Tutorial on Monte Carlo Simulations – Day 1

First we installed gnuplot: `sudo apt-get install gnuplot`
and then looked at several cases of **uniformly distributed random variables**.
The steps we (should have!) followed are sketched below.

```
1) list of random numbers btw 0 and 1
echo|awk '{for(i=0;i<10;i++)print rand()}'


# average
echo|awk '{n=10;for(i=0;i<n;i++)a+=rand();print a/n}'


# how does the average approach 0.5 for increasing n
echo|awk '{for(n=10;n<=1.e04;n=n**2){for(i=0;i<n;i++)a+=rand();print n,a/n}}'


# average + error
echo|awk '{for(n=10;n<=1.e04;n=n**2){for(i=0;i<n;i++){x=rand();s+=x;s2+=x**2}\
     print n,s/n,sqrt(s2/n-(s/n)**2)/sqrt(n)}}'
# "exact" result = sqrt(<x^2>-<x>^2)/sqrt(n)
echo|awk '{for(n=10;n<=1.e04;n=n**2){print n,0.5,sqrt(1/3-(1/2)**2)/sqrt(n)}}'


2) square, circle, disk
gnuplot:


# SQUARE
!echo|awk '{for(i=0;i<500;i++)print -1+2*rand(),-1+2*rand()>"square"}'
plot "square"   # not so nice
set size square; plot "square" w p pt 5 lc 4 ps 0.5    # nicer :)


# CIRCLE
!echo|awk '{pi=atan2(0,-1);for(i=0;i<200;i++){theta=2*pi*rand();\
     x=cos(theta);y=sin(theta);print x,y>"circle"}}'
plot "circle" w p pt 7


# DISK
!echo|awk '{pi=atan2(0,-1);for(i=0;i<500;i++){theta=2*pi*rand();r=rand();\
```

```
            x=r*cos(theta);y=r*sin(theta);print x,y>"disk"}}'
plot "disk" w p pt 5 ps 0.5
# looks bad, increase # points to see it better
!echo|awk '{pi=atan2(0,-1);for(i=0;i<5000;i++){theta=2*pi*rand();r=rand();\
            x=r*cos(theta);y=r*sin(theta);print x,y>"baddisk"}}'
plot "baddisk" w p pt 5 ps 0.5
# -> how do we fix it?? back to the square...
!echo|awk '{for(i=0;i<5000;i++){x=-1+2*rand();y=-1+2*rand();\
            if(x^2+y^2<=1)print x,y>"gooddisk"}}'
plot "gooddisk" w p pt 5 ps 0.5   # so "throwing darts" works much better :)
# NOTE: may also be fixed by redefining x, y coordinates to have the
# correct Jacobian, i.e. from  dx dy = r dr dtheta  to  dx' dy' ~ dr dtheta
# for x'=sqrt(r)*cos(theta) (similarly for y') get dx' dy' = 0.5 dr dtheta
# -> fine (factor 0.5 related to the fact that area will be = pi)
!echo|awk '{pi=atan2(0,-1);for(i=0;i<5000;i++){theta=2*pi*rand();r=rand();\
            x=sqrt(r)*cos(theta);y=sqrt(r)*sin(theta);print x,y>"gooddisktoo"}}'
plot "gooddisktoo" w p pt 5 ps 0.5


3) use above "code" to estimate pi
# direct method uses fewer calls to rand() :) on the other hand "rejection"
# method naturally provides estimate of pi (area of the disk) -> MC integral
(exit gnuplot)
echo|awk '{for(n=10;n<=1.e08;n=n**2){accept=0;for(i=0;i<n;i++){\
            x=-1+2*rand();y=-1+2*rand();if(x^2+y^2<=1)accept++}print 4*accept/n}}'
# NOTE: last data point takes a while... and is not a lot better than 22/7 :(
echo|awk '{printf"%8.8f %8.8f\n",atan2(0,-1),22/7}'
```