Flavour Physics – Lecture 4

Chris Sachrajda

School of Physics and Astronomy
University of Southampton
Southampton SO17 1BJ
UK

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UNIVERSITY OF Southampton
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and Astronomy

Programme

1. Lecture 1: Introduction to Flavour Physics
2. Lecture 2: Lattice Computations in Flavour Physics
3. Lecture 3: Light-quark physics
4. Lecture 4: Heavy-quark physics
    1. Introduction
    2. Approaches to lattice simulations of heavy-quark physics
    3. Lattice input into the Unitarity Triangle Analysis
        1. Calculation of f_B and f_{B_s}
        2. $B_s \to \mu^+\mu^-$
        3. $B^0 - \bar{B}^0$ mixing
        4. Determination of V_{ub} and V_{cb}
        5. The Golden Mode $B \to K_S J/\Psi$
    4. Nonleptonic B-decays

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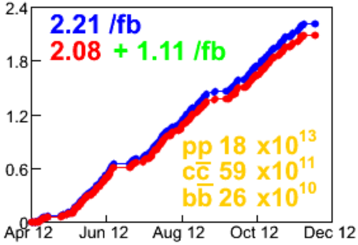
- The b-quark is particularly suitable for detailed studies of the limits of the standard model and in searches for signatures of physics BSM.
  - It is sufficiently heavy ($m_B \simeq 5.3\,$GeV) to have a huge number (hundreds) of decays modes.
  - It is sufficiently light that it can be produced copiously.

LHCb delivered and recorded luminosity in 2012, (+1.11 fb indicates recorded luminosity in 2010-11). The number of proton-proton (pp) collisions visible at LHCb, as well as the numbers of cc and bb quark pair produced within LHCb acceptance in 2010-2012 are also shown.



- The c-quark also leads to some interesting physics.

- $m_b \sim 4-5$ GeV whilst typical lattice spacings are $a^{-1} \sim$2-3 GeV $\Rightarrow$ we cannot simulate the propagation of b-quarks directly in QCD.
  - We are however getting closer, but simulations with $a^{-1} \geq 5$ GeV or so problematical.
- One approach may be to extrapolate results from lighter masses ($m_Q \leq m_c$ say), but then the scaling laws should be understood.
- The most common approach is to use effective theories:
  - HQET;
    World effort led by R.Sommer and the Alpha Collaboration.
  - *Fermilab/Relativistic Heavy Quark (RHQ)* Actions;
  - NRQCD.
- I will focus here on heavy-light physics, rather than on the physics of quarkonia.

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- B-physics is playing a central rôle in flavourdynamics and it is useful to exploit the symmetries which arise when $m_Q \gg \Lambda_{\mathrm{QCD}}$.
- The *Heavy Quark Effective Theory* (HQET) is proving invaluable in the study of heavy quark physics.
  - For scales $\ll m_Q$ the physics in HQET is the same as in QCD.
  - For scales $O(m_Q)$ and greater, the physics is different but, in principle, can be *matched* onto QCD using perturbation theory.
  - The approach of the Alpha Collaboration is to match the HQET, including the $O(1/m_b)$ corrections, nonperturbatively.
  - The non-perturbative physics in the same in the HQET as in QCD.

Consider the propagator of a (free) heavy quark: $\xrightarrow[p]{} \; = \; i\,\frac{\not p + m}{p^2 - m_Q^2 + i\epsilon}$.

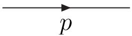
- If the momentum of the quark p is not far from its mass shell,

$$p_\mu = m_Q v_\mu + k_\mu \,,$$

where $|k_\mu| \ll m_Q$ and v_μ is the (relativistic) four velocity of the hadron containing the heavy quark ($v^2 = 1$), then

$$\xrightarrow[p]{} \; = \; i\,\frac{1+\not v}{2}\,\frac{1}{v\cdot k + i\epsilon} + O\left(\frac{|k_\mu|}{m_Q}\right).$$

HQET Cont.



$$= i\,\frac{1+\not v}{2}\,\frac{1}{v\cdot k+i\epsilon} + O\left(\frac{|k_\mu|}{m_Q}\right).$$

- $(1+\not v)/2$ is a projection operator, projecting out the *large* components of the spinors.
- This propagator can be obtained from the gauge-invariant action

$$\mathscr{L}_{HQET} = \bar h (iv\cdot D)\frac{1+\not v}{2}h$$

  where h is the spinor field of the heavy quark.
- $\mathscr{L}_{HQET}$ is independent of m_Q, which implies the existence of symmetries relating physical quantities corresponding to different heavy quarks (in practice the b and c quarks or Scaling Laws).
- The light degrees of freedom are also not sensitive to the spin of the heavy quark, which leads to a spin-symmetry relating physical properties of heavy hadrons of different spins.

- Consider, for example, the correlation function:

$$\int d^3x \, \langle 0| J_H(x) \, J_H^\dagger(0) \, |0\rangle \, ,$$

  - $J_H^\dagger$ and J_H are interpolating operators which can create or annihilate a heavy hadron H.
  - Here I take H to be a pseudoscalar or vector meson.
  - The hadron is produced at rest, with four velocity $v = (1, \vec{0})$.
  - For example take $J_H = \bar{h}\gamma^5 q$ for the pseudoscalar meson and $J_H = \bar{h}\gamma^i q$ $(i = 1,2,3)$ for the vector meson. This means that the correlation function will be identical in the two cases except for the factor

$$\gamma^5 \, \frac{1+\gamma^0}{2} \, \gamma^5 = \frac{1-\gamma^0}{2}$$

    when H is a pseudoscalar meson, and

$$\gamma^i \, \frac{1+\gamma^0}{2} \, \gamma^i = -3 \, \frac{1-\gamma^0}{2}$$

    when it is a vector meson.

- Correlation functions $\sim \exp(-iM_H t) \Rightarrow$ the pseudoscalar and vector mesons are degenerate (up to relative corrections of $O(\Lambda_{QCD}^2/m_Q)$):

$$M_P = M_V + O(\Lambda_{QCD}^2/m_Q)\,.$$

(or $M_V^2 - M_P^2 = \text{constant.}$)

- Heavy quark scaling laws (e.g. $f_P \sim 1/\sqrt{M_P}$) can be derived similarly.
- The difficulty is to go beyond the leading order and to determine the $O(1/m_b)$ corrections to physical quantities. There are more operators in the action, at $O(1/m_b)$:

$$\mathscr{L} = \bar{h}\,(D_4 + \delta_m)\,h + \omega_{\text{spin}}\bar{h}\,\sigma_{ij}F_{ij}\,h - \omega_{\text{kin}}\bar{h}\,\underline{D}^2\,h\,.$$

  - The coefficients δ_m, ω_{spin} and ω_{kin} have to be determined by matching the lattice HQET onto QCD.
  - The higher dimensional operators $\bar{h}\,\underline{D}^2\,h$ mixes with the leading operator $\bar{h}\,D_4\,h$ with coefficients which diverge linearly with the UV cut-off.
- Higher dimensional operators also have to be added to the operators, e.g. in the evaluation of f_B terms proportional to

$$\frac{1}{m_b}\bar{h}\gamma_5(\underline{\gamma}\cdot\overrightarrow{\underline{D}})q \quad \text{and} \quad \frac{1}{m_b}\bar{h}\gamma_5(\underline{\gamma}\cdot\overleftarrow{\underline{D}})q$$

have to be added to the axial current.

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The aim is to construct lattice actions for simulations which will remove $O((m_Q a)^n)$ ($\forall n$) and $O(\Lambda_{\rm QCD} a)$ discretization errors.

- Start with lattice QCD and imagine adding all possible “irrelevant" terms necessary to *improve* the action a la Symanzik.
- Use the equations of motion to reduce the number of terms to the minimum required to achieve the required precision.

$$S=\sum_{n,n'}\bar\psi_{n'}\left(\gamma_4 D_4+\zeta\vec\gamma\cdot\vec D+m_0-\frac{r_t}{2}D_4^2-\frac{r_s}{2}\vec D^2+c_B\sum_{i,j}\frac{i}{4}\sigma_{ij}F_{ij}+c_E\sigma_{4i}F_{4i}\right)_{n'n}\psi_n.$$

- All higher dimensional operators which might be added to the action can be reduced to those above, up to terms of $O((\Lambda_{\rm QCD} a)^2)$.

This idea was first proposed by the Fermilab Group,

A.X.El-Khadra, A.S.Kronfeld & P.B.Mackenzie [hep-lat/9604004]

Relativistic Heavy Quark Action(s) - Cont

$$S=\sum_{n,n'}\bar{\psi}_{n'}\left(\gamma_4 D_4+\zeta\vec{\gamma}\cdot\vec{D}+m_0-\frac{r_t}{2}D_4^2-\frac{r_s}{2}\vec{D}^2+c_B\sum_{i,j}\frac{i}{4}\sigma_{ij}F_{ij}+c_E\sigma_{4i}F_{4i}\right)_{n'n}\psi_n.$$

- In fact only 3 of the above 6 parameters (ζ, m_0, $r_{t,s}$, $c_{E,B}$) need to be determined in order to ensure on-shell improvement.
- Example of the reduction of the number of parameters:

$$\begin{aligned}\bar{\psi}\sigma_{4i}F_{4i}\psi &= \bar{\psi}\gamma_4\gamma_i\left[D_4D_i-D_iD_4\right]\psi\\ &= -2m_0\,\bar{\psi}\,\vec{\gamma}\cdot\vec{D}\,\psi-4\bar{\psi}\,\vec{D}^2\,\psi\end{aligned}$$

  Thus a change in the coefficient c_E can be compensated by a change in the coefficients ζ and r_s .
- The Columbia group pointed out that a further parameter can be eliminated by making changes of variables in the fermion functional integral such as:

$$\psi\to\left(1+\chi\sigma_{4i}[D_i,D_4]\right)\psi$$

  They propose to set $r_s=r_t=1$ and then tune the 3 parameters, m_0,ζ and $c_P\equiv c_E=c_B$. N.Christ, M.Li & H-W.Lin [heo-lat/0608006] .
- This has now been done non-perturbatively for both c and b quarks by using m_D (m_B), $m_D^*-m_D$, ($m_B^*-m_B$) and the dispersion relations to fix the parameters.

  RBC-UKQCD arXiv:1206.2554

The Fermilab approach and its generalizations are very interesting.

- It allows for the calculation of a variety of important quantities in heavy-quark physics (e.g. leptonic decay constants, form-factors for semileptonic decays, rare decay amplitudes).
- We need to add all possible improvement terms to the operators whose matrix elements we are computing with matching coefficients which have to be determined.
- A non-perturbative procedure for evaluating the matching coefficients still has to be developed (existing results were obtained using perturbatively determined coefficients at fixed a).
- The coefficients of the neglected operators are functions of $m_Q a$ and one might worry that they become large, particularly for b-physics. However, the theory does have the correct static limit.

- In the physics of heavy quarkonia the appropriate expansion parameter is the velocity ($p \sim v$ and $K \sim v^2$) and NRQCD is designed to facilitate this expansion.

NRQCD	Heavy Quark Expansion
$E = m(1 + O(v^2))$	$E = m + O(\Lambda_{\mathrm{QCD}})$
$p = O(m * v)$	$p = O(\Lambda_{\mathrm{QCD}})$
Expand in v.	Expand in Λ_{QCD}/m.

- NRQCD is also used in computations of quantities in Heavy-Light physics, where the HQET counting is relevant.

- In practice the groups doing the calculations do their book-keeping and systematics differently:
- HQET
  - The kinetic term $\vec{D}^2/2m$ is treated as a perturbation.
  - The coefficients are being determined nonperturbatively (see above).
  - The continuum limit is taken.
- NRQCD
  - The kinetic term $\vec{D}^2/2m$ is treated as a full part of the action (and included in the propagator).
  - The coefficients are being determined perturbatively (estimates of higher order corrections are included in the errors).
    For example in the computation of the B-Meson semileptonic form-factors, the higher-order coefficients were all set to their tree-level values.
    E.Gulez et al. (HPQCD Collaboration) [hep-lat/0601021]
  - The calculations are performed at fixed a (ma is simply treated as a number).

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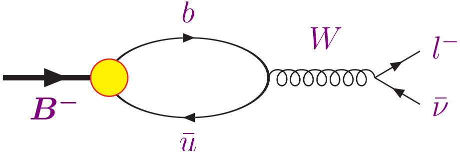
- Two collaborations of theorists and experimenters have been formed to perform global analyses of the CKM matrix, checking the consistency of the Standard Model and hence searching for inconsistencies which would signal the presence of New Physics.
    1. CKMFitter http://ckmfitter.in2p3.fr/ and
    2. UTfit http://utfit.org/UTfit/.
- When quoting results, I will use UTfit, see for example:

  C.Tarantino, "Lattice flavor physics with an eye to SuperB," PoS LATTICE **2012**, 012 (2012).
- In the UTfit analysis, there are five constraints which rely on Lattice QCD results: ε_K, Br$(B\to\tau\nu)$, Δm_d, $\Delta m_d/\Delta m_s$ and V_{ub}/V_{cb}.
    - The last four are B-physics observables and their determination is the subject of this section.

- The difficulty in making predictions for weak decays of hadrons is in controlling the non-perturbative strong interaction effects.
- As a particularly simple example consider the leptonic decays of pseudoscalar mesons in general and of the B-meson in particular.



- Non-perturbative QCD effects are contained in the matrix element

$$\langle 0|\,\bar b\gamma^\mu(1-\gamma^5)u\,|B(p)\rangle\ .$$

- Lorentz Inv. + Parity $\Rightarrow$ $\langle 0|\,\bar b\gamma^\mu u\,|B(p)\rangle = 0$.
- Similarly $\langle 0|\,\bar b\gamma^\mu\gamma^5 u\,|B(p)\rangle = if_B p^\mu\ .$

All QCD effects are contained in a single constant, f_B, the B-meson's *(leptonic) decay constant.* ($f_\pi \simeq 132\,$MeV)

- There is no such decay of B_s, but it is nevertheless useful to calculate the matrix element $\langle 0|\bar{b}\gamma^\mu\gamma^5 s\,|B_s(p)\rangle = if_{B_s}p^\mu$. It enters in the prediction for $B_s \to \mu^+\mu^-$.
- In practice it is convenient to calculate f_{B_s} for which the chiral extrapolation depends only on the light quarks in the sea and the ratio f_{B_s}/f_B for which there is a partial cancellation of statistical and discretisation errors.
  1. European Twisted Mass Collaboration - arXiv:1107.1441
     $N_f = 2$ Twisted Mass Fermions and tree-level improved Symanzik gauge action.
     Simulations are performed in the charm-region and extrapolations made to the static limit.
  2. HPQCD Collaboration - arXiv:1110.4510
     Staggered light quarks (MILC configurations) and “highly improved discretization of the relativistic quark action"
  3. FermiLab and MILC Collaborations - arXiv:1112.3051
     Staggered light quarks (MILC configurations) and Fermilab heavy quarks.
  4. HPQCD Collaboration - arXiv:1202.4914
     Staggered light quarks (MILC configurations) and NRQCD b-quarks.
  5. HPQCD Collaboration - arXiv:1302.2644
     Staggered light quarks (MILC configurations) at physical light-quark masses and improved NRQCD b-quarks.

Collaboration	arXiv	f_{B_s}	f_{B_s}/f_B	f_B
ETMC	1107.1441	(232 ± 10) MeV	1.19± 0.05	(195 ± 12) MeV
HPQCD	1110.4510	(225 ± 4) MeV		
FNAL&MILC	1112.3051	(242.0 ± 9.5) MeV	1.229± 0.026	(196.9 ± 8.9) MeV
HPQCD	1202.4914	(228 ± 10) MeV	1.188± 0.018	(191 ± 9) MeV
HPQCD	1302.2644	(224 ± 5) MeV	1.205± 0.007	(186 ± 4) MeV
Alpha	1210.7932	$(219\pm(12)_{\rm stat})$ MeV		$(193\pm9\pm4)$ MeV

- Note the relatively small number of groups performing these computations.
- In 1997, Jonathan Flynn and I summarised the rather primitive quenched results as

$$f_{B_s}=195\pm35\,\mathrm{MeV}\qquad \frac{f_{B_s}}{f_B}=1.14\pm0.08\qquad f_B=170\pm35\,\mathrm{MeV}\,.$$

hep-lat/9710057

- The 2012 UTA from UTfit uses

$$f_{B_s}=233\pm10\,\mathrm{MeV}\qquad \frac{f_{B_s}}{f_B}=1.20\pm0.02\qquad f_B=194\pm9\,\mathrm{MeV}\,.$$

- For many years the experimental upper bound for this FCNC decay has been several orders of magnitude above the SM prediction.
- Many extensions of the standard model give loop corrections which enhance the width.
- The LHC experiments have lowered the limit very significantly (unfortunately!)

$$\mathrm{Br}(B_s \to \mu^+\mu^-) < 4.2 \times 10^{-9}\,.$$

P.Eorola, arXiv:1209.3440

and even provided the first evidence of a measurement

$$\mathrm{Br}(B_s \to \mu^+\mu^-) = 3.2^{+1.5}_{-1.2} \times 10^{-9}\,.$$

LHCb, arXiv:1211.2674

- From the UTA analysis, the Standard Model prediction is

$$\mathrm{Br}(B_s \to \mu^+\mu^-) = (3.5 \pm 0.3) \times 10^{-9}\,.$$

See also the recent refined analysis which gave the SM result as

$$\mathrm{Br}(B_s \to \mu^+\mu^-) = (3.23 \pm 0.27) \times 10^{-9}\,.$$

Buras et al., arXiv:1208.0934

- In the experimental determination of Br$(B_s \to \mu^+\mu^-)$ it is necessary to know the fraction of b-quarks which hadronise into a B_s meson (f_s).
- To determine f_s some normalisation channels must be used e.g.

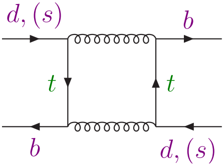
Fleischer, Serra, Tuning, arXiv:1004.3982, 1012,2784

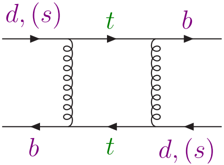
$$\frac{\mathrm{Br}(\bar{B}_s^0 \to D_s^+\pi^-)}{\mathrm{Br}(\bar{B}^0 \to D^+K^-)}\,.$$

- Factorisation à la BBNS then relates this ratio to the ratio of form factors in the semileptonic decays $B_s^0 \to D_s^+\ell^-\bar{\nu}$ and $B^0 \to D^+\ell^-\bar{\nu}$.
- The FNAL-MILC collaborations have recently computed this ratio in a lattice simulation finding

$$\frac{f_s}{f_d} = 0.28 \pm 0.04\,.$$

J.Bailey et al., arXiv:1202.6346





- In $B^0 - \bar{B}^0$ mixing, the top quark dominates and hence from the measured mass differences $\Rightarrow V_{td}$ and V_{ts}.
- The non-perturbative QCD effects are contained in the matrix element of the $\Delta B = 2$ operator:

$$\langle \bar{B} | O^{\Delta B=2} | B \rangle = \langle \bar{B} | \bar{b} \gamma^\mu (1-\gamma^5) d \; \bar{b} \gamma_\mu (1-\gamma^5) d | B \rangle \equiv \frac{8}{3} m_B^2 f_B^2 B_B(\mu) \, .$$

The uncertainty in this matrix element dominates that in the final answer for $|V_{td}|$.

- PDG2012 use $\Delta m_d = (0.507 \pm 0.004)$, $\Delta m_s = (17.719 \pm 0.043)$ and take the "lattice values" $f_{B_d} \sqrt{\hat{B}_{B_d}} = (211 \pm 12)$ MeV and $f_{B_s} \sqrt{\hat{B}_{B_s}} = (248 \pm 15)$ MeV to obtain

$$|V_{td}| = (7.4 \pm 0.8) \times 10^{-3} \quad \text{and} \quad |V_{ts}| = (42.9 \pm 2.6) \times 10^{-3} \, .$$

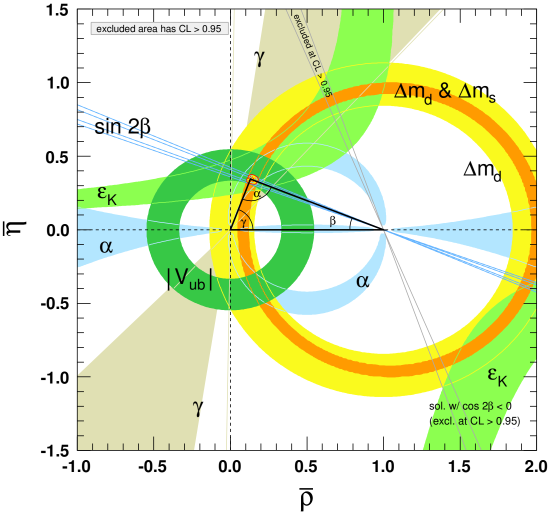
- The uncertainties are reduced in the lattice calculation of the ratio

$$\xi = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}} = 1.237 \pm 0.032 \quad \Rightarrow \left|\frac{V_{td}}{V_{ts}}\right| = 0.211 \pm 0.001 \pm 0.006\,,$$

  where the numerical values have been taken from PDG2012.

- $V_{td} \propto 1 - \bar{\rho} - i\bar{\eta}$ and so knowledge of $|V_{td}|$ fixes a circle centred on (1,0) on which the vertex A must lie.

PDG2012 Unitarity Triangle



- The evaluation of the B-parameter of neutral B-meson mixing proceeds as for B_K with an appropriate treatment of the heavy quarks.
- UTA analysis uses the previous HPQCD result as input: arXiv:0902.1815

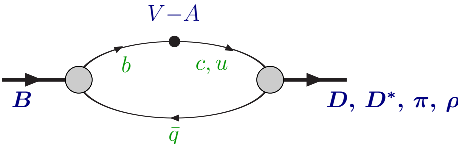
$$\hat{B}_s = 1.33 \pm 0.06 \qquad \frac{B_{B_s}}{B_B} = 1.05 \pm 0.07\,.$$

- New generation of computations are underway:

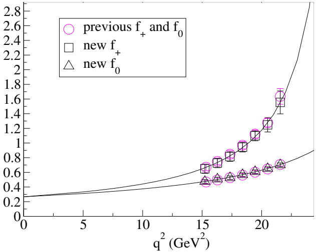
$$\frac{B_{B_s}}{B_B} = 1.03 \pm 0.02 \qquad \xi \equiv \frac{f_{B_s}\sqrt{B_{B_s}}}{f_B\sqrt{B_B}} = 1.21 \pm 0.06. \qquad \text{ETMC, arXiv:1211.0565}$$

$$\xi \equiv \frac{f_{B_s}\sqrt{B_{B_s}}}{f_B\sqrt{B_B}} = 1.268 \pm 0.063. \qquad \text{FNAL-MILC, arXiv:1205.7013}$$

- It is possible to extract V_{ub} and V_{cb} from either exclusive or inclusive semileptonic decays.
- The inclusive decays rely on the fact that the b-quark is heavy and therefore that perturbation theory can be used.
- In practice however, cuts have to be imposed to separate $b \to c$ decays from $b \to u$ ones which introduces technical complications. Nevertheless this is a standard method.
- Lattice simulations can be used to calculate the semileptonic decays $B \to (\pi, \rho, D, D^*)\ell\nu$ and a comparison with the experimental partial widths.



- V_{ub} can be determined from the semileptonic $B \to \pi$ or $B \to \rho$ transitions.
- In order to avoid large discretization errors the final state meson should have a small momentum $\Rightarrow$ large q^2.
- A new generation of calculations is underway to supercede the classic ones of FNAL-MILC and HPQCD.



B-Meson Semileptonic Form Factors

(NRQCD + MILC Staggered Configurations)

E.Gulez et al. (HPQCD Collab.) [hep-lat/0601021]

- The new modern calculations include
  1. HPQCD using NRQCD with HISC light valence quarks. arXiv:1210.6992
  2. ALPHA using HQET with $1/m_b$ corrections. arXiv:1210.3478
  3. RBC–UKQCD working with RHQ on DWF light quarks. arXiv:1211.0956
- All three were presented at the 2012 Lattice Conference, so we look forward to improved phenomenology with final results for the form factors very soon.
- Based on the old results for the form factors the UTA analysis take as input

$$|V_{ub}|_{\text{excl}} = (3.28 \pm 0.31) \times 10^{-3} \,.$$

- There is an interesting tension with the value obtained from the measured branching ratio for the decay $B \to \tau\bar{\nu}$:

$$\text{Br}(B \to \tau\bar{\nu}) = (1.67 \pm 0.30) \times 10^{-4} \,.$$

HFAG, arXiv:1010.1589

  Taking (another) average value $f_B = (190.6 \pm 4.6)$ MeV, PDG 2012 find $V_{ub} = (5.10 \pm 0.47) \times 10^{-3}$.
- But Belle at ICHEP 2012 quote a lower branching ratio and $V_{ub} = (3.87 \pm 0.53 \pm 0.09) \times 10^{-3}$.

- The situation is made more murky by the value obtained from the studies of inclusive decays.
- The PDG 2012 Review quotes the inclusive value

$$|V_{ub}|_{\text{incl}} = (4.41 \pm 0.15^{+0.15}_{-0.19})\, 10^{-3}\,,$$

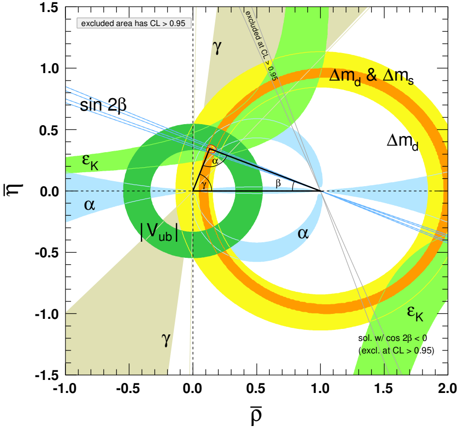
to be compared to the exclusive result

$$|V_{ub}|_{\text{excl}} = (3.28 \pm 0.31) \times 10^{-3}\,.$$

- The value which PDG2012 quote for the average within the SM (i.e. using unitarity) is

$$|V_{ub}| = (3.51^{+0.15}_{-0.14}) \times 10^{-3}\,.$$

- $|V_{ub}|^2 = \rho^2 + \eta^2$ and so a precise determination of $|V_{ub}|$ determines a circle on which the vertex A of the UT must lie.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excluded at CL > 0.95
γ
Δm_d & Δm_s
sin 2β
Δm_d
ε_K
α
β
γ
$|V_{ub}|$
α
α
ε_K
γ
sol. w/ cos 2β < 0
(excl. at CL > 0.95)
$\bar{\eta}$
$\bar{\rho}$
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0

- V_{cb} is known with better precision that V_{ub}, because the experimental cuts are at higher energies where the OPE is more reliable and the form factors are close to 1 because of the heavy-quark symmetry.
- The two decay channels which are used to extract $|V_{cb}|_{\text{excl}}$ are

$$B \to D^* \ell \nu \quad \text{and} \quad B \to D \ell \nu\,,$$

  with the vector D^* channel better measures and under better theoretical control in the HQET. The only unquenched calculation was by FNAL/MILC (2008-2010).
- UTA analysis input is $|V_{cb}|_{\text{excl}} = (39.0 \pm 0.9) \times 10^{-3}$ and the HFAG give $|V_{cb}|_{\text{incl}} = (41.9 \pm 0.8) \times 10^{-3}$. Small discrepency.
- A new set of calculations is underway by the Fermilab and MILC collaborations, and some preliminary results were presented at Lattice 2012. arXiv:1211.2247

- We start by introducing the framework for Mixing Induced CP-Violating Decays.
- In order to study CP-violation we need to be sensitive to the weak phase $\Rightarrow$ *interference*.
- The strong interactions also generate phases, so, in general, we need to be able to control the hadronic effects.
- For the golden-mode $B \to J/\Psi K_S$ this is possible to a great degree of accuracy $\Rightarrow$ precise determination of $\sin(2\beta)$. I will now review the theoretical background behind this statement.
- The two neutral mass-eigenstates are given by

$$|B_L\rangle = \frac{1}{\sqrt{p^2+q^2}}\left(p\,|B^0\rangle + q\,|\bar{B}^0\rangle\right)$$

and

$$|B_H\rangle = \frac{1}{\sqrt{p^2+q^2}}\left(p\,|B^0\rangle - q\,|\bar{B}^0\rangle\right).$$

where p and q are complex parameters.

- The 2×2 mass-matrix takes the form

$$M - \frac{i\Gamma}{2} = \begin{pmatrix} A & p^2 \\ q^2 & A \end{pmatrix} .$$

where A, p and q are complex parameters.

- Starting with a B^0 meson at time $t = 0$, its subsequent evolution is governed by the Schrödinger equation:

$$|B^0_{\text{phys}}(t)\rangle = g_+(t)\,|B^0\rangle + \left(\frac{q}{p}\right) g_-(t)|\bar{B}^0\rangle \,, \quad \text{where}$$

$$\begin{aligned} g_+(t) &= \exp\left[-\frac{\Gamma t}{2}\right] \exp[-iMt] \cos\left(\frac{\Delta M\, t}{2}\right), \\ g_-(t) &= \exp\left[-\frac{\Gamma t}{2}\right] \exp[-iMt]\, i \sin\left(\frac{\Delta M\, t}{2}\right) \text{ and } M = (M_H + M_L)/2. \end{aligned}$$

- Starting with a $\bar{B}^0$ meson at $t = 0$, the time evolution is

$$|\bar{B}^0_{\text{phys}}(t)\rangle = (p/q)\, g_-(t)|B^0\rangle + g_+(t)\,|\bar{B}^0\rangle.$$

- Let f_{CP} be a CP-eigenstate and $A, \bar{A}$ be the amplitudes

$$A \equiv \langle f_{CP}|\mathscr{H}|B^0\rangle \quad \text{and} \quad \bar{A} \equiv \langle f_{CP}|\mathscr{H}|\bar{B}^0\rangle.$$

- Defining

$$\lambda \equiv \frac{q}{p}\,\frac{\bar{A}}{A}$$

we have

$$\langle f_{CP}\,|\,\mathscr{H}\,|\,B^0_{\rm phys}\rangle = A\,[g_+(t) + \lambda\, g_-(t)] \quad \text{and} \quad \langle f_{CP}\,|\,\mathscr{H}\,|\,\bar{B}^0_{\rm phys}\rangle = A\,\frac{p}{q}\,[g_-(t) + \lambda\, g_+(t)].$$

- The time-dependent rates for initially pure B^0 or $\bar{B}^0$ states to decay into the CP-eigenstate f_{CP} at time t are given by:

$$\begin{aligned}
\Gamma(B^0_{\rm phys}(t)\to f_{CP}) &= |A|^2\, e^{-\Gamma t}\times\left[\frac{1+|\lambda|^2}{2} + \frac{1-|\lambda|^2}{2}\cos(\Delta M\, t) - {\rm Im}\,\lambda\,\sin(\Delta M\, t)\right]\\
\Gamma(\bar{B}^0_{\rm phys}(t)\to f_{CP}) &= |A|^2\, e^{-\Gamma t}\times\left[\frac{1+|\lambda|^2}{2} - \frac{1-|\lambda|^2}{2}\cos(\Delta M\, t) + {\rm Im}\,\lambda\,\sin(\Delta M\, t)\right].
\end{aligned}$$

Decays of Neutral B-Mesons into CP-Eigenstates (Cont.)

- The time-dependent asymmetry is defined as:

$$\mathscr{A}_{f_{CP}}(t) \equiv \frac{\Gamma(B^0_{\rm phys}(t)\to f_{CP})-\Gamma(\bar{B}^0_{\rm phys}(t)\to f_{CP})}{\Gamma(B^0_{\rm phys}(t)\to f_{CP})+\Gamma(\bar{B}^0_{\rm phys}(t)\to f_{CP})} = \frac{(1-|\lambda|^2)\cos(\Delta M\, t)-2{\rm Im}\,\lambda\,\sin(\Delta M\, t)}{1+|\lambda|^2}.$$

- If $|q/p|=1$ (which is the case if $\Delta\Gamma \ll \Delta M$) and $|\bar{A}/A|=1$ (examples of this will be presented below), then $|\lambda|=1$ and the first term on the right-hand side above vanishes.
- The form of the amplitudes A and $\bar{A}$ is:

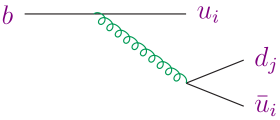
$$A=\sum_i A_i\, e^{i\delta_i}\, e^{i\phi_i} \quad \text{and} \quad \bar{A}=\sum_i A_i\, e^{i\delta_i}\, e^{-i\phi_i}$$

  - Sum is over all the contributions to the process;
  - the A_i are real;
  - the δ_i are the strong phases;
  - the ϕ_i are the phases from the CKM matrix.

$$A = \sum_i A_i\, e^{i\delta_i}\, e^{i\phi_i} \quad \text{and} \quad \bar{A} = \sum_i A_i\, e^{i\delta_i}\, e^{-i\phi_i}$$

- In the most favourable situation, all the contributions have a single CKM phase (ϕ_D say) and
$$\frac{\bar{A}}{A} = \exp(-2i\phi_D).$$
- Since $\Gamma_{12} \ll M_{12}$, $q/p = \sqrt{M_{12}^*/M_{12}} \equiv \exp(-2i\phi_M)$, and
$$\lambda = \exp(-2i(\phi_D + \phi_M)).$$
Thus
$$\mathrm{Im}\,\lambda = -\sin(2(\phi_D + \phi_M)).$$
- From the box diagrams:
$$\left(\frac{q}{p}\right)_{B_d} = \frac{V_{td}V_{tb}^*}{V_{td}^*V_{tb}} \quad \text{and} \quad \left(\frac{q}{p}\right)_{B_s} = \frac{V_{ts}V_{tb}^*}{V_{ts}^*V_{tb}}.$$

- Consider processes in which the b-quark decays through the subprocess $b \to d_j u_i \bar{u}_i$. The corresponding tree-level diagram is



for which

$$\frac{\bar{A}}{A} = \frac{V_{ib}V_{ij}^*}{V_{ib}^*V_{ij}} \, .$$

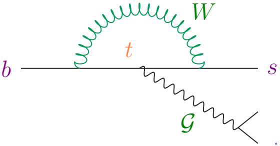
- $B_d \to J/\Psi K_S$ – In this case

$$\lambda(B \to J/\Psi K_S) = \frac{V_{td}V_{tb}^*}{V_{td}^*V_{tb}} \, \frac{V_{cs}V_{cd}^*}{V_{cs}^*V_{cd}} \, \frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}} \Rightarrow \mathrm{Im}\lambda = -\sin(2\beta)$$

  - The first factor is $(q/p)_{B_d}$;
  - the second factor is the analogous one for the final state kaon;
  - the third factor is $\bar{A}/A$, with $u_i = c$ and $d_j = s$.
  - Recall that

$$\beta = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) .$$

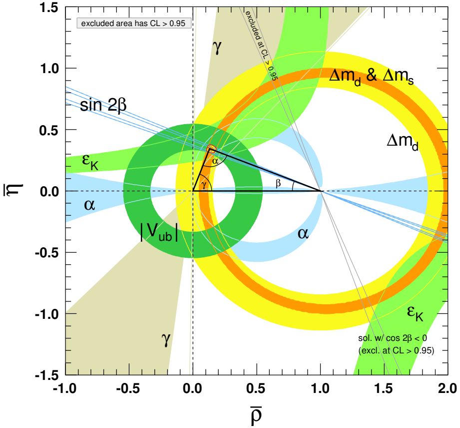
- There is also a small penguin contribution to this process:



  - Phase is that of $V_{tb}V_{ts}^*$, which is is equal (to an excellent approximation) to that of $V_{cb}V_{cs}^*$.
  - Thus we have a single weak phase and hence hadronic uncertainties are negligible in the determination of the $\sin(2\beta)$ from this process (*golden mode*).
  - This is an (almost) ideal situation but one which is very rare.
  - PDG 2012 average the results from BaBar and Belle and obtain

$$\sin(2\beta) = 0.679 \pm 0.020\,.$$

PDG2012 Unitarity Triangle



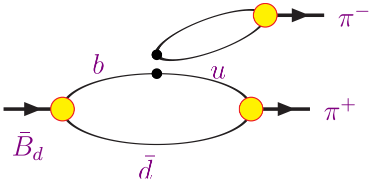
UTfit Input Values and SM predictions

Observable	Input value	SM prediction	Pull
$\varepsilon_K \times 10^3$	2.23 ± 0.01	1.96 ± 0.20	1.4
$\Delta m_s\ \mathrm{ps}^{-1}$	17.69 ± 0.08	18.0 ± 1.3	< 1
$\lvert V_{cb}\rvert \times 10^3$	41.0 ± 1.0	42.3 ± 0.9	< 1
$\lvert V_{ub}\rvert \times 10^3$	3.82 ± 0.56	3.62 ± 0.14	< 1
$\mathrm{Br}(\mathrm{B}\tau\nu) \times 10^4$	1.67 ± 0.30	0.82 ± 0.08	2.7
$\sin 2\beta$	0.68 ± 0.02	0.81 ± 0.05	2.4
α	$91^\circ \pm 6^\circ$	$88^\circ \pm 4^\circ$	< 1
γ	$76^\circ \pm 11^\circ$	$68^\circ \pm 3^\circ$	< 1

C.Tarantino, “Lattice flavor physics with an eye to SuperB,” PoS LATTICE **2012**, 012 (2012).

1. Lecture 1: Introduction to Flavour Physics
2. Lecture 2: Lattice Computations in Flavour Physics
3. Lecture 3: Light-quark physics
4. Lecture 4: Heavy-quark physics
    1. Introduction
    2. Approaches to lattice simulations of heavy-quark physics
    3. Lattice input into the Unitarity Triangle Analysis
        1. Calculation of f_B and f_{B_s}
        2. $B_s \to \mu^+\mu^-$
        3. $B^0 - \bar{B}^0$ mixing
        4. Determination of V_{ub} and V_{cb}
        5. The Golden Mode $B \to K_S J/\Psi$
    4. Nonleptonic B-decays

- A huge amount of information has been obtained about decay rates and CP-asymmetries for $B \to M_1 M_2$ decays (over 100 channels).
- With just a few exceptions (e.g. CP-asymmetry in $B \to J/\Psi K_s$) our ability to deduce fundamental information about CKM matrix elements is limited by our inability to quantify the non-perturbative strong interaction effects.
- Most approaches were based on **Naive Factorization:**

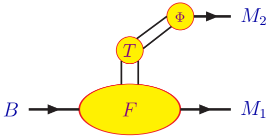


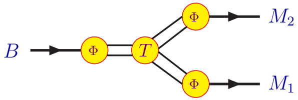
$$\langle \pi^+\pi^- \,|\, (\bar{u}b)_{V-A}\; (\bar{d}u)_{V-A} \,|\, \bar{B}_d \rangle \to \langle \pi^- \,|\, (\bar{d}u)_{V-A} \,|\, 0 \rangle \; \langle \pi^+ \,|\, (\bar{u}b)_{V-A} \,|\, \bar{B}_d \rangle$$

- $\langle \pi^- \,|\, (\bar{d}u)_{V-A} \,|\, 0 \rangle$ is known (f_π).
- $\langle \pi^+ \,|\, (\bar{u}b)_{V-A} \,|\, \bar{B}_d \rangle$ is known in principle ($F_0^{B\to\pi}(m_\pi^2)$).
- No rescattering in the final state. No strong phase-shifts.
- μ dependence does not match on the two sides.

- In 1999 we realized that in the limit $m_b \to \infty$, the long distance effects *factorise* into simpler universal quantities:

M.Beneke, G.Buchalla, M.Neubert, CTS, (BBNS)





$$\langle M_1, M_2 | O_i | B \rangle = \sum_j F_j^{B\to M_1}(m_2^2) \int_0^1 du\, T_{ij}^I(u) \Phi_{M_2}(u) + (M_1 \leftrightarrow M_2)$$
$$+ \int_0^1 d\xi\, du\, dv\, T_i^{II}(\xi, u, v)\, \Phi_B(\xi)\, \Phi_{M_1}(v)\, \Phi_{M_2}(u)$$

Implications of Factorization

- The significance and usefulness of the factorization formula stems from the fact that the non-perturbative quantities which appear on the RHS are much simpler than the original matrix elements which appear on the LHS.
  They either reflect universal properties of a single meson state (the light-cone distribution amplitudes) or refer to a $B \to$ meson transition matrix element of a local current (form-factor).
- Conventional (naive) factorization is recovered as a rigorous prediction in the infinite quark-mass limit (i.e. neglecting $O(\alpha_s)$ and $O(\Lambda_{\rm QCD}/m_b)$ corrections).
- Perturbative corrections to naive factorization can be computed systematically. The results are, in general, non-universal (i.e. process dependent).
- All strong interaction phases are generated perturbatively in the heavy quark limit.
- The factorization formulae are valid up to $O(\Lambda_{\rm QCD}/m_b)$ corrections.
- Many observables of interest for CP-violation become accessible. The precision of the calculations is limited by our knowledge of the wave-functions and of the power corrections.
- For a comprehensive study of 96 PP and PV decay modes see

Beneke and Neubert, hep-ph/0308039.

- The main limitation of the factorization framework is due to the fact that m_b is not so large, so that CKM and chiral enhancements to non-factorizable $O(\Lambda_{\text{QCD}}/m_b)$ terms are important.
- At present we do not know how to begin computing $B \to M_1 M_2$ matrix elements!
  - Many intermediate states contribute.
- What can lattice simulations contribute to the factorization formula:
  - Parton distribution amplitudes of light mesons (at least the low moments) $\surd$.
  - $B \to M$ form-factors $\surd$.
  - Parton distribution amplitudes of B-meson X.
- I now briefly explain why we have not been able to compute ϕ^B or its moments.

$$\phi^B_{\alpha\beta}(\tilde{k}_+) = \int dz_-\, e^{i\tilde{k}_+ z_-}\, \langle 0\,|\,\bar{u}_\beta(z)[z,0]b_\alpha(0)\,|\,B\rangle\big|_{z^+, z_\perp = 0}$$

- ϕ^B is convoluted with the perturbative hard-scattering amplitude $T_i^{II} \Rightarrow$ we need

$$\frac{\sqrt{2}}{\lambda_b} = \int_0^\infty \frac{d\tilde{k}_+}{\tilde{k}_+}\, \phi^B_+(\tilde{k}_+)\,.$$

  (In higher orders of perturbation theory factors containing $\log(\tilde{k}_+)$ appear.)
- At large $\tilde{k}_+$, $\phi^B(\tilde{k}_+) \sim 1/\tilde{k}_+$, but the convolution is finite.
- Positive moments of $\phi^B(\tilde{k}_+)$, which can be written in terms of local operators, diverge as powers of $1/a \Rightarrow$ need a technique to subtract these divergences with sufficient precision.
- We need new theoretical ideas for the lattice to contribute to $B \to M_1 M_2$ decays.

Conclusions

- Precision flavour physics is a complementary approach to the large $p_\perp$ studies at the LHC in exploring the limits of the standard model.
- The improved precision of Lattice QCD simulations is making this approach really viable.
- In addition to the improved precision in the evaluation of "standard" quantities, it is important to continue extending the range of physical quantities which can be studied.
- There is a huge amount of work to be done!

UNIVERSITY OF Southampton
School of Physics and Astronomy

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