

Understanding the concept of resolving power in the Fabry–Perot interferometer using a digital simulation

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Abstract

The resolution concept in connection with the Fabry–Perot interferometer is difficult to understand for undergraduate students enrolled in physical optics courses. The resolution criterion proposed in textbooks for distinguishing equal intensity maxima and the deduction of the resolving power equation is formal and non-intuitive. In this paper, we study the practical meaning of the resolution criterion and resolution power using a computer simulation of a Fabry–Perot interferometer. The light source in the program has two monochromatic components, the wavelength difference being tunable by the user. The student can also adjust other physical parameters so as to obtain different simulation results. By analysing the images and graphics of the simulation, the resolving power concept becomes intuitive and understandable.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The multiple beam interferometer was proposed by Charles Fabry (1867–1945) and Alfred Perot (1863–1925) in 1899. The multiple beam interferences generated between two glass plates, internally covered by a highly reflective film and illuminated by an extended quasi-monochromatic source, produce an intensity distribution consisting of concentric circles. It can be demonstrated, using the mathematical law that describes the intensity profile of the interference pattern, that a given wavelength generates a specific set of maxima. Consequently,

circles resulting from different wavelengths can be discriminated. This capability makes the Fabry–Perot interferometer a valuable tool in high-resolution spectroscopy.

The spectral resolving power $|\lambda/\Delta\lambda|$ quantifies the capability of an interferometer to resolve two close wavelengths. It is defined as the ratio of the wavelength of the source λ to the minimum difference of wavelengths $\Delta\lambda$ that generates two circle series that can be discriminated. To calculate the resolving power formula, it is necessary to know the law that describes the interference pattern and set a separation criterion to decide when the circles are viewed separately [1, 2].

Our experience in teaching optics to second year university physics students shows that the explanation of the resolution concept is barely understood. Very often, students merely retain the formula and are unable to grasp the underlying concepts. Because of this reason, we propose to use a Java applet to show how the Fabry–Perot interferometer works. These simulations make possible to modify the parameters involved in the physical problem, so the effect of the changes in the variables is visualized automatically.

As the mathematical description of the interferometer is relatively simple, the implementation of a computer simulation of this device is a straightforward task. Consequently, it is possible to find in the Internet multiple computer implementations of the interferometer (see, for instance, [3, 4]). Nevertheless, these Fabry–Perot applets do not allow you to analyse problems related with resolving power because they are not able to deal with more than one wavelength. For this reason, a simple application to emphasize this topic [5] has been developed.

In section 2, we explain how to calculate the spectral resolving power in a Fabry–Perot spectrometer and in section 3, we describe a visual methodology to determine the resolving power from the data generated by the applet. We compare visual results with those obtained directly from the formula presented in section 2.

2. Fabry–Perot resolution

2.1. How a Fabry–Perot interferometer works

The Fabry–Perot interferometer consists of two glass plates with parallel plane surfaces, separated at a distance d . The media between the glass plates is air ($n = 1$). If a monochromatic wave impinges upon the plate at angle ϵ , multiple reflections are generated. If the inner surfaces are covered by a highly reflective film, the reflections in the glass plates are negligible. Therefore, only the interferences produced by the multiple beams in the air plate are observable. The intensity of the interference patterns is described by the following expression:

$$I = \frac{a^2}{1 + \frac{4r^2}{(1-r^2)^2} \sin^2\left(\frac{\delta}{2}\right)}, \quad (1)$$

where a is the amplitude of the incident wave, r is the reflection coefficient of the coating film and δ is the phase difference between two consecutive waves. Let Δ be the optical length difference. In this case, the optical length is $\Delta = 2d \cos \epsilon$. As usual, the optical length and phase difference are related by $\delta = \frac{2\pi}{\lambda} \Delta$.

The intensity depends on the thickness d , the reflection coefficient r , the wavelength λ , the intensity a^2 of the incident plane wave and the incidence angle ϵ . If the light comes from an extended source from all possible directions ϵ , and taking into account that the geometry of the light distribution only depends on ϵ , the intensity pattern should exhibit rotational symmetry.

The intensity maximum ($I_{\max} = a^2$) is obtained if the following condition is verified:

$$\sin^2 \frac{\delta}{2} = 0 \quad \text{thus} \quad \delta = 2m\pi \quad \text{and} \quad \Delta = 2d \cos \epsilon = m\lambda, \quad (2)$$

where $m = 0, \pm 1, \pm 2, \dots$. The intensity profile therefore exhibits a sequence of maxima and minima and, consequently, the interference pattern is characterized by a set of light circles. The index m in the previous equation labels each circle. For instance, the maximum m value is found when ϵ tends to zero. In particular, if a maximum is found when the incidence angle is zero ($\epsilon = 0$), then $m = 2d/\lambda$.

2.2. The Rayleigh criterion

As we have explained before, the intensity pattern is a function of the wavelength. Each wavelength coming from a light source generates a set of light circles. The resolving power is a measure of the ability to discriminate between sets of circles generated by different wavelengths. Moreover, it is also necessary to define mathematically a separation criterion between two very close maxima. This criterion represents one's visual ability to distinguish two concentric circles.

Different resolution criteria are described in physical optics textbooks. Several authors such as Born and Wolf [1] and Hecht [2] use Rayleigh's criterion [6]. It was first introduced by Lord Rayleigh in 1879 to determine whether two diffraction spots can be distinguished or not. The criterion states that two intensity maxima are separated if the maximum value of the first spot is superimposed on the first minimum of the second spot.

Rayleigh's resolution limit seems to be rather arbitrary and is based on resolving capabilities of the human eye. This limit was set to guarantee a clear distinction between two close spots using the eye. When visual inspection is replaced by detectors, other less restrictive criteria can be used. For instance, Sparrow's criterion states that two diffraction spots are just distinguished when the minimum between the two intensity maxima of the composite intensity is undetectable [7].

Rayleigh's criterion cannot be directly applied to the Fabry–Perot intensity profile because of the slow decrease of these values. Consequently, the minimum is located far from the maximum. To avoid this problem, an alternative definition is proposed: let $I_\lambda(x)$ and $I_{\lambda+\Delta\lambda}(x)$ equal the intensity profiles along a diameter, generated by wavelengths λ and $\lambda + \Delta\lambda$ respectively [1]. Two maxima are resolved if the minimum value of $I_\lambda(x) + I_{\lambda+\Delta\lambda}(x)$ verifies the following condition:

$$\min\{I_\lambda(x) + I_{\lambda+\Delta\lambda}(x)\} = 0.81 \max\{I_\lambda(x)\}. \quad (3)$$

When the Rayleigh criterion is applied to diffraction in a slit, we obtain the condition shown in equation (3).

2.3. The Taylor criterion

Other authors of optical textbooks (Klein–Furtak [8], Pérez [9], Frañón [10], Fowles [11]) use the full-width half-maximum criterion (FWHM). This states that the separation of the maxima is equal to the half-maximum width. This criterion is also known as the Taylor criterion (TC). In practice, both criteria give very similar results. For instance, Hecht [2] introduces the Rayleigh criterion but, after an approximation, uses the TC. We also use TC in this paper because it is easier to handle the mathematics involved.

Figure 1 shows an example of the use of this criterion for different values of the reflective coefficient r . In figure 1(a) ($r = 0.65$), the intensity maxima profiles are not separated

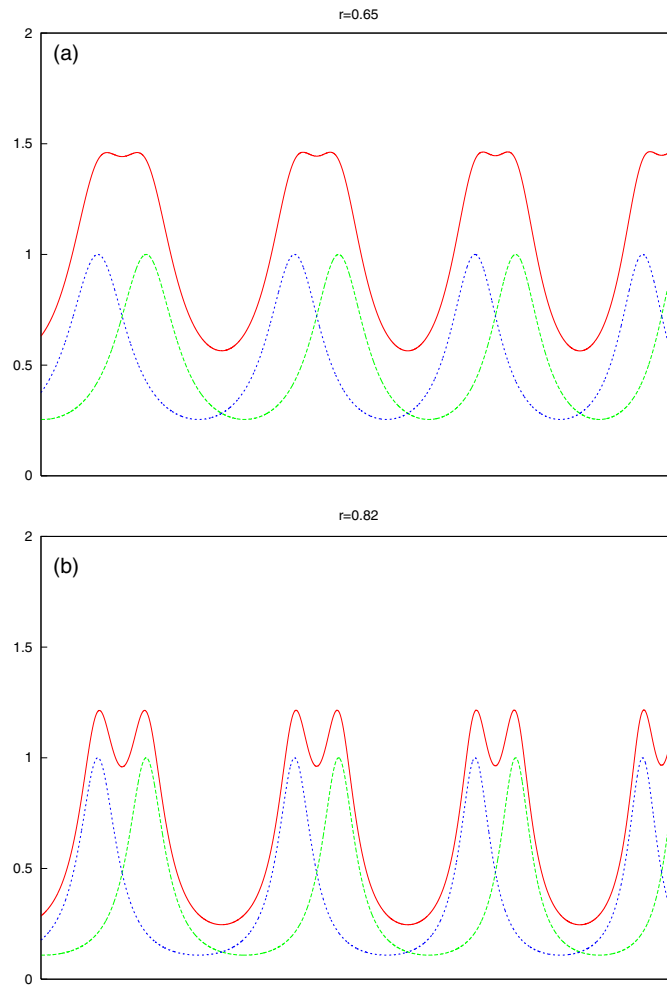


Figure 1. Taylor criterion as a function of the reflective coefficient. The figure shows the intensity profile (equation (1)) for two close wavelengths (blue and green curves). The composite intensity is shown in red. (a) $r = 0.65$, the maxima cannot be discriminated and (b) $r = 0.82$, the maxima fulfil the criterion.

enough to be visually differentiated whereas in figure 1(b) ($r = 0.82$), the maxima fulfil the requirements of the TC; thus, the interference fringes are considered separated.

Using equations (1) and (2) (intensity and position of maxima), and applying the TC, a formula that gives the minimum wavelength difference ($\Delta\lambda$) for the fringes produced by two monochromatic sources to be separated can be deduced as

$$\text{SRP} = \left| \frac{\lambda}{\Delta\lambda} \right| = \frac{\pi m r}{1 - r^2}. \quad (4)$$

This equation is called the spectral resolving power (SRP) of the interferometer. The derivation of this formula is shown in the appendix.

For a specific wavelength, the SRP depends on the maximum order m and the reflective coefficient r . The SRP gives higher values for fringes near the centre of the interferometric pattern and for values of r tending to 1, which means that closer wavelengths can be discriminated.

3. Procedure

We propose to analyse the meaning of the SRP concept, using the applet. First, we study the dependence of SRP with coefficient r , for a fixed m . Initially, we set the source wavelength ($\lambda_1 = 500.1$ nm) and the thickness ($d = 7.5$ mm). From equation (2) we get $m = 29993$. We use these values just as an example.

Taking into account that the best resolving conditions can be found for central fringes (maximum m values), we adjust the screen size to display only the inner circle of the interferometric pattern (focal length 1 = 500 mm, focal length 2 = 500 mm, source size = 10 mm, screen size = 5 mm.) If, for instance, we want to discriminate a second wavelength $\lambda_2 = 500.103$ nm, then $\Delta\lambda = 0.003$ nm. These are arbitrary values and have been selected to show the ability of the Fabry–Perot interferometer to distinguish two really close wavelengths.

Now, we increase the value of r starting from $r = 0.6$. As shown in figure 2(a), the circles generated by λ_1 and λ_2 cannot be separated visually. In practice, two fringes are distinguished according to Rayleigh's criterion if they can be clearly discriminated by the observer's eye. When the values of r reach the interval $0.75 \leq r \leq 0.78$, the fringes appear clearly separated (see figure 2(b)). Figure 2(c) displays the intensity profile. The plot shows that the separation of the maxima is equal to the half-maximum width when $r = 0.76$.

Now, we can verify this result with the use of the equation predicted by theory (equation (4)). Setting the variables $\lambda = 500.1$ nm, $\Delta = 0.003$ nm and $m = 29993$, and isolating r , we obtain the following equation:

$$r^2 + \left(\pi m \left| \frac{\Delta\lambda}{\lambda} \right| \right) r - 1 = 0. \quad (5)$$

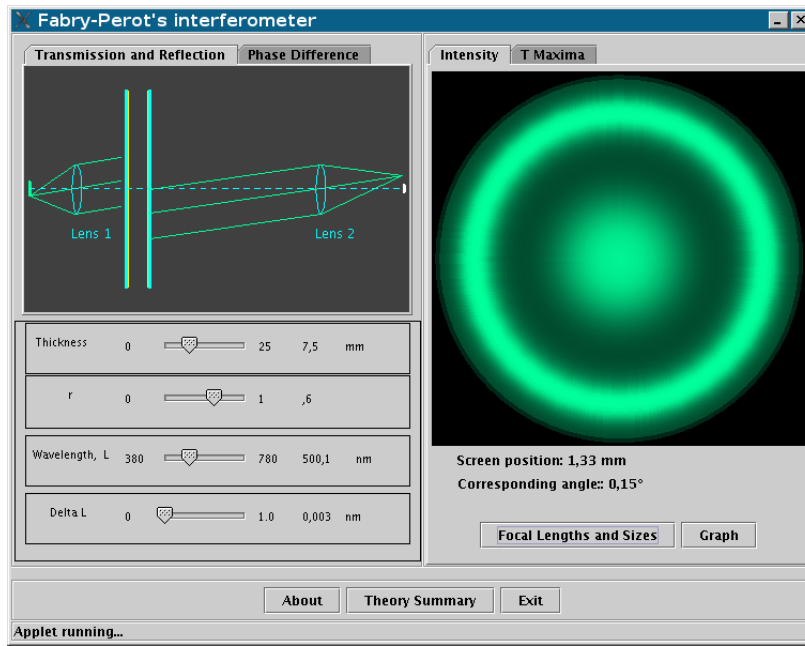
Solving equation (5), we get a value of $r = 0.757$, very similar to that obtained visually using the applet.

We suggest a second exercise to show how the SRP varies with the thickness d . Here, the variables are set to the following values: focal length 1 = 100 mm, focal length 2 = 1000 mm, source size = 5 mm, screen size = 35 mm, $r = 0.7$. The wavelengths selected are $\lambda_1 = 429.248$ nm and $\lambda_2 = 429.286$ nm, which correspond to two equal intensity lines of the emission spectrum of sodium (Na II) (see, for instance, [12]). We start from $d = 0.3$ mm, and the thickness increases until 0.7. Figure 3 shows the interferogram at $d = 0.4, 0.45, 0.5, 0.55, 0.6$ and 0.65 mm. The inner circles appear to be distinguished at $d = 0.55$ mm or $d = 0.6$ mm. Again, numerical verification agrees with the visual inspection of simulated interferograms because the limit value of d that verifies the Rayleigh criterion is $d = 0.56$ mm.

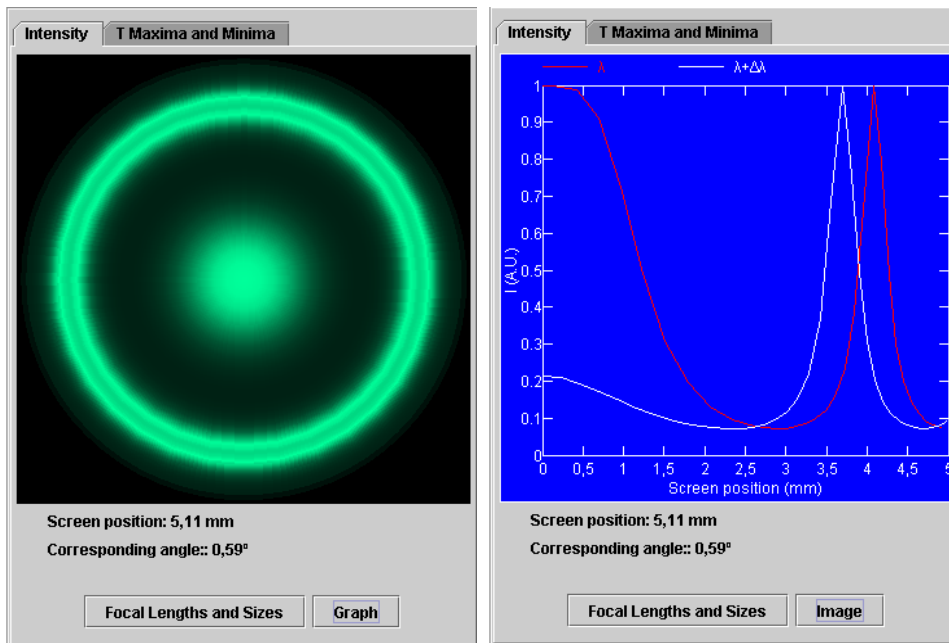
4. Concluding remarks

The most common physical optics textbooks analyse the spectral resolution power concept in interferometry from the point of view of Rayleigh's resolution criterion. The use of this criterion is appropriate when qualitative observation of some physical phenomena is carried out using the human visual system. Nowadays, undergraduate experiments often include data acquisition systems, so the use of Rayleigh criterion seems to be rather arbitrary and the students find it confusing to apply.

Computer simulations of interferometric devices allow the user to handle the variables involved in the physical problem. A digital implementation of the Fabry–Perot interferometer can be a helpful tool to analyse the connection between spatial resolution power, resolution criteria and their mathematical formalism. In our opinion, the use of such programs, combined with a suitable methodology, helps students to understand these concepts better.



(a)



(b)

(c)

Figure 2. Results: (a) program window showing two unresolved fringes ($r = 0.6$) and (b) limit of visual resolution ($r = 0.76$). The two circles are clearly distinguishable. (c) Intensity profile of (b).

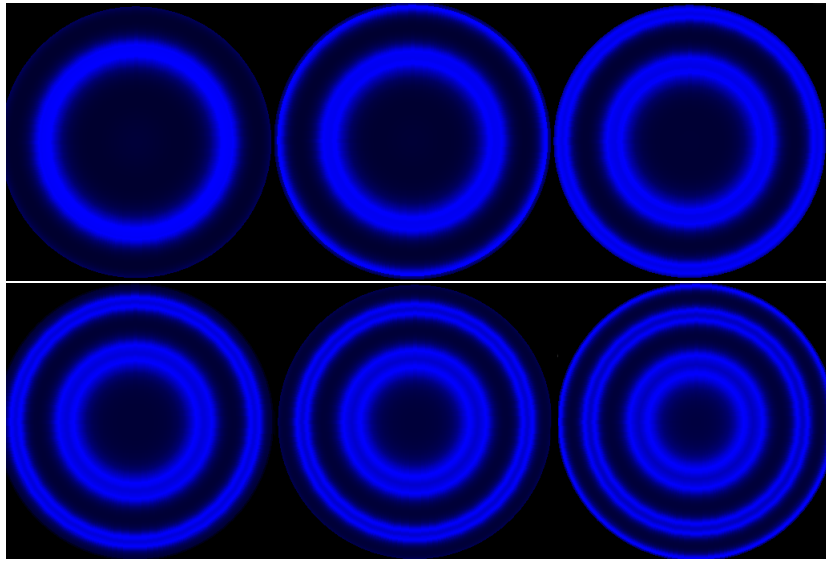


Figure 3. Interferograms at $d = 0.4, 0.45, 0.5, 0.55, 0.6$. Note that for a fixed m , the radius of the fringes decreases when the thickness increases.

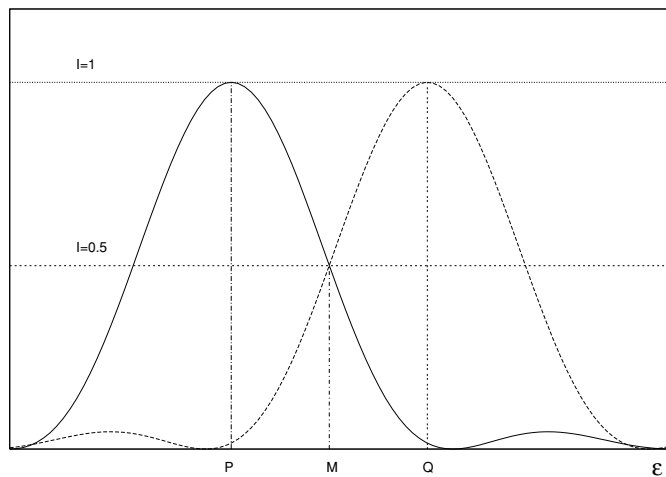


Figure 4. Full width at half-maxima criterion.

In this paper, we have suggested two exercises to study the dependence of the spatial resolution power with the reflectance r and the thickness d in a Fabry–Perot interferometer. We suggest these exercises as classroom activities or as homework.

The analysis of the results obtained shows the practical meaning of the Rayleigh criterion. Far from being an arbitrary condition, Rayleigh's formula truly captures, in a mathematical form, the resolution limit of the human visual system. The applet clearly shows that the two fringes can be distinguished by the eye when the Rayleigh criterion is fulfilled, which helps students to appreciate the resolving power of the Fabry–Perot interferometer. Moreover, the

students enjoy this approach because they understand better and more easily the underlying concepts.

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Appendix: Deduction of the spectral resolving power formula

The intensity of the interference patterns is described by

$$I = \frac{a^2}{1 + \frac{4r^2}{(1-r^2)^2} \sin^2\left(\frac{\delta}{2}\right)}, \quad (\text{A.1})$$

where

$$\delta = \frac{2\pi}{\lambda} \Delta = \frac{2\pi}{\lambda} 2d \cos \epsilon. \quad (\text{A.2})$$

The maximum of intensity ($I_{\max} = a^2$) is obtained if the following condition is verified:

$$\sin^2 \frac{\delta}{2} = 0, \quad \delta = 2m\pi \quad \text{then} \quad \Delta = 2d \cos \epsilon = m\lambda \quad \text{for} \quad m = 0, \pm 1, \pm 2, \dots \quad (\text{A.3})$$

Now, we want to determine the minimum $\Delta\lambda$ that generates two fringes separated to the extent that they fulfil the Taylor criterion (figure 4).

Let $\Delta\delta$ be the change in a phase associated with the change in the wavelength $\Delta\lambda$: $\delta_Q = \delta_P + \Delta\delta$. As P corresponds to a maximum, $\delta_P = 2m\pi$. Taking into account that the phase variation is linear for small changes in the phase, then

$$\delta_M = \delta_P + \Delta\delta/2 = 2m\pi + \Delta\delta/2. \quad (\text{A.4})$$

The resolving criterion states that $I_M = I_P/2 = a^2/2$, and introducing the condition in equation (A.1),

$$\sin^2 \frac{\delta_M}{2} = \frac{(1-r^2)^2}{4r^2} \quad \text{then} \quad \sin \frac{\Delta\delta}{4} = \frac{1-r^2}{2r} \quad (\text{A.5})$$

and approximating $\sin(\Delta\delta/4)$ with $\Delta\delta/4$, then

$$\frac{\Delta\delta}{2} = \frac{1-r^2}{r}. \quad (\text{A.6})$$

Differentiating equation (A.2),

$$\Delta\delta = -\frac{2\pi}{\lambda^2} 2d \cos \epsilon \Delta\lambda, \quad (\text{A.7})$$

and using the relation $2d \cos \epsilon = m\lambda$ results in $\Delta\delta = -\frac{2\pi}{\lambda^2} m \Delta\lambda$. Replacing $\Delta\delta$ as a function of the reflective coefficient (equation (A.6)), we get the formula of the spectral resolving power:

$$\left| \frac{\lambda}{\Delta\lambda} \right| = \frac{\pi m r}{1-r^2}. \quad (\text{A.8})$$

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