

Experimental Determination of Planck's Constant

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Abstract

Experimental determination of Planck's constant $h = 6.626 \times 10^{-34} J \cdot s$ is performed using Light Emitting Diodes (LEDs). Spectrophotometry and basic circuit analysis techniques are employed to obtain empirical data with respect to the energy of the emitted photons of light and thus used to calculate Planck's constant. It is found using three LEDs of different colors that the experimental value observed is $6.180 \times 10^{-34} J \cdot s$ with a $\sigma = 3.092 \times 10^{-36}$.

1 Introduction

The origins of discrete packets of energy, or quantum, are a result of the work done by the German physicist Max Planck. Towards the end of the 19th century the classical theory of black body radiation was proving inadequate as evident by what is known today as the ultraviolet catastrophe[1]. In October of 1900 Planck developed his black body formula, marking the true start of quantum theory. Planck's original formula as published was[2]:

$$u(\lambda, T) = \frac{C_1}{\lambda^5} \left(\frac{1}{e^{C_2/\lambda T} - 1} \right) \quad (1)$$

where u is a function of frequency f and temperature T and $h = 6.626 \times 10^{-34} J \cdot s$ is Planck's constant and $k = 1.380 \times 10^{-23} J/K$ is Boltzmann's constant, $C_1 = 8\pi ch$ and $C_2 = \frac{hc}{k}$. The success of Planck's formula for black body radiation came from his concept that blackbody radiation was the product of submicroscopic oscillators that he referred to as resonators. The significance of this concept was that the mechanical frequency f of these

resonators could only be integral multiples of hf thus ultimately resulting in the quantization of discrete energy:

$$E_{resonator} = nhf \quad n = 1, 2, 3, \dots \quad (2)$$

where n is the principal quantum number and the corresponding emission of radiation of some frequency f was the result of one of his resonators dropping down to the next lowest energy state.

Light Emitting Diodes (LEDs) are semiconductor devices characteristically defined by their ability to emit electromagnetic radiation in the visible spectrum when a potential is applied to the semiconductor materials. LEDs are composed of p-type (electron acceptor) and n-type (electron donor) materials that form a physical connection referred to as the p-n junction[3]. Excited electrons transitioning from the conducting band down to the valence band (hole-electron recombination) releases a quanta of energy equal to that of the quanta required to create the electron hole pair as a photon of discrete energy ($E = hf$) [4]. An energy band diagram shows the relationship of these hole-electron energy transitions within the semiconductor material (Fig. 1).

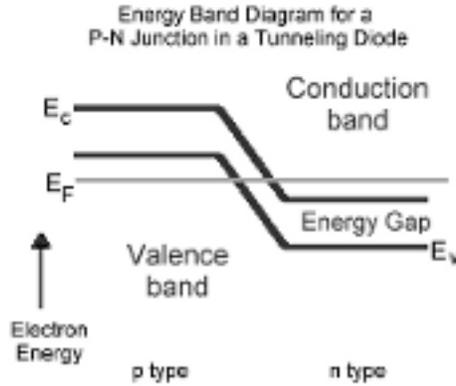


Figure 1: Typical Energy Band Diagram for a P-N Junction Diode. The electron transition from E_c to E_f releases a photon with an E equal to the difference between E_c and E_v . E_f represents threshold or forward voltage.

The relationship between the wavelength of the emitted photon, the applied potential and discrete quanta of energy E is:

$$E = hf = \frac{hc}{\lambda} = e\Delta V \quad (3)$$

where $e = 1.602 \times 10^{-19}C$ is the magnitude of the electron charge and ΔV is the magnitude of the discrete step in applied voltage over the LED.

The experimental determination of Planck's constant is then easily obtained by determining the wavelength of the emitted radiation from an LED and applied voltage over the LED concurrently, solving Eq. (3) for h to yield:

$$h = \frac{\lambda e \Delta V}{c} \quad (4)$$

and substituting the empirical values.

2 Experimental Methods

Data was collected for three different LEDs using a simple circuit (Fig. 2) to measure the potential over the LED while collecting spectral data using a PASCO spectrophotometer. The specifications of the LEDs are listed in Table 1.

Table 1: LED Information.

Color	λ (nm)	Forward V_f	Semiconductor Element [5]
Super Red	633	2.2 V	InGaAIP GaAsP/
Yellow Super	585	2.1 V	GaP
Pure Green	560	2.1 V	InGaAIP

Spectral analysis of the optical radiation emitted by each LED was performed by diffraction grating spectrophotometry. A PASCO spectrophotometer¹ configured with a PASCO high sensitivity light sensor² and rotary motion sensor³ provided light intensity with respect to angular position. The collimated light from the hydrogen discharge tube was propagated through a diffraction grating with a grating line spacing of $1673nm$. The line spacing of the diffraction grating was determined using a PASCO red diode laser⁴

¹PASCO Spectrophotometer, Model No. OS-8537

²PASCO High Sensitivity Light Sensor, Model No. CI-6604

³PASCO Rotary Motion Sensor, Model No. CI-6538

⁴PASCO Red Diode Laser, Model No. OS-8525A

with a wavelength of $650nm$. Observation of the first order fringes in the laser diffraction pattern was obtained to determine the deviation θ from the central order maximum. Diffraction grating line spacing d was then calculated using the known wavelength and θ :

$$d = \frac{\sin \theta}{m\lambda} \quad (5)$$

where $m = 1$ to yield $1.673 \times 10^{-6}m$ for the value of d .

Angular position data was obtained by the rotary motion sensor coupled with the revolving degree plate used to rotate the light sensor. Angular position calibration was performed over five trials to obtain the mean ratio ($0.0169s^{-1}$) between the degree plate and rotary motion sensor armature, thus the angular position of spectral data y is given by $y = \frac{x}{59.238}s^{-1}$ where x is the position reported by the rotary motion sensor with a sample frequency of 1400 Hz.

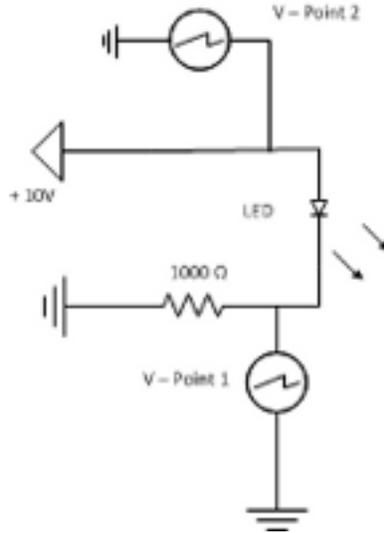


Figure 2: The schematic for the circuit used in the experiment.

A bench DC power supply was used for the potential source. PASCO voltage probes were connected at points 1 and 2 as labeled in the schematic (Fig. 2) to determine the voltage across the $1k\Omega$ resistor in series with the LED (point 1) and the total voltage across the the LED and $1k\Omega$. The difference between the two voltages yields the actual voltage across the LED or ΔV .

3 Experimental Results

A Current/Voltage (IV) profile was obtained for each LED by plotting the voltage from point 1 vs. point 2 to establish the forward voltage V_f of the LED and the resulting discrete voltage steps, $\Delta V = 0.05V$. The difference between the potentials from point 1 and 2 represent the sum of the discrete energy steps, thus the wavelength of the emitted light will decrease as the photons emitted will have higher energy proportional to the increasing potential. The current through the LED is proportional to the ΔV as given by Ohm's law:

$$I = \frac{\Delta V}{R} = \frac{\Delta V}{1000\Omega} \quad (6)$$

Spectrophotometric determination of the wavelength of light emitted from the LEDs was obtained from the first order ($m = 1$) bright fringes of the observed spectrum at the peak current as allowed by the circuit, Eq. (5). The ΔV was recorded at this point and used to determine the photon energy E as given by:

$$E_{photon} = e\Delta V \quad (7)$$

where $e = 1.602 \times 10^{-19}C$. The respective empirical data for each LED is provided in Table 2. The observed spectrum for the three LEDs is provided in the appendix with their respective IV graphs.

Table 2: LED Wavelength and Photon Energies.

LED	Wavelength	E_{photon}	h
Color	$\lambda \text{ nm}$	$(1 \times 10^{-19}J)$	$(1 \times 10^{-34})J \cdot s$
Green	562.7	3.270	6.138
Yellow	586.2	3.167	6.211
Red	629.9	2.956	6.192

4 Conclusion

Overall it is found that the experimentally determined value for Planck's constant $h = 6.180 \times 10^{-34} \text{ J} \cdot \text{s}$ is within acceptable limits as compared to the accepted value $h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s}$ with a difference of 6.7%. Regarding random error and measurement uncertainty, total differentials proved to be insignificant with regard to the the final precision of the experimental value of h as evident by the magnitude of in the final experimental value.

Further investigation and refinement of experimental execution and techniques would most likely decrease random error. Furthermore the acquisition of additional data for a variety of LEDs within each color group would also help to decrease random error by statistical elimination of inherent manufacturing inconsistencies in the LEDs.

References

- [1] Raymond A. Serway and Jr. John W. Jewett. *Principles of Physics*. Brooks/Cole, Belmont, CA, fourth edition, 2006.
- [2] Raymond A. Serway, Clement J. Moses, and Curt A. Moyer. *Modern Physics*. Saunders College Publishing, Philadelphia, PA, 1989.
- [3] Malcolm Plant. *Teach Yourself Electronics*. McGraw-Hill, Blacklick, OH, 2003.
- [4] S. M. Sze. *Physics of Semiconductor Devices*. John Wiley & Sons, New York, 1969.
- [5] S. M. Sze. *Physics of Semiconductor Devices*. John Wiley & Sons, New York, 1969.

A Spectral and IV Data

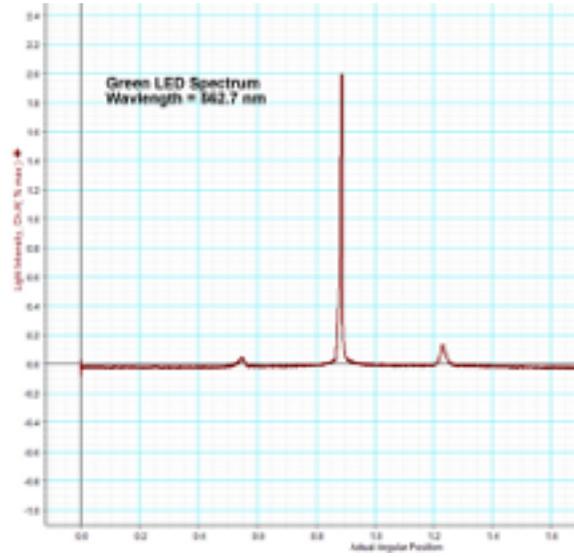


Figure 3: Observed spectrum, green LED.

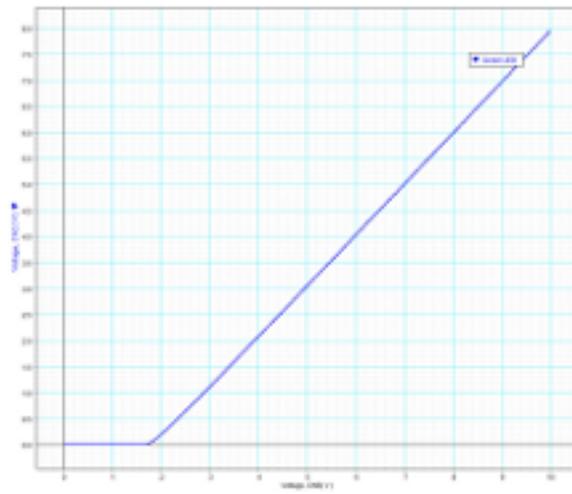


Figure 4: IV Plot, green LED.

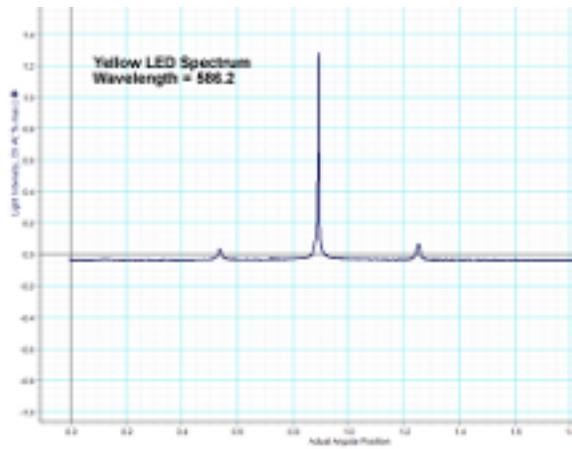


Figure 5: Observed spectrum, yellow LED.

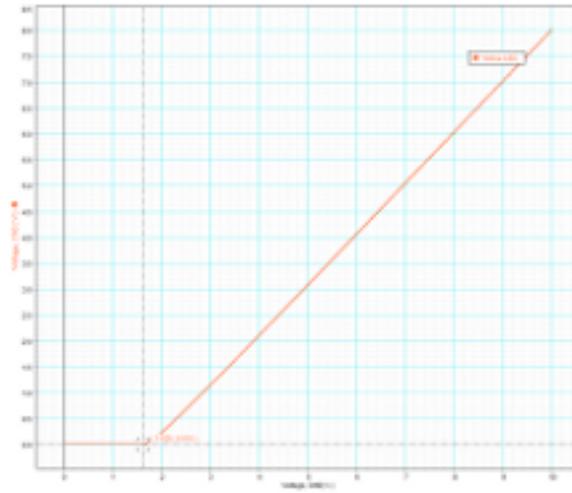


Figure 6: IV Plot, yellow LED.

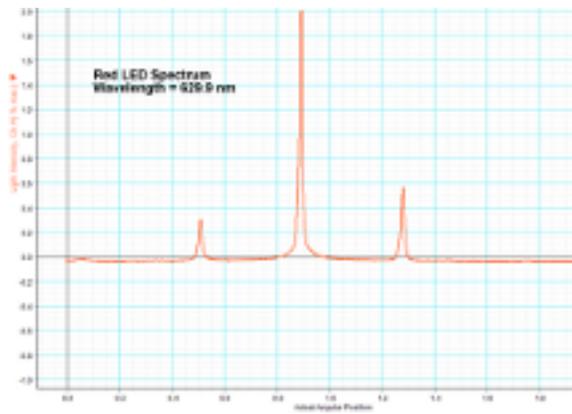


Figure 7: Observed spectrum, red LED.

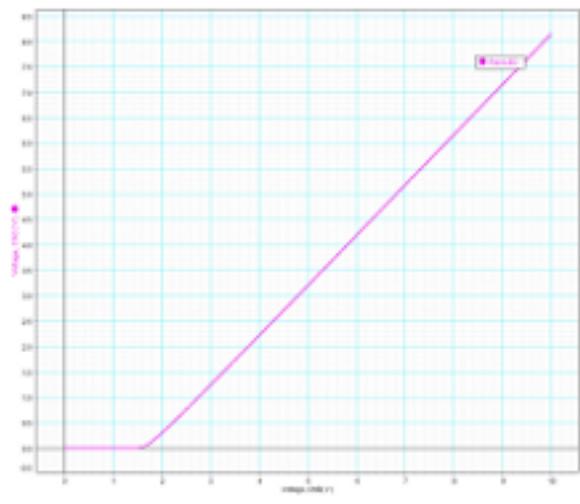


Figure 8: IV Plot, red LED.