

## Tutorial on Monte Carlo Simulations – Day 3

We are now ready to simulate the simple harmonic oscillator.

### Problem

Using the Metropolis method, write a program to sample the configurations of the  $x_i$  [or  $q(i)$ ] variables, i.e. the position of the particle for the different values of (discretized) time. As usual, the potential is given by

$$V(x) = \frac{1}{2}m\omega^2 x^2 .$$

Take the time extent to be  $Na$  ( $N$  time steps with finite separation  $a$ ) and implement periodic boundary conditions. The action is best represented in lattice units, i.e. in terms of dimensionless parameters  $ma$  and  $\omega a$  and dimensionless variables  $x_i/a$ . We have

$$S = ma \frac{1}{2} \sum_{i=0}^{N-1} \left\{ [(x_{i+1} - x_i)/a]^2 + (\omega a)^2 (x_i/a)^2 \right\} .$$

Use your program to study  $S$ ,  $\bar{x} \equiv \sum_i x_i/N$ ,  $C(t) \equiv \langle x_t x_0 \rangle$ . Look at (Monte Carlo) time histories for these quantities and compute their averages and errors. Are the samples uncorrelated? study this by varying the length of the Metropolis proposal for updating  $x_i$  and the number of Metropolis hits.