

Optical bistability in sideband output modes induced by a squeezed vacuum

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We consider N two-level atoms in a ring cavity interacting with a broadband squeezed vacuum centered at frequency ω_s and an input monochromatic driving field at frequency ω . We show that, besides the central mode (at ω), an infinity of *sideband modes* are produced at the output, with frequencies shifted from ω by multiples of $2(\omega - \omega_s)$. We analyze the optical bistability of the two nearest sideband modes, red shifted and blue shifted.

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I. INTRODUCTION

Optical bistability (OB) has been the subject of intense research since its prediction and observation in the 1970s [1–3]. In Ref. [4] a model consisting of a system of homogeneously broadened two-level atoms driven by a coherent resonant field proved to give a successful description of OB. Due to its potential applications in optical devices there have been a lot of efforts to observe and understand the phenomenon of optical bistability in two-level atoms [6–18].

The effects of the squeezed vacuum field on the absorptive OB for a system of two-level atoms in a ring cavity (see Fig. 1), with different relaxation rates of the in-quadrature and in-phase components, were originally calculated in Ref. [8]. The authors verified that the squeezed vacuum strongly affects the OB through the increase of the atomic decay time and through the introduction of a relative phase between the input pumping and squeezed vacuum fields. Although several aspects of squeezed vacuum effects on OB have been considered [9–12], no explicit calculations were done, to our knowledge, to the situation where the frequencies of the input fields, pump (ω), and broadband squeezed vacuum (carrier ω_s) are detuned. In Refs. [9–12], exact resonance between pump and squeezed field frequencies, $\omega = \omega_s$, is assumed in order to maximize the squeezing effects. Nonetheless, consideration of detuning, $\omega \neq \omega_s$, is the source of interesting physics as will be shown in this paper. Here, we analyze the effects of that detuning over the OB in the output field, produced by a system of two-level atoms in a cavity. We show that, besides the central mode at ω , the output field contains an infinity of sideband modes at frequencies shifted from ω by multiples of $2(\omega - \omega_s)$. We analyze the OB of the two nearest sideband modes, red shifted and blue shifted.

The paper is organized as follows: In Sec. II we introduce the model we use and derive the system dynamical equations. In Sec. III we obtain the stationary solutions for the output field. In Sec. IV we discuss the results and present our conclusions. Finally, in the Appendix we derive the many-

body master equation and apply the mean-field approximation for a dilute atomic gas.

II. MODEL

We consider an input pump coherent signal of undepleted electric-field amplitude E_{in} and a broadband squeezed vacuum, with frequency distribution centered at ω_s , interacting with N two-level atoms. The Hamiltonian of the system is given by

$$H = \frac{1}{2} \omega_0 S_0 + F^* e^{i\omega t} S_- + F e^{-i\omega t} S_+ + \sum_k \omega_k b_k^\dagger b_k + \sum_k (g_k b_k S_+ + g_k^* b_k^\dagger S_-) \quad (1)$$

(we have set $\hbar = 1$), where the first term stands for the two-level atomic system (transition frequency ω_0), the two following terms represent the interaction between the atoms and the input pump field amplitude E_{in} , $F = \mu E_{in}$ (μ is the atomic dipole moment), the fourth term corresponds to the squeezed vacuum modes, and the last one is for the interaction between atoms and squeezed vacuum field. Operator b_k (b_k^\dagger) annihilates (creates) squeezed field quanta of frequency ω_k , and g_k is the coupling constant. The atomic collective operators are

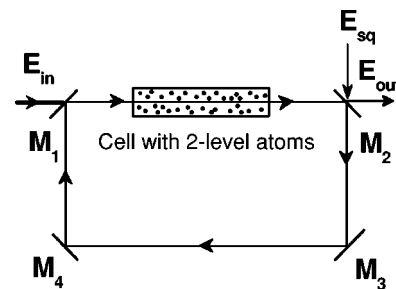


FIG. 1. Schematic ring cavity with N two-level atoms in a cell. Input signal at the left, output at the right, and injection of squeezed vacuum from above. M_1 to M_4 specify the mirrors.

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$$S_0 = \sum_{i=1}^N s_0(i), \quad S_{\pm} = \sum_{i=1}^N s_{\pm}(i), \quad (2)$$

where $s_0(i)$ and $s_{\pm}(i)$ are single-particle operators satisfying the commutation relations $[s_0(i), s_{\pm}(j)] = \pm 2\delta_{i,j}s_{\pm}(i)$ and $[s_{+}(i), s_{-}(j)] = \delta_{i,j}s_0(i)$. Although the atoms do not interact directly with each other and the coherent field is assumed undepleted, they become correlated to each other only due to their coupling with the squeezed vacuum field.

In the mean-field approximation and in a rotating frame at frequency ω , the atomic system is described by a one-body master equation, obtained by calculating the trace over the squeezed vacuum degrees of freedom (see the Appendix for a detailed derivation),

$$\begin{aligned} \frac{d\rho(t)}{dt} = & \frac{1}{i}[H_{eff}, \rho(t)] - \left\{ \left[e^{i\theta} e^{i\epsilon t} \left(\frac{\gamma}{2} - i\nu \right) 2 \sinh r \right. \right. \\ & \times \cosh rs_+ \rho s_+ + \text{H.c.} \left. \right] + \frac{\gamma}{2} \sinh^2 r (s_- s_+ \rho - 2s_+ \rho s_- \\ & + \rho s_- s_+) + \frac{\gamma}{2} \cosh^2 r (s_+ s_- \rho - 2s_- \rho s_+ + \rho s_+ s_-) \left. \right\}. \quad (3) \end{aligned}$$

The term in braces represents the phase-sensitive damping due to the squeezed vacuum, r is the squeezing parameter, $\epsilon = 2(\omega - \omega_s)$ is twice the detuning between input pump and squeezed vacuum fields, γ is the damping constant, and θ is a phase reference of the squeezed vacuum field. H_{eff} is an effective nonlinear mean-field single-particle Hamiltonian, describing the motion of one atom in the sample,

$$\begin{aligned} H_{eff} = & \frac{1}{2}(\delta - \nu \cosh 2r)s_0 + \mu \left\{ \left[E_{in} + \frac{N-1}{\mu} \left(-\nu + i\frac{\gamma}{2} \right) \right. \right. \\ & \left. \left. \times \langle s_+ \rangle \right] s_- + \text{H.c.} \right\}, \quad (4) \end{aligned}$$

where $\langle s_{\pm} \rangle = \text{Tr}(\rho s_{\pm})$, $\delta = \omega_0 - \omega$ is the detuning between atomic transition and pump field frequencies, and ν is the dynamic frequency shift; being much smaller than γ , it will

be neglected [19]. From second term in the Hamiltonian (4) we see that effectively a single generic atom is excited by the input field E_{in} plus a polarization field,

$$\epsilon_{pol}(t) = \frac{N-1}{\mu} \left(-\nu + i\frac{\gamma}{2} \right) \langle s_+ \rangle, \quad (5)$$

due to the other $(N-1)$ atoms.

The equations of motion for the atomic operators' mean values are

$$\langle \dot{s}_0 \rangle = 2i\mu [\epsilon_T(t) \langle s_- \rangle - \epsilon_T^*(t) \langle s_- \rangle^*] - \gamma (\langle s_0 \rangle \cosh 2r + 1), \quad (6)$$

$$\langle \dot{s}_- \rangle = -i\Omega \langle s_- \rangle + i\mu \epsilon_T^*(t) \langle s_0 \rangle - Q e^{i\epsilon t} \langle s_- \rangle^*, \quad (7)$$

where $\langle \dot{s}_+ \rangle = \langle \dot{s}_- \rangle^*$, $\Omega \equiv \delta - i(\gamma/2) \cosh 2r$, $Q \equiv (\gamma/2) e^{i\theta} \sinh 2r$, and

$$\epsilon_T(t) = E_{in} + \epsilon_{pol}(t) \quad (8)$$

is the total effective field experienced by a single atom. The second term in Ω is due to the commutation relations in the Heisenberg equations. Furthermore, in the induced atomic polarization field,

$$\epsilon_{pol}(t) \equiv \Lambda \langle \dot{s}_- \rangle^*(t) / \mu \quad \left(\Lambda = i\frac{\gamma}{2} N_{eff} \right), \quad (9)$$

we have assumed an effective number of atoms N_{eff} contributing effectively to this field ($N_{eff} \ll N$).

In the following section we obtain the stationary effective field amplitude $\epsilon_T(t)$ as a function of E_{in} and system parameters.

III. STATIONARY SOLUTIONS

For no detuning between vacuum squeezed and pump fields, $\epsilon = 0$, there is no explicit time dependence in Eq. (7), and the equilibrium solutions ($\langle \dot{s}_- \rangle = 0, \langle \dot{s}_0 \rangle = 0$) $\langle s_- \rangle^{eq}$ and $\langle s_0 \rangle^{eq}$ are easily obtained as functions of the output field ϵ_T , which, together with Eq. (8), enables to recover the well-known result [8,10],

$$E_{in} = \epsilon_T - \frac{\gamma\mu\Lambda(\Omega\epsilon_T - iQ^*\epsilon_T^*)}{4\mu^2\Omega_I|\epsilon_T|^2 - 2\mu^2[Q\epsilon_T^2 + Q^*(\epsilon_T^*)^2] - \gamma[|\Omega|^2 - |Q|^2]\cosh 2r}. \quad (10)$$

The bistable behavior becomes evident from plotted output field amplitude modulus $|\epsilon_T|$ as a function of the same for the pump field $|E_{in}|$, as displayed in Fig. 2. Above a critical value of N_{eff} an S-shaped curve is produced, meaning that there are two possible output fields for a single input one. More-

over, the S-shaped curve is quite sensible to the phase difference between input and squeezed vacuum fields, as stressed in Refs. [8,10].

We are interested in the situation $\epsilon \neq 0$, when Eqs. (6) and (7) are no more autonomous, so the asymptotic stationary

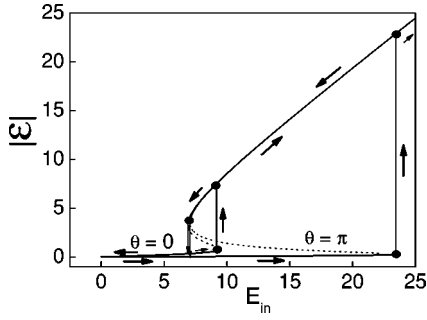


FIG. 2. Output vs input field amplitudes. Sensitivity to the phase θ is manifest.

solutions are periodical time-dependent series,

$$\langle s_- \rangle = \sum_{n=-\infty}^{\infty} a_n e^{in\epsilon t}, \quad \langle s_0 \rangle = \sum_{n=-\infty}^{\infty} b_n e^{in\epsilon t}, \quad (11)$$

whose coefficients can be determined from Eqs. (6) and (7). The output field amplitude is also expanded as an infinite series,

$$\epsilon_T(t) = \sum_{n=-\infty}^{\infty} \mathcal{E}_n e^{in\epsilon t}. \quad (12)$$

In a nonrotating frame, the total output field amplitude is a superposition of an infinite and countable number of modes,

$$E_T(t) = \epsilon_T(t) e^{-i\omega t} = \mathcal{E}_0 e^{-i\omega t} + \mathcal{E}_{+1} e^{-i(\omega-\epsilon)t} + \mathcal{E}_{-1} e^{-i(\omega+\epsilon)t} + \dots, \quad (13)$$

at frequencies $\omega_n = \omega \pm n\epsilon$ for $n=0, 1, 2, \dots$

Inserting the series (11) and (12) into Eqs. (6) and (7) and equating coefficients with same time-dependent factor $e^{in\epsilon t}$, one gets the following equations in terms a_n and b_n :

$$E_{in} = \mathcal{E}_0 - \Lambda a_0^* \quad \text{for } n=0, \quad (14)$$

$$\mathcal{E}_n = \Lambda a_{-n}^* \quad \text{for } n \neq 0. \quad (15)$$

$$i(n\epsilon + \Omega)a_n + Q a_{-n+1}^* = i \sum_{m=-\infty}^{\infty} \mathcal{E}_{m-n}^* b_m, \quad (16)$$

$$(in\epsilon + \gamma \cosh 2r)b_n = 2i \sum_{m=-\infty}^{\infty} (\mathcal{E}_{n-m} a_m - \mathcal{E}_{m-n}^* a_{-m}^*) - \gamma \delta_{n,0}. \quad (17)$$

After a lengthy but straightforward algebraic manipulation of Eqs. (15)–(17), one obtains an equation involving only the coefficients a_n and the central output field amplitude \mathcal{E}_0 ,

$$\begin{aligned} G_n(\epsilon)a_n + F_n(\epsilon)a_{-n}^* + Q a_{-n+1}^* + i \frac{\mathcal{E}_0^*}{\cosh 2r} \delta_{n,0} \\ = -2 \sum_{l(\neq n)} \left\{ \frac{\mathcal{E}_0^*}{Y_n(\epsilon)} [\Lambda a_{l-n}^* a_l - \Lambda^* a_{n-l} a_{-l}^*] + \frac{\Lambda^*}{Y_l(\epsilon)} [\mathcal{E}_0 a_{n-l} a_n \right. \\ \left. - \mathcal{E}_0^* a_{n-l} a_{-n}^*] + \sum_{m(\neq n)} \frac{1}{Y_m(\epsilon)} [|\Lambda|^2 a_{n-m} a_{l-n} a_l \right. \\ \left. - (\Lambda^*)^2 a_{n-m} a_{n-l} a_{-l}^*] \right\}, \quad (18) \end{aligned}$$

where

$$G_n(\epsilon) = i \left[n\epsilon + \Omega + \frac{\gamma \Lambda^* (1 - \delta_{n,0})}{Y_0(\epsilon)} \right] + 2 \frac{|\mathcal{E}_0|^2}{Y_n(\epsilon)}, \quad (19)$$

$$F_n(\epsilon) = -2 \frac{(\mathcal{E}_0^*)^2}{Y_n(\epsilon)}, \quad (20)$$

and

$$Y_n(\epsilon) = in\epsilon + \gamma \cosh 2r. \quad (21)$$

On the left-hand side (LHS) of Eq. (18) N_{eff} enters only in $G_n(\epsilon)$, while on right-hand side (RHS) it enters the terms involving the products of a_n 's. For field intensities of sideband modes much weaker than the central mode, we neglect the nonlinear terms on the RHS of Eq. (18). This allows us to rewrite the LHS in terms of a finite difference equation for a_n ,

$$B_n(\epsilon)a_n + C_n(\epsilon)a_{n+1} + D_n(\epsilon)a_{n-1} = E_0(\epsilon)\delta_{n,0} + H_1(\epsilon)\delta_{n,1}, \quad (22)$$

where

$$B_n(\epsilon) = G_n(\epsilon) - \frac{F_n(\epsilon)F_{-n}^*(\epsilon)}{G_{-n}^*(\epsilon)} - \frac{|Q|^2}{G_{-n+1}^*(\epsilon)}, \quad (23)$$

$$C_n(\epsilon) = -\frac{Q^* F_n(\epsilon)}{G_{-n}^*(\epsilon)}, \quad (24)$$

$$D_n(\epsilon) = -\frac{Q F_{-n+1}^*(\epsilon)}{G_{-n+1}^*(\epsilon)}, \quad (25)$$

$$E_n(\epsilon) = -\frac{i}{\cosh 2r} \left[\frac{\mathcal{E}_0 F_n(\epsilon)}{G_{-n}^*(\epsilon)} + \mathcal{E}_0^* \right], \quad (26)$$

$$H_n(\epsilon) = -i \frac{Q \mathcal{E}_0}{G_{-n+1}^*(\epsilon) \cosh 2r}. \quad (27)$$

Even in this very linear approximation the n dependence in the coefficients (23)–(27) does not allow obtaining an exact closed solution to Eq. (22) for $\epsilon \neq 0$. In the present analysis, we are going to determine only the first three sideband coefficients a_0 and $a_{\pm 1}$. From Eq. (22) one gets the following system of equations:

$$\begin{aligned}
 B_0 a_0 + C_0 a_1 + D_0 a_{-1} &= E_0, \\
 B_1 a_1 + C_1 a_2 + D_1 a_0 &= H_1, \\
 B_{-1} a_{-1} + C_{-1} a_0 + D_{-1} a_{-2} &= 0,
 \end{aligned} \tag{28}$$

which is not closed because a_0 and $a_{\pm 1}$ are coupled to $a_{\pm 2}$ that, by their turn, are coupled to higher-order coefficients. Instead of simply disregarding a_2 and a_{-2} in Eqs. (28), we consider a better approximation by estimating them from truncated continued fractions. Setting

$$x_n \equiv \frac{a_n}{a_{n-1}}, \quad y_{-n} \equiv \frac{a_{-(n+1)}}{a_{-n}}, \tag{29}$$

for $a_{n-1} \neq 0, a_{-n} \neq 0$, and $n \neq 0, 1$, we can write Eq. (22) as two equations,

$$x_n = \frac{-D_n}{B_n + C_n x_{n+1}}, \quad n \neq 0, 1, \tag{30}$$

$$y_{-n} = \frac{-C_{-(n+1)}}{B_{-(n+1)} + D_{-(n+1)} y_{-(n+1)}}, \tag{31}$$

for positive integers n . For $n=2$ in Eq. (30), $n=1$ in Eq. (31) and truncation of the continued fractions, up to a second-order iteration, yields (a higher-order iteration does not affect significantly the result)

$$\frac{a_2}{a_1} = x_2^{(2)} = - \frac{D_2}{B_2 - \frac{C_2 D_3}{B_3 - \frac{C_3 D_4}{B_4}}}, \tag{32}$$

$$\frac{a_{-2}}{a_{-1}} = y_{-1}^{(2)} = - \frac{C_{-2}}{B_{-2} - \frac{D_{-2} C_{-3}}{B_{-3} - \frac{D_{-3} C_{-4}}{B_{-4}}}}. \tag{33}$$

Substituting $a_2 = x_2^{(2)} a_1$, $a_{-2} = y_{-1}^{(2)} a_{-1}$, we get the coefficients

$$a_0 = \frac{E_0 - \frac{C_0 H_1}{B_1 + C_1 x_2^{(2)}}}{B_0 - \frac{D_0 C_{-1}}{B_{-1} + D_{-1} y_{-1}^{(2)}} - \frac{C_0 D_1}{B_1 + C_1 x_2^{(2)}}}, \tag{34}$$

$$a_1 = \frac{H_1 - D_1 a_0}{B_1 + C_1 x_2^{(2)}}, \quad a_{-1} = - \frac{C_{-1} a_0}{B_{-1} + D_{-1} y_{-1}^{(2)}}. \tag{35}$$

IV. RESULTS AND CONCLUSIONS

Using the solutions for the amplitudes a_0 and $a_{\pm 1}$, Eqs. (34) and (35), we can analyze the functional dependence of the output fields \mathcal{E}_0 and \mathcal{E}_{\pm} as a function of the input field E_{in} in modulus. To simplify the illustration of the bistable behavior, we assume θ being the phase difference between pump and squeezed input fields. The output field amplitudes $|\mathcal{E}_0|$,

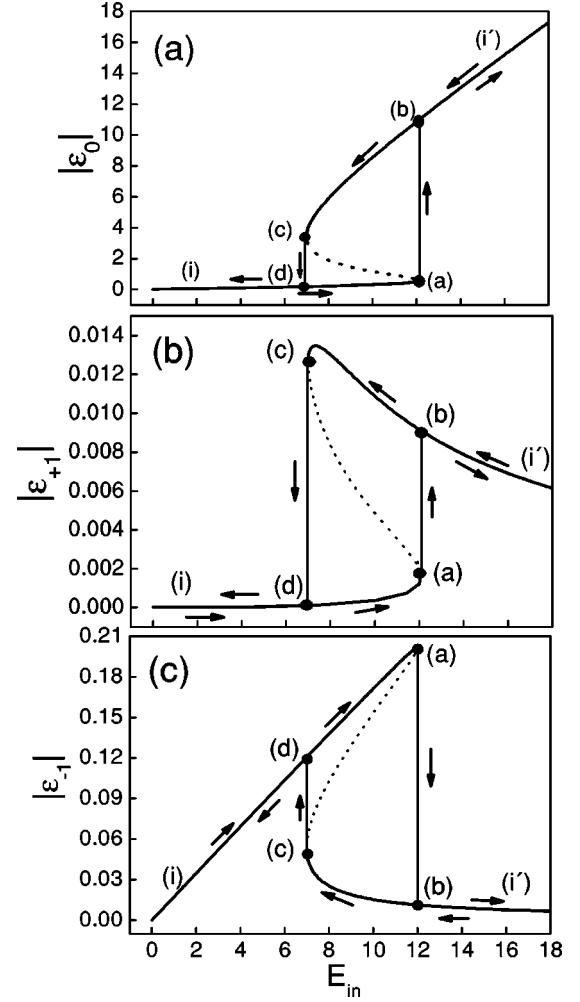


FIG. 3. Output modes vs input field E_{in} . Dashed lines are for the unstable branches. Arrows indicate the direction of variation of output amplitudes with increasing (decreasing) E_{in} . (a) Central mode amplitude \mathcal{E}_0 . The jumps goes from (a) \rightarrow (b) [(c) \rightarrow (d)] increasing [decreasing] the output amplitude. (b) red-shifted sideband mode \mathcal{E}_{+1} . The jumps are in same direction as in (a). (c) Blue-shifted sideband mode \mathcal{E}_{-1} . The jump goes from (a) \rightarrow (b) [(c) \rightarrow (d)] with decreasing [increasing] amplitude of the output field. Both jumps occur in direction opposite to those in (a) and (b). The parameters are set as $N_{eff}=101$, $\epsilon/\gamma=2.0$, $r=0.5$, $\delta=0$, and $\theta=\pi$.

$|\mathcal{E}_{+1}|=|\Lambda a_{+1}^*|$, and $|\mathcal{E}_{-1}|=|\Lambda a_{-1}^*|$ are plotted as functions of E_{in} in Figs. 3(a)–3(c), respectively. The parameters are set as $N_{eff}=101$, $\epsilon/\gamma=2.0$, $r=0.5$, $\delta=0$, and $\theta=\pi$. We verified that the OB loses the phase sensitivity, varying more significantly with r , because the coefficients a_0 , a_1 , and a_{-1} now depend on $|Q|^2$, instead of Q . Although the sideband field intensities are much weaker than the central one, they also display a bistable behavior, with turning points occurring at the same input field intensity. The dashed lines correspond to the unstable branches; the arrows indicate the path followed by the output field variation as the input is increased or decreased. The bistable behavior of the central mode [Fig. 3(a)] is similar to the case where $\epsilon=0$; however, the sideband modes \mathcal{E}_{+1} [Fig. 3(b)] and \mathcal{E}_{-1} [Fig. 3(c)] that are, respectively, red shifted and blue shifted with respect to the central mode

show some qualitative differences. Differently from the central mode, at strong pump amplitude modulus, the sideband modes show a monotonic decrease in the amplitude modulus at the output. The sideband modes also present the following different features in the switchings, or jumps from low to high amplitude (and vice versa) in comparison with the central mode.

(i) By increasing the input field intensity the $(a) \rightarrow (b)$ switch is from low to high amplitude in modes \mathcal{E}_0 and \mathcal{E}_{+1} , see Figs. 3(a) and 3(b); however it is inverted in mode \mathcal{E}_{-1} , switching from high to low amplitude, see Fig. 3(c).

(ii) By reverting the path, going from high to low input intensity the switches occur from high to low output amplitudes, $(c) \rightarrow (d)$, in modes \mathcal{E}_0 and \mathcal{E}_{+1} , Figs. 3(a) and 3(b), while it is from low to high in mode \mathcal{E}_{-1} . Essentially, the sideband modes show inverse behavior, with respect to the switchings.

(iii) In comparison to the central mode, the sidebands present a higher contrast in the jumps from higher to lower amplitude.

A possible application of the above results could be the simultaneous transmission of a message by the output field through three different channels (the three modes), where the triplicated information could be useful for error control. Additionally, the codification in the blue-shifted sideband $(0, 1, 1, 0, 0, 1, \dots)$ is the inverse of that in the other mode $(1, 0, 0, 1, 1, 0, \dots)$, so the sideband modes could transmit information as the codification occurring in the DNA double-strand macromolecule, where one strand sequence is the inverse of the other.

In conclusion, we have shown that the frequency detuning between input pump and squeezed vacuum fields, interacting with two-level atoms, gives rise to a multiple-mode output field with frequencies that are multiples of ϵ . By analyzing the closest (red shifted and blue shifted) sideband modes to the central one, we did verify different features in the bistable behavior. Although the obtention of pump and squeezed fields with controllable phase difference could be, at the moment, experimentally difficult, because both should derive from a common source, we believe that the reported physical effects could be useful in optical devices and in the transmission of information.

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APPENDIX: MASTER EQUATION

In a referential frame rotating at frequencies $\omega - \omega_k$ the Hamiltonian (1) becomes

$$H = H_{0S} + V(t), \quad (\text{A1})$$

where

$$H_{0S} = \frac{\delta}{2} S_0 + F^* S_- + F S_+, \quad (\text{A2})$$

the collective operators are defined in Eq. (2) $\delta = \omega - \omega_0$, and

$$V(t) = \sum_k (g_k b_k S_+ e^{i(\omega - \omega_k)t} + \text{H.c.}). \quad (\text{A3})$$

Following the usual procedure [19], by eliminating the reservoir degrees of freedom one obtains a premaster equation for the system density operator $\rho(t)$,

$$\frac{d\rho(t)}{dt} = \frac{1}{i} [H_{0S}, \rho(t)] - \int_0^t dt' \text{Tr}_{\mathcal{R}} [V(t), [V(t'), \rho(t') \rho_R]], \quad (\text{A4})$$

where ρ_R is the state of the reservoir at thermal equilibrium. Substituting the interaction (A3) in Eq. (A4) one gets

$$\begin{aligned} \frac{d\rho(t)}{dt} = & -i[H_{0S}, \rho(t)] - \int_0^t dt' \{ \xi_{11}(t, t') [S_+, [S_+, \rho(t')]] \\ & + \xi_{12}^*(t, t') [S_-, S_+ \rho(t')] \} \end{aligned} \quad (\text{A5})$$

$$+ \xi_{21}(t, t') [S_+, S_- \rho(t')] + \text{H.c.} \}. \quad (\text{A6})$$

The coefficients $\xi_{ij}(t, t')$ are characterized by the kind of reservoir,

$$\begin{aligned} & \int_0^t dt' \xi_{11}(t, t') \rho(t') \\ & = \int_0^t dt' \sum_{\omega', \omega''} g_{\omega'} g_{\omega''}^* e^{i(\omega - \omega')t + i(\omega - \omega'')t'} \langle b_{\omega'} b_{\omega''} \rangle_R \rho(t'), \end{aligned} \quad (\text{A7})$$

$$\begin{aligned} & \int_0^t dt' \xi_{12}(t, t') \rho(t') \\ & = \int_0^t dt' \sum_{\omega', \omega''} g_{\omega'}^* g_{\omega''} e^{-i(\omega - \omega')t + i(\omega - \omega'')t'} \langle b_{\omega'}^\dagger b_{\omega''} \rangle_R \rho(t'), \end{aligned} \quad (\text{A8})$$

$$\begin{aligned} & \int_0^t dt' \xi_{21}(t, t') \rho(t') \\ & = \int_0^t dt' \sum_{\omega', \omega''} g_{\omega'} g_{\omega''}^* e^{i(\omega - \omega')t - i(\omega - \omega'')t'} \langle b_{\omega'} b_{\omega''}^\dagger \rangle_R \rho(t'), \end{aligned} \quad (\text{A9})$$

where $\langle b_{\omega'} b_{\omega''} \rangle_R = \text{Tr}_R(\rho_R b_{\omega'} b_{\omega''})$. For a squeezed reservoir

$$\langle b_{\omega'} b_{\omega''} \rangle_R = -e^{i\theta} \sinh r \cosh r \delta[\omega'' - (2\omega_s - \omega')], \quad (\text{A10})$$

$$\langle b_{\omega'}^{\dagger} b_{\omega''} \rangle_R = \sinh^2 r \delta(\omega' - \omega''),$$

$$\langle b_{\omega'} b_{\omega''}^{\dagger} \rangle_R = \cosh^2 r \delta(\omega' - \omega''), \quad (\text{A11})$$

where r is the squeeze parameter, θ is the reference phase of the squeezed field, and ω_s is the central resonant frequency of the squeezing device. Going from sums to integrals in Eqs. (A7)–(A9) and using expressions (A10) and (A11), one gets, for example,

$$\begin{aligned} \int_0^t dt' \xi_{11}(t, t') \rho(t') &= -e^{i\theta} \sinh r \cosh r \int_0^{\infty} d\omega' D(\omega') g(\omega') \\ &\quad \times g(2\omega_s - \omega') e^{i(\omega - \omega')t} \\ &\quad \times \int_0^t dt' e^{i(\omega - 2\omega_s + \omega')t'} \rho(t'), \end{aligned}$$

where $D(\omega)$ is the reservoir density of modes. Making the change $t - t' = \tau$ and invoking the Markov approximation $\rho(t - \tau) \approx \rho(t)$ we obtain

$$\begin{aligned} \int_0^t d\tau e^{-i(\omega - 2\omega_s + \omega')\tau} \rho(t - \tau) &\cong \int_0^{\infty} d\tau e^{-i(\omega - 2\omega_s + \omega')\tau} \rho(t) \\ &= \rho(t) \left[\pi \delta(\omega - 2\omega_s + \omega') \right. \\ &\quad \left. - i\text{P} \frac{1}{\omega - 2\omega_s + \omega'} \right], \end{aligned}$$

and

$$\int_0^t dt' \xi_{11}(t, t') \rho(t') \cong \tilde{\xi}_{11}(t) \rho(t)$$

with

$$\begin{aligned} \tilde{\xi}_{11}(t) &= -e^{i\theta} \sinh r \cosh r e^{2i(\omega - \omega_s)t} \left[\pi D(2\omega_s - \omega) g(2\omega_s \right. \\ &\quad \left. - \omega) g(\omega) - i\text{P} \int_0^{\infty} d\omega' \frac{D(\omega') g(\omega') g(2\omega_s - \omega')}{\omega - 2\omega_s + \omega'} \right], \end{aligned}$$

where P stands for the Cauchy principal value. For $|\omega_s - \omega| \ll \omega$ the two terms in the brackets are assumed being approximately constant, so we define the damping constant γ and the dynamical frequency shift ν :

$$\gamma \equiv 2\pi D g^2, \quad \nu \equiv \text{P} \int_0^{\infty} d\omega' \frac{D(\omega') g(\omega') g(2\omega_s - \omega')}{\omega - 2\omega_s + \omega'},$$

therefore

$$\tilde{\xi}_{11}(t) = -e^{i\theta} \sinh r \cosh r e^{2i(\omega - \omega_s)t} \left(\frac{\gamma}{2} - i\nu \right).$$

Following the same procedure one obtains the other coefficients

$$\tilde{\xi}_{12} = \left(\frac{\gamma}{2} - i\nu \right) \sinh^2 r, \quad \tilde{\xi}_{21} = \left(\frac{\gamma}{2} - i\nu \right) \cosh^2 r,$$

which are time independent.

Thus the master equation for an N -atom system becomes

$$\begin{aligned} \frac{d\rho_N(t)}{dt} &= \frac{1}{i} [H_{0S}^{(N)}, \rho_N(t)] - \{ \tilde{\xi}_{11}(t) [S_+, [S_+, \rho_N(t)]] \\ &\quad + \tilde{\xi}_{12}^* [S_-, S_+ \rho_N(t)] + \tilde{\xi}_{21} [S_+, S_- \rho_N(t)] + \text{H.c.} \}, \end{aligned} \quad (\text{A12})$$

while for a system of p -atom system, $p < N$, it is

$$\begin{aligned} \frac{d\rho_p(t)}{dt} &= -i [H_{0S}^{(p)}, \rho_p(t)] - \left\{ \tilde{\xi}_{11}(t) \sum_{i,j=1}^p [s_+(i), [s_+(j), \rho_p(t)]] \right. \\ &\quad + \tilde{\xi}_{12}^* \left(\sum_{i,j=1}^p [s_-(i), s_+(j) \rho_p(t)] + (N-p) \right. \\ &\quad \left. \times \sum_{i=1}^p [s_-(i), \text{Tr}_{p+1} s_+(p+1) \rho_{p+1}(t)] \right) \\ &\quad + \tilde{\xi}_{21} \left(\sum_{i,j=1}^p [s_+(i), s_-(j) \rho_p(t)] + (N-p) \right. \\ &\quad \left. \times \sum_{i=1}^p [s_+(i), \text{Tr}_{p+1} s_-(p+1) \rho_{p+1}(t)] \right) + \text{H.c.} \left. \right\}. \end{aligned} \quad (\text{A13})$$

For a dilute system the atomic correlations may be disregarded, so we shall consider a single generic atom ($p=1$) moving in a mean field produced by all the others, with the two-atom density operator factorized as $\rho_2 \approx \rho_1 \otimes \rho_1$. In this approximation Eq. (A13) reduces to

$$\begin{aligned} \frac{d\rho_1(t)}{dt} &= \frac{1}{i} [H_{0S}^{(1)}, \rho_1(t)] - \{ \tilde{\xi}_{11}(t) [s_+, [s_+, \rho_1(t)]] \\ &\quad + \tilde{\xi}_{12}^* ([s_-, s_+ \rho_1(t)] + (N-1) \langle s_+ \rangle [s_-, \rho_1(t)]) \\ &\quad + \tilde{\xi}_{21} ([s_+, s_- \rho_1(t)] + (N-1) \langle s_- \rangle [s_+, \rho_1(t)]) + \text{H.c.} \}, \end{aligned} \quad (\text{A14})$$

with the single-particle Hamiltonian

$$H_{0S}^{(1)} = \frac{\delta}{2} s_0 + F^* s_- + F s_+ \quad (\text{A15})$$

and $\langle s_{\pm} \rangle = \text{Tr}(s_{\pm} \rho_1)$ is the mean value. Rearranging the terms in Eq. (A14) and dropping the subscript 1 in ρ_1 we can write Eq. (A14) as

$$\begin{aligned} \frac{d\rho(t)}{dt} = & -i[H_{eff}, \rho(t)] - \left\{ 2 \left[e^{i\theta} e^{2i(\omega-\omega_s)t} \left(\frac{\gamma}{2} - i\nu \right) \sinh r \right. \right. \\ & \times \cosh rs_+ \rho s_+ + \text{H.c.} \left. \right] + \frac{\gamma}{2} \sinh^2 r (s_- s_+ \rho - 2s_+ \rho s_- \\ & \left. \left. + \rho s_- s_+) + \frac{\gamma}{2} \cosh^2 r (s_+ s_- \rho - 2s_- \rho s_+ + \rho s_+ s_-) \right\}. \end{aligned} \quad (\text{A16})$$

The single-particle effective Hamiltonian in Eq. (16) is given by

$$\begin{aligned} H_{eff} = & \frac{1}{2} (\delta - \nu \cosh 2r) s_0 + (F^* s_- + F s_+) \\ & + (N-1) \left[\left(-\nu + i \frac{\gamma}{2} \right) \langle s_+ \rangle s_- + \text{H.c.} \right]. \end{aligned} \quad (\text{A17})$$

It contains nonlinear terms due to the mean-field effect of the remaining $N-1$ atoms. The frequency shift $\nu \cosh 2r$ is due to the interaction with the reservoir. The second term in the RHS of Eq. (A16) stands for the dissipative part due to the decay in the squeezed vacuum [19]. By setting $\omega = \omega_s$, identifying $\sinh r \cosh r \rightarrow \bar{m}$, $\sinh^2 r \rightarrow \bar{n}$, and $\cosh^2 r \rightarrow \bar{n} + 1$, the dissipative term of the master equation takes the same form as considered in Ref. [8].

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- [1] A. Szöke, V. Daneu, J. Goldhar, and N. A. Kurnit, *Appl. Phys. Lett.* **15**, 376 (1969).
- [2] S. L. McCall, *Phys. Rev. A* **9**, 1515 (1974).
- [3] H. M. Gibbs, S. L. McCall, and T. N.C. Vekatesan, *Phys. Rev. Lett.* **36**, 1135 (1976).
- [4] R. Bonifacio and L. A. Lugiato, *Opt. Commun.* **19**, 172 (1976).
- [5] R. Bonifacio and L. A. Lugiato, *Lett. Nuovo Cimento Soc. Ital. Fis.* **21**, 505 (1978); **21**, 510 (1978); **21**, 517 (1978).
- [6] E. Abraham and S. D. Smith, *Rep. Prog. Phys.* **45**, 815 (1982).
- [7] P. W. Milonni, M. L. Shih, and J. R. Ackerhalt, *Chaos in Laser-Matter Interactions* (World Scientific, Singapore, 1987), Vol 6, p. 183.
- [8] S. F. Haas and M. Sargent III, *Opt. Commun.* **5**, 366 (1990).
- [9] P. Galatola, L. A. Lugiato, M. Porreca, and P. Tombesi, *Opt. Commun.* **81**, 175 (1991).
- [10] J. Bergou and D. Zhao, *Phys. Rev. A* **52**, 1550 (1995).
- [11] S. S. Hassan, H. A. Batarfi, R. Saunders, and R. K. Bullough, *Eur. Phys. J. D* **8**, 403 (2000); **8**, 417 (2000).
- [12] M. F.M. Ali, S. S. Hassan, and S. M.A. Maize, *J. Opt. B: Quantum Semiclassical Opt.* **4**, 388 (2002).
- [13] R. Bonifacio, B. W.J. McNeil, N. Piovella, and G. R.M. Robb, *Phys. Rev. A* **61**, 023807 (2000).
- [14] D. Erenso, R. Adams, H. Deng, R. Vyas, and S. Singh, *Phys. Rev. A* **64**, 043806 (2001).
- [15] G. R.M. Robb, B. W.J. McNeil, R. Bonifacio, and N. Piovella, *Opt. Commun.* **194**, 151 (2001).
- [16] P.-W. Zhai, X.-M. Su, and J.-Y. Gao, *Phys. Lett. A* **289**, 27 (2001).
- [17] M. A. Antón, O. G. Calderón, and F. Carreño, *Phys. Lett. A* **311**, 297 (2003).
- [18] A. Joshi, A. Brown, H. Wang, and M. Xiao, *Phys. Rev. A* **67**, 041801 (2003).
- [19] C. W. Gardiner, *Phys. Rev. Lett.* **56**, 1917 (1986).