Collective atomic recoil lasing

CLAUS ZIMMERMANN, DIETMAR KRUSE, CHRISTOPH VON CUBE, SEBASTIAN SLAMA, BENJAMIN DEH and PHILIPPE COURTEILLE
Physikalisches Institut, Eberhard Karls Universität Tübingen, Germany

(Received 9 December 2003)

Abstract. A cloud of ultra-cold atoms is loaded into the attractive potential of a light wave that is generated by two counter-propagating modes of a high-finesse ring resonator. The two modes are coupled by the atoms due to coherent Rayleigh scattering and generate a potential which acts back on the motion of the atoms. This feedback leads to a new frequency component and can be described in terms of the long time proposed collective atomic recoil laser (CARL). This model is investigated experimentally and extended by introducing an optical friction force acting on the atoms. This allows for steady state operation of the CARL. Furthermore, it leads to a threshold behaviour of the CARL that translates into a novel type of phase transition: while passing the threshold the initially homogeneous atomic distribution is bunched in space and velocity. With this behaviour the system turns out to acquire some of the main features of the so-called Kuramoto model which provides a very general description of a network of limit cycle oscillators.

1. Overview

Light fields acting on the motion of atoms is a well-known and intensively investigated topic. The experiment here deals with conservative optical potentials which arise if a laser beam is tuned to the red of an atomic resonance. The induced atomic electrical dipole moment follows the inducing electrical field more or less in phase such that the dipole is dragged into regions of high light intensity. As a consequence the atom can be trapped in the focus of a laser beam or in the anti-nodes of a standing light wave. If an atom oscillates near the potential minimum of such an optical dipole trap the atomic momentum changes in time and thus the momentum of the trapping light field must also change too due to momentum conservation. This effect is typically small and there are, up to now, only few experiments which concentrate on this back action of the atoms onto the light field [1]. If, as in our experiment, the dipole trap is generated by two counter-propagating modes of a high-finesse ring cavity the back action can be observed directly as a frequency shift between the two modes. We arrive at a feedback situation where the properties of the trapping light field are determined by the motion of the trapped atoms.

To understand this frequency shift we look at a single atom which is illuminated by a running light wave from the left (figure 1). The light is far detuned to the red of the atomic resonance such that there is no substantial...
excitation of the upper atomic level. The interaction between the atoms and the light is then dominated by coherent scattering [2]. The part of the light that is scattered to the left will superimpose the incident light and form a standing wave. Since the scattered light is phase delayed by $\pi/2$, the atom finds itself placed at the slope of the freshly generated standing wave. As a consequence it is drawn to the right into the anti-node of the wave. In the photon picture the same process is explained by the recoil that is transferred from the backscattered photon to the atom. While the atom is accelerated to the right the scattering continues and the scattered waves add to the light field, which transforms into a standing wave that starts moving to the right, following the atom. Loosely speaking, one can say that the atom surfs on a self-generated standing wave. This moving standing wave can again be decomposed into two counter-propagating waves, however, now the two components are shifted in frequency by an amount that coincides with the Doppler effect. This frequency shift is observed experimentally.

In the experiment we deal with more than 1 million atoms all interacting with the same light wave. Thus, a net frequency shift can only emerge if the atoms react collectively. In fact it turns out that a fraction of the atomic ensemble starts to move in a synchronized way. They form bunches and move at the same increasing velocity. This effect is known as collective atomic recoil lasing (CARL) and was first proposed almost ten years ago [3]. The ongoing acceleration will eventually lead to a frequency shift that cannot be supported any more by the experiment which means that the effect must remain transient. As an extension of the original CARL proposal we have introduced a velocity-dependent friction force by exposing the atoms to an optical molasses, which is a standard tool for experiments with ultra-cold atoms. Now, a constant velocity is reached when the friction force levels the CARL acceleration. We not only obtain an enhanced signal and a steady state situation, the optical molasses also leads to a novel threshold behaviour: the CARL starts operating only above a certain power in the pump beam, very

Figure 1. The CARL principle: an atom is accelerated by a self-generated optical dipole potential.
analogous to a normal laser. The onset of CARL is connected to a phase transition of the atomic ensemble, which shows a sudden bunching and velocity locking if the threshold is passed. Before we discuss this in more detail the experimental setup now needs to be explained.

2. Experimental setup

In the experiment [4] we use an optical ring resonator in order to enhance the build-up of a counter-propagating wave. The resonator with a finesse of 80,000 to 150,000 is formed by two curved mirrors (200 mm radius of curvature) and a plane input coupler with 25 ppm transmission. The free spectral range of 3.5 GHz leads to a cavity linewidth between 25 kHz and 45 kHz. The forward propagating mode (pump beam, $\alpha_+$) is fed with light from a self-build titanium sapphire laser which is tuned 2 nm to the red of the rubidium D1 line and locked to the resonance of the ring resonator with a Pound-Drever locking scheme. The frequency of the laser is controlled by a piezo-mounted mirror as part of the laser resonator and an acousto-optic modulator between the laser and the ring resonator. Typically 10 W are propagating inside the cavity. The reverse mode of the resonator (probe beam, $\alpha_-$) can also be fed by the laser but in most of the experiments described here this input branch is blocked by a shutter.

The light that leaks out of the cavity through one of the curved mirrors is used to record a beat signal between the pump and the probe beam. This signal provides access to the frequency shift explained in the introduction. Rubidium atoms are loaded from a standard magneto-optical trap (MOT) into the dipole trap formed by the waist of the pump beam between the two curved mirrors ($w_0 = 125 \, \mu m$). The MOT is operated inside a vacuum chamber (base pressure: $< 10^{-10} \, \text{mbar}$) by means of a continuously operated rubidium dispenser (SAES) and 100 mW total light power from an injection locked diode laser system. The dipole trap is

![Figure 2. Experimental apparatus.](image-url)
activated during loading of the MOT. After blocking the MOT light, up to $10^8$ atoms are captured in the dipole trap at a temperature of about 200 $\mu$K and a peak density of up to $10^{11}$ cm$^{-3}$. In the experiments described below the density ($10^9$ cm$^{-3}$) and the atom number ($10^6$) are typically much smaller such that the field coupling between the two resonator modes due to the atoms is about ten times smaller than the optical decay rate of the resonator. The trapped cloud decays rapidly due to inelastic collisions but stabilizes at a reduced density after some 10 ms. Then the lifetime of the atoms inside the dipole trap is only limited by collisions with the background gas to about 500 ms. This value can easily be increased by reducing the current in the rubidium dispenser and investing more time for loading the MOT. However, the experiments are all carried out within a few milliseconds and a longer lifetime is not required. The atomic cloud can be directly observed by time-of-flight images after the trap has been turned off. A resonant laser beam illuminates the atomic cloud after it has expanded for some milliseconds. The atoms generate a shadow which is recorded with a CCD camera. From the shape and the opacity of the shadow image the density distribution and the number of atoms can be derived. In addition, we use a Raman spectrometer, which allows one to record the atomic velocity distribution by means of recoil-induced resonances [5]. This method can be applied inside the dipole trap without destroying the atomic cloud [6].

3. Some observations and interpretations

In a first experiment [7] the shutter is open and the atoms are loaded into the standing wave dipole trap. The beat signal is monitored and at $t = 0$ the shutter is closed. Without trapped atoms, the signal rapidly decays to a constant value within less than 10 $\mu$s. However, with atoms loaded into the dipole trap, a rapid oscillation is observed (figure 3). Its frequency increases in time while its amplitude drops. Qualitatively, one expects such a behaviour from the above picture. The atoms are accelerated by the standing wave which is maintained due to coherent scattering. In time, the increasing frequency shift tunes the probe beam off resonance and the signal drops. Nevertheless it is possible to still detect a signal after several milliseconds, which exceeds the optical lifetime of the resonator by several orders of magnitude.

![Figure 3](image-url)

---

**Figure 3.** Beat signal between the two counter-propagating modes of the resonator after the shutter has been closed and with atoms loaded from a magneto-optical trap.
For quantitative interpretation, a set of non-linear differential equations is available [8] that describes the amplitudes of the pump and the probe beam, the frequency difference between them and the motion of the individual atoms within the atomic cloud. The atomic ensemble is further characterized by the bunching parameter $b$ which is 0 for a homogenous spatial distribution and 1 for bunched atoms, i.e. a distribution where the atoms are all at the same position modulo one period of the standing wave dipole potential. The model is one-dimensional and ignores the transverse motion of the atoms. With this simplification CARL can be described very well, as well as some other interesting effects such as bistable behaviour as has been recently observed in a similar experiment [9]. Yet, for our experiment the 1D model describes the observation surprisingly well. Figure 4 presents the result of a numerical simulation with 100 atoms. For comparison with the experiment the coupling due to a single atom is scaled up appropriately. The simulation also yields a value for the bunching parameter $b$. Since the atoms are initially loaded into a standing wave the bunching is preset. It remains partially preserved when the shutter is closed.

In the second experiment [7] the shutter is closed from the beginning and a homogenous distribution of atoms is loaded into the pump beam trap. A beat note is now observed only after the molasses has been turned on. Within microseconds the signal appears (figure 5(a)). The CARL acceleration is almost instantaneously counterbalanced by the friction force and the beat frequency is constant in time (viscous CARL). Furthermore, the beat amplitude exceeds that of figure 4 by almost an order of magnitude. The simulation is now done with an additional

![Figure 4](image1.png)

Figure 4. Comparison between experiment and theory for a standing wave loaded with atoms from a magneto-optical trap. At $t = 0$ the shutter has been closed.

![Figure 5](image2.png)

Figure 5. Viscous CARL. The atoms are loaded into a running wave dipole trap. At $t = 0$ the molasses is activated and the atoms are submitted to a friction force. This leads to dynamical instability which results in CARL operation at a constant frequency. The results of the numerical simulation are presented in (b). The molasses is modelled by adding a simple friction term to the basic CARL equations.
friction term in the equations of motion for the atoms (figure 5(b)). The bunching parameter $b$ approaches 1 as soon as the molasses is activated. In this experiment the beat signal and the atomic bunching are spontaneously bootstrapping from thermal fluctuations.

Although the beat signal can be reproduced nicely by the simulations the mere introduction of a simple damping term is not sufficient for a good model. It would lead to perfect bunching and a vanishing temperature of the atomic ensemble because all fluctuations would be damped completely. In the experiment the finite temperature which is inherent to an optical molasses plays an important role and gives rise to a threshold behaviour that is described in the next section.

4. Threshold, phase transition and Kuramoto model

The viscous CARL properties depend on the pump power. In the experiment the pump power is varied while the beat signal is recorded. The most striking observation is the existence of a threshold behaviour: CARL operation sets in only above a critical pump power. Above threshold the amplitude and the frequency increase with pump power in a characteristic way. Following the simple picture of the introduction one would not expect a threshold which, in fact, is a consequence of the interplay between the introduced friction and a stochastic driving force. In the optical molasses the atomic motion is damped by a velocity-dependent optical force, but there are also stochastic heating effects due to the photonic character of the quantized light field. The balance between damping and heating defines the temperature of the atoms in the molasses [10]. For our experiment the main consequence now is that the stochastic heating inside the molasses counteracts the atomic synchronization required for CARL. Only for sufficiently strong pumping power the tendency for bunching overcomes the stochastic diffusion. This effect can be incorporated into the theory by introducing a Focker–Planck equation to describe the atomic ensemble [11]. Besides the damping constant for the atomic motion a diffusion constant appears as a new parameter of the system which describes the stochastic force acting on the atoms. The threshold can now be described theoretically and simulations are possible (figure 6).

These predictions agree very well with the experimental observations [12]. The value of the threshold varies with the atom number and the detuning of the dipole trap from atomic resonance. Both change the coherent scattering strength that couples the pump and the probe beam. It is interesting to look at the CARL frequency at threshold. It turns out that it does not depend on the scattering strength but only on the temperature and the viscosity of the optical molasses. Preliminary data confirm this prediction. This also offers a new access to determine the temperature inside the dipole trap during the experiment.

The threshold behaviour and the connected sudden synchronization of the atomic motion can be interpreted also as a realization of the so-called Kuramoto model [13]. It investigates the very general situation of an ensemble of limit cycle oscillators which are coupled each with all others. The model can be applied to very different phenomena ranging from arrays of Josephson junctions or phase-locked lasers, to biological systems like cardiac pacemaker cells or chorusing crickets [14]. If the intrinsic resonance frequencies of the (uncoupled) oscillators only differ within a certain range the model predicts frequency and phase synchronization of part of the oscillators provided that the coupling strength
exceeds a critical value. The threshold value is given by the spread of oscillator frequencies. The connection to our CARL system is established by the so-called mean field approach. It assumes that the collective action on one of the oscillators by all others can be properly modelled with a global force field that all oscillators are submitted to. The force field is again given by the average over all oscillator excursions. In our case the global field is realized by the optical dipole potential due to the light inside the resonator. The oscillators are formed by the atoms whereby the position of the atom is mapped to the phase of the oscillators. Frequency synchronization of the oscillators is translated into equal velocity of all atoms that participate in the CARL process. Phase synchronization is equivalent to a bunching parameter close to one. The spread in the intrinsic resonance frequencies can be mapped to the stochastic force, which is different for each atom at a given moment. For CARL the collective motion is self-determined: the friction defines a steady atomic centre-of-mass velocity to which the individual atomic velocities may lock. Diffusion is the source of disorder, which rules the phase transition by competing with the dynamical coupling.

5. Outlook

Some questions remain to be solved for the current experiment. As mentioned above, not all atoms participate in the CARL process. The initial ensemble

<table>
<thead>
<tr>
<th>beat frequency ( \Delta \omega/2\pi ) (kHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>probe power ( P_+ ) (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>0.1</td>
</tr>
<tr>
<td>0.1</td>
</tr>
<tr>
<td>0.2</td>
</tr>
<tr>
<td>0.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>bunching parameter ( b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>0.5</td>
</tr>
<tr>
<td>0.5</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>1</td>
</tr>
</tbody>
</table>
separates into two fractions which can be clearly distinguished in velocity space by taking spectra of recoil-induced resonances (see experimental setup and [6]). In this context it is interesting to measure the bunching parameter directly by observing Bragg scattering at the atomic ensemble [15]. It is also interesting to enter the ‘bad cavity limit’ where the CARL beat signal is smaller than the cavity line width. In this case one would expect a superradiant behaviour with a CARL amplitude that depends quadratically on the atom number. The experimental challenge is to reduce the cavity lifetime without falling below the CARL threshold. This should be possible if one goes to smaller temperatures. Other open aspects are recently discussed cavity cooling schemes [16,17] which should occur as an additional damping mechanism.

For the future it is certainly fascinating to enter the quantum world and develop a ‘Quantum Kuramoto’ model. For instance, a new regime is reached if the temperature drops below the so-called recoil limit where the atomic de Broglie wave exceeds the optical wavelength. An atom is no longer localized at a single lattice site and a band model of CARL is required. Finally, going below the critical temperature for Bose–Einstein condensation would open a whole new range of experiments. First theoretical studies predict novel entanglement mechanisms which might be accessible experimentally [18].

Atomic ensembles in high finesse ring resonators appear to be fascinating new systems which are just about to be investigated. They combine very different topics ranging from classical phase transitions and instabilities, to optical cooling, cavity quantum electrodynamics and even quantum many body physics. It will certainly be exciting to explore these systems in the future.

Acknowledgements
We gratefully acknowledge numerous fruitful and inspiring discussions with R. Bonifacio, A. Hemmerich, J. Javaloyes, N. Piovella, H. Ritsch, G. Robb, V. Vuletic. The work is supported in parts by the Landesschwerpunktsprogramm Baden-Württemberg.

References
Collective atomic recoil lasing
