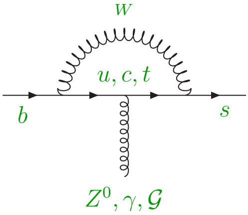
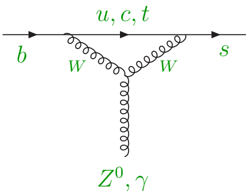
Examples of penguin diagrams relevant for $b \to s$ transitions:





We will discuss several of the physical processes induced by these loop-effects. The Glashow-Illiopoulos-Maiani (GIM) mechanism $\Rightarrow$ FCNC effects vanish for degenerate quarks ($m_u = m_c = m_t$). For example unitarity implies

$$V_{ub}V_{us}^* + V_{cb}V_{cs}^* + V_{tb}V_{ts}^* = 0$$

$\Rightarrow$ each of the above penguin vertices vanish.

- **Parity**

$$(\vec{x},t) \to (-\vec{x},t).$$

The vector and axial-vector fields transform as:

$$V_\mu(\vec{x},t) \to V^\mu(-\vec{x},t) \;\text{ and }\; A_\mu(\vec{x},t) \to -A^\mu(-\vec{x},t).$$

  - The vector and axial-vector currents transform similarly.

Left-handed components of fermions $\psi_L = (\frac{1}{2}(1-\gamma^5)\psi)$ transform into right-handed ones $\psi_R = (\frac{1}{2}(1+\gamma^5)\psi)$, and vice-versa.

  - Since CC weak interactions in the SM only involve the left-handed components, parity is not a good symmetry of the weak force.
  - QCD and QED are invariant under parity transformations.

- **Charge Conjugation** – Charge conjugation is a transformation which relates each complex field ϕ with $\phi^\dagger$.

  Under C the currents transform as follows:

  $$\bar\psi_1\gamma_\mu\psi_2 \to -\bar\psi_2\gamma_\mu\psi_1 \quad \text{and} \quad \bar\psi_1\gamma_\mu\gamma_5\psi_2 \to \bar\psi_2\gamma_\mu\gamma_5\psi_1,$$

  where ψ_i represents a spinor field of type (flavour or lepton species) i.

- **CP** – Under the combined CP-transformation, the currents transform as:

  $$\bar\psi_1\gamma_\mu\psi_2 \to -\bar\psi_2\gamma^\mu\psi_1 \quad \text{and} \quad \bar\psi_1\gamma_\mu\gamma_5\psi_2 \to -\bar\psi_2\gamma^\mu\gamma_5\psi_1.$$

  The fields on the left (right) hand side are evaluated at $(\vec{x},t)$ ($(-\vec{x},t)$).

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- Consider now a charged current interaction:

$$(W^1_\mu - iW^2_\mu)\,\bar{U}^i\gamma^\mu(1-\gamma^5)V_{ij}D^j + (W^1_\mu + iW^2_\mu)\,\bar{D}^j\gamma^\mu(1-\gamma^5)V^*_{ij}U^i,$$

U^i and D^j are up and down type quarks of flavours i and j respectively.

- Under a CP transformation, the interaction term transforms to:

$$(W^1_\mu + iW^2_\mu)\,\bar{D}^j\gamma^\mu(1-\gamma^5)V_{ij}U^i + (W^1_\mu - iW^2_\mu)\,\bar{U}^i\gamma^\mu(1-\gamma^5)V^*_{ij}D^j$$

- CP-invariance requires V to be real
(or more strictly that any phases must be able to be absorbed into the definition of the quark fields).
- For CP-violation in the quark sector we therefore require 3 generations.

Nobel prize in 2008 to Kobayashi and Maskawa.

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- The property of asymptotic freedom $\Rightarrow$ quark and gluon interactions become weak at short distances, i.e. distances $\ll 1$ fm.

  Nobel prize in 2004 to Gross, Politzer and Wilczek.

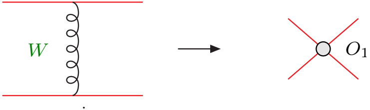
- Thus at short distances we can use perturbation theory.
- Schematically weak decay amplitudes are organized as follows:

$$\mathscr{A}_{i\to f} = \sum_j C_j(\mu)\,\langle f\,|\,O_j(0)\,|\,i\rangle_\mu$$

  where

  - The C_j contain the short-distance effects and are calculable in perturbation theory;
  - the long-distance *non-perturbative* effects are contained in the matrix elements of composite local operators $\{O_i(0)\}$ which are the quantities which are computed in lattice QCD simulations;
  - the renormalization scale μ can be viewed as the scale at which we separate the short-distances from long-distances.

- Quarks interact strongly ⇒ we have to consider QCD effects even in weak processes.
- Our inability to control (non-perturbative) QCD Effects is frequently the largest systematic error in attempts to obtain fundamental information from experimental studies of weak processes!
- Tree-Level:



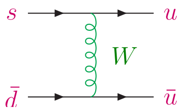
- Since $M_W \simeq 80\,$GeV, at low energies the momentum in the W-boson is much smaller than its mass ⇒ the four quark interaction can be approximated by the local Fermi β-decay vertex with coupling

$$\frac{G_F}{\sqrt{2}} = \frac{g_2^2}{8M_W^2}\ .$$

Operator Product Expansions and Effective Hamiltonians Cont.

- *Asymptotic Freedom* $\Rightarrow$ we can treat QCD effects at short distances, $|x| \ll \Lambda_{QCD}^{-1}$ ($|x| < 0.1$ fm say) or corresponding momenta $|p| \gg \Lambda_{QCD}$ ($|p| > 2$ GeV say), using perturbation theory.
- The natural scale of strong interaction physics is of $O(1\ \text{fm})$ however, and so in general, and for most of the processes discussed here, non-perturbative techniques must be used.
- For illustration consider $K \to \pi\pi$ decays, for which the tree-level amplitude is proportional to

$$\frac{G_F}{\sqrt{2}} V_{ud}^* V_{us} \langle \pi\pi | (\bar{d}\gamma^\mu (1-\gamma^5) u) (\bar{u}\gamma_\mu (1-\gamma^5) s) | K \rangle \ .$$



- We therefore need to determine the matrix element of the operator

$$O_1 = (\bar{d}\gamma^\mu (1-\gamma^5) u) (\bar{u}\gamma_\mu (1-\gamma^5) s) \ .$$

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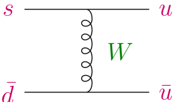
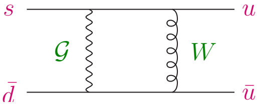




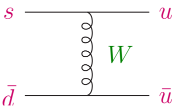
- Gluonic corrections generate a second operator $(\bar{d}T^a\gamma^\mu(1-\gamma^5)u)\,(\bar{u}T^a\gamma_\mu(1-\gamma^5)s)$, which by using Fierz Identities can be written as a linear combination of O_1 and O_2 where
$$O_2 = (\bar{d}\gamma^\mu(1-\gamma^5)s)\,(\bar{u}\gamma_\mu(1-\gamma^5)u)\,.$$
- OPE $\Rightarrow$ the amplitude for a weak decay process can be written as
$$A_{if} = \frac{G_F}{\sqrt{2}}V_{CKM}\sum_i C_i(\mu)\langle f|\,O_i(\mu)\,|i\rangle\ .$$
- μ is the renormalization scale at which the operators O_i are defined.
- Non-perturbative QCD effects are contained in the matrix elements of the O_i, which are independent of the large momentum scale, in this case of M_W.
- The Wilson coefficient functions $C_i(\mu)$ are independent of the states i and f and are calculated in perturbation theory.
- Since physical amplitudes manifestly do not depend on μ, the μ-dependence in the operators $O_i(\mu)$ is cancelled by that in the coefficient functions $C_i(\mu)$.

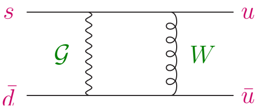
- The *effective Hamiltonian* for weak decays takes the form

$$\mathscr{H}_{\mathrm{eff}} \equiv \frac{G_F}{\sqrt{2}} V_{CKM} \sum_i C_i(\mu)\, O_i(\mu)\ .$$

- We shall see below that for some important physical quantities (e.g. ε'/ε), there may be as many as ten operators, whose matrix elements have to be estimated.
- Lattice simulations enable us to evaluate the matrix elements non-perturbatively.
- In weak decays the large scale, M_W, is of course fixed. For other processes, most notably for deep-inelastic lepton-hadron scattering, the OPE is useful in computing the behaviour of the amplitudes with the large scale (e.g. with the momentum transfer).

Towards more insight into the structure of the OPE.

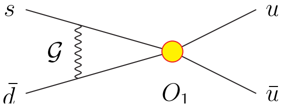




- For large loop-momenta k the right-hand graph is ultra-violet convergent:

$$\int_{k\ \text{large}} \frac{1}{k}\,\frac{1}{k}\,\frac{1}{k^2}\,\frac{1}{k^2-M_W^2}\,d^4k\,,$$

  ($1/k$ for each quark propagator and $1/k^2$ for the gluon propagator.)
  We see that there is a term $\sim\log(M_W^2/p^2)$, where p is some infra-red scale.
- In the OPE we do not have the W-propagator.



$$\text{Power Counting}:\quad \int_{k\ \text{large}} \frac{1}{k}\,\frac{1}{k}\,\frac{1}{k^2}\,d^4k \quad\Rightarrow\quad \textbf{divergence} \quad\Rightarrow\quad \mu-\text{dependence}.$$

- Infra-dependence is the same as in the full field-theory.

$$\log\left(\frac{M_W^2}{p^2}\right) = \log\left(\frac{M_W^2}{\mu^2}\right) + \log\left(\frac{\mu^2}{p^2}\right)$$

- The ir physics is contained in the matrix elements of the operators and the uv physics in the coefficient functions:

$$\begin{aligned} \log\left(\frac{M_W^2}{\mu^2}\right) &\to C_i(\mu) \\ \log\left(\frac{\mu^2}{p^2}\right) &\to \text{matrix element of } O_i \end{aligned}$$

- In practice, the matrix elements are computed in lattice simulations with an ultraviolet cut-off of 2 – 4 GeV. Thus we have to resum *large logarithms* of the form $\alpha_s^n \log^n(M_W^2/\mu^2)$ in the coefficient functions $\Rightarrow$ factors of the type

$$\left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\right]^{\gamma_0/2\beta_0} .$$

$$\left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\right]^{\gamma_0/2\beta_0}$$

- γ_0 is the one-loop contribution to the *anomalous dimension* of the operator (proportional to the coefficient of $\log(\mu^2/p^2)$ in the evaluation of the one-loop graph above) and β_0 is the first term in the β-function, $(\beta \equiv \partial g/\partial \ln(\mu) = -\beta_0 \, g^3/16\pi^2)$.
- In general when there is more than one operator contributing to the right hand side of the OPE, the mixing of the operators $\Rightarrow$ matrix equations.
- The factor above represents the sum of the *leading logarithms*, i.e. the sum of the terms $\alpha_s^n \log^n(M_W^2/\mu^2)$. For almost all the important processes, the first (or even higher) corrections have also been evaluated.
- These days, for most processes of interest, the perturbative calculations have been performed to several loops (2,3,4), $\mathrm{N}^n\mathrm{LO}$ calculations.