Quantum Nondemolition Measurements

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Quantum nondemolition (QND) measurement is actually a special type of measurement in quantum system in which the uncertainty of the measured observable does not increase from its measured value during the evolution of the system. This necessarily requires that the measurement process preserve the physical integrity of the measured system, and moreover places requirements on the relationship between the measured observable and the self-Hamiltonian of the system.

I. INTRODUCTION

Quantum mechanics tells us that, whenever a person measures some property of a particle in a microworld, this measurement will disturb the particle state somewhat in unpredictable way. The more accurate the measurement is, the bigger and more unpredictable the disturbance [2]. Most devices capable of detecting a single particle and measuring its position strongly modify the particle's state in the measurement process, e.g. photons are destroyed when striking a screen. A very careful measurement of the east-west position of an electron, with imprecision $\triangle x$, can be guaranteed to disturb its east-west momentum by not much more than $\triangle p = \hbar/(2\triangle x)$, where $\hbar$ is the reduced Planck constant [1].

We need to keep in mkind that "nondemolition" does not imply that the wave function fails to collapse. Much of the investigation into QND measurements was motivated by the desire to avoid the standard quantum limit in the experimental detection of gravitational waves [3]. The general theory of QND measurements was laid out by Braginsky, Vorontsov, and Thorne.

II. TECHNICAL DEFINITIONS

A. Gravit-wave antenna

Gravity-wave detectors consist of aluminum (or sapphire or silicon or niobium) bars, weighing between 10 kilograms and 10 tons, which are driven into motion by passing waves of gravity, the motions are very tiny. For the gravity waves that theorists predict are bathing the earth, a displacement $\delta x \sim 10^{-19} cm$ might be typical [10]. And this displacement may oscillate, due to oscillations of the gravity wave, with a period $P \sim 10^{-3} s$. To see the details of the gravity wave, one must thus make repeated measurements of the bar's position with precision $\triangle x \leq 10^{-19} cm$, and with time intervals between measurements of $\tau \leq 10^{-3} s$ but one never before tried to make measurements of such enormous precision as $\delta x \sim 10^{-19} cm$. If the bar is suspended freely like a pendulum, as it is in some detectors, then over time intervals $\tau \sim 10^{-3} s$ it will behave as though it were not suspended at all. It will be as free to move horizontally as the electron described above and like the electron it will be subject to the laws of quantum mechanics: an "initial" measurement of the bar's east-west position with precision $\triangle x_i \sim 10^{-19} cm$ will inevitably disturb the bar's east-west momentum by $\triangle p \geq \hbar/2\triangle x_i$, and correspondingly will disturb its velocity by $\triangle v = \triangle p/m \geq \hbar/2m\triangle x_i$, where m is the bar's mass.

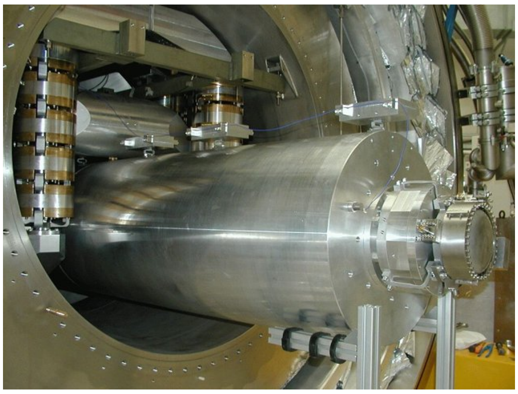

Figure 1: The AURIGA gravitational wave detector consists of a 3-meter aluminum cylinder, cooled to a few thousandths of a degree above absolute zero. The bar is kept carefully isolated from other vibrations, to help measure tiny gravitational disturbances - or possibly quantum-gravitational effects.

During the time interval $\tau \sim 10^{-3} s$ between measurements, the mass will move away from its initial position by an amount, $\triangle x_m = \triangle v\tau \geq \hbar\tau/2m\triangle x_i$, which is unpredictable because $\triangle v$ is unpredictable. Putting in values, we find $\triangle x_m \geq 5 \times 10^{-19} cm$ which is somewhat larger than the desired precision of our sequence of measurements. If the next measurement reveals a position changed by as much as $5 \times 10^{-19} cm$, we have no way of knowing whether the change was due to a passing gravity wave or to the unpredictable, quantum mechanical disturbance made by our first measurement. In effect, our first measurement plus subsequent free motion of the bar has "demolished" all possibility of making a second measurement of the same precision, $\triangle x \sim 10^{-19} cm$, as the first, and of thereby monitoring the bar and detecting

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the expected gravity waves.

In principle one can circumvent this problem by making the bar much heavier than 10 tons (recall that $\triangle x_m$ is inversely proportional to the mass). However, this is impractical. Another solution is to shorten the time between measurements (recall that $\triangle x_m$ is directly proportional to τ). However, this will weaken the gravitational-wave signal even more than it reduces the unpredictable movement of the bar. Alternatively, find some way to circumvent the effects of the Heisenberg uncertainty principle that is, some way to prevent the inevitable disturbance due to the first measurement, plus subsequent free motion, from demolishing the possibility of a second accurate measurement: a quantum nondemolition (QND) method.

One QND method which could work in principle is this: instead of measuring the position of the 10-ton bar, measure its momentum with a small enough initial error, $\triangle p_i \sim 10^{-9}$ gr.cm/s, to detect the expected gravity waves. Thereby inevitably disturb the bar's position by an unknown amount $\triangle x \geq \hbar(2\triangle p_i) \sim 5\times10^{-19}cm$. Wait a time $\tau \sim 10^{-3}s$ then make another momentum measurement. As the bar moves freely between the measurements, its momentum remains fixed. The uncertainty $\triangle x$ in the bar's position does not by free evolution produce a new uncertainty $\triangle p_m$ in the momentum.

B. Quantum nondemolition measurement

Let $\hat{A}$ be an observable for a system S with self-Hamiltonian $\hat{H}_S$. The system S is measured by an apparatus R which is coupled to S through interactions Hamiltonian $\hat{H}_{RS}$ for only brief moments. Otherwise, S evolves freely according to $\hat{H}_S$. A precise measurement of $\hat{A}$ is one which brings the global state of S and R into the approximate form

$$|\psi\rangle \approx \sum_i |A_i\rangle_S |R_i\rangle_R \tag{1}$$

where $|A_i\rangle_S$ are the eigenvectors of A corresponding to the possible outcomes of the measurement, and $|R_i\rangle_R$ are the corresponding states of the apparatus which record them. Allow time-dependence to denote the Heisenberg picture observables:

$$\hat{A}(t_i) = e^{-it\hat{H}_S}\hat{A}e^{it\hat{H}_S} \tag{2}$$

Mathematically, a sequence of measurements of $\hat{A}$ are said to be a QND measurements if and only if

$$\left[\hat{A}(t_i), \hat{A}(t_j)\right] = 0 \tag{3}$$

If this property holds for any choice of t_i and t_j, then $\hat{A}$ is called a continuous QND observable. If this only holds for certain discrete times, then $\hat{A}$ is said to be a strobescopic QND observable. If $\hat{A}$ is conserved during the free evolution , $d\hat{A}/dt = 0$, then it is guranteed to satisfy Eq. 3 for all t_i, t_j and therefore to be continous QND observable.

C. Free particle and Harmonic oscillator

For example, in the case of a free particle, the energy and momentum are conserved and indeed continous QND observables, but the position is not: $\hat{x}(t+\tau) = \hat{x}(t) + \hat{p}\tau/m$.

$$[\hat{x}(t), \hat{x}(t+\tau)] = \frac{i\hbar\tau}{m} \tag{4}$$

On the other hand, for harmonic oscillator the position and momentum satisfy the commutation relations

$$[\hat{x}(t), \hat{x}(t+\tau)] = \frac{i\hbar}{m\omega}\sin\omega\tau \tag{5}$$

$$[\hat{p}(t), \hat{p}(t+\tau)] = i\hbar m\omega\sin\omega\tau \tag{6}$$

Relations (5) and (6) imply that $\hat{x}$ and $\hat{p}$ are not continuous QND observables. However, if one makes the measurements at times seperated by an integral numbers of half-periods ($\tau = k\pi/\omega$), then the commutators in Eqs. 5 and 6 vanish. This means that $\hat{x}$ and $\hat{p}$ are stroboscopic QND observables [4, 5]. Stroboscopic QND measurments of $\hat{x}$ and $\hat{p}$ drive the oscillator into a state where x is known precisely. For example at $t = k\pi/\omega$, $\hat{x}$ and $\hat{p}$ are known precisely but at other times are highly uncertain. For an oscillator the conserved quantities which are QND observables at all times, include the energy [6] and the real and complex parts of the amplitude [7].

$$\hat{X}_1 = \hat{x}(t)\cos\omega t - \left(\frac{\hat{p}(t)}{m\omega}\right)\sin\omega t \tag{7}$$

$$\hat{X}_2 = \hat{x}(t)\sin\omega t - \left(\frac{\hat{p}(t)}{m\omega}\right)\cos\omega t \tag{8}$$

High precision measurement of $\hat{X}_1$and $\hat{X}_2$ are called back-action-evading measurements [8, 9] because they enable the measured component of the amplitude $\hat{X}_1$ to avoid back-action contamination by the measuring device at the price of strongly contaminating the other component $\hat{X}_2$.

$$\triangle X_1 \triangle X_2 \geq \frac{\hbar}{2m\omega} \tag{9}$$

D. State-preparation measurement

Let $\hat{A}$ be a QND observable which is monitored by a sequence of perfect QND measurements at times t_0, t_1, t_2, ... Since $\hat{A}(t_0)$ and $\hat{A}(t_j)$ commut, one can perform a state-preparation measurementat time t_0 which puts the system into a simultaneous eigenstate $|\psi_0\rangle$ of the observable $\hat{A}(t_0)$, $\hat{A}(t_1)$, $\hat{A}(t_2)$ and so on. From the results of the first measurement, one can compute the eigenvalues $A(t_0)$, $A(t_1)$, $A(t_2)$, ... Later as the system evolves freely, its state $|\psi_0\rangle$ remains fixed in time, while its observable $\hat{A}$ evolves through the values $\hat{A}(t_0)$, $\hat{A}(t_1)$ and so on. Subsequent perfect measurements of $\hat{A}$ at times t_1, t_2, ... must give the known eigenvalues $A(t_1)$, $A(t_2)$ and must leave the state of the system $|\psi_0\rangle$ unchanged.

If $\hat{A}$ is a continuous QND observable, then the QND measurements can be made continuously, and each measurement can last as long or as short a time as one wishes. If $\hat{A}$ is a stroboscopic QND observable, then each measurement must be made very quickly (stroboscopically) to avoid contamination.

E. QND measurement error

ecause of this back action, the measurement error must always exceed an ultimate quantum limit. We shall derive that limit under the special assumption that in the Heisenberg picture $\hat{A}$ and $\hat{C}$ are time-independent (either because they are constants of the motion such as $\hat{X}_1$, and $\hat{X}_2$, or because they are time) evolving observables evaluated at some fixed moment of time. We assume that the "readout observable" of the last quantum stage, $\hat{Q}_R$, which couples into the first classical stage, is expressible as

$$\hat{Q}_R = f\left(\alpha\hat{A} + \beta\hat{C}\right) \tag{10}$$

Where

$$\left[\hat{A}, \hat{C}\right] = 2i\gamma\hbar \tag{11}$$

with γ a real number. The time evolution of the readout observable $\hat{Q}_R$ is embodied in the function f and/or in the real parameters α and β. Typically,α and β will be sinusoidal functions of time which are used to code and separate the $\hat{A}$ and $\hat{C}$ signals. We assume that the first classical stage (usually an amplifier) is equally sensitive to signals at the $\hat{A}$ and $\hat{C}$ frequencies. Then no matter how accurately the first classical stage monitors $\hat{Q}_R$, it must give errors in $\hat{A}$ and $\hat{C}$ related by

$$\triangle A = \left(\frac{\overline{\beta}}{\overline{\alpha}}\right)\triangle C \tag{12}$$

Where $\overline{\alpha}$ and $\overline{\beta}$ are the rms values of α and β. These relative errors imply the ultimate quantum limit

$$\triangle A \geq \left[\left(\frac{\overline{\beta}}{\overline{\alpha}}\right)\gamma\hbar\right]^{1/2} \tag{13}$$

III. APPLICATION: PHOTONS COUNTING

In a way, the quantum world seems to know when it's being watched. When physicists make measurements on photons and other quantum-scale particles, the measurements always disturb the system in some way. Although an ideal disturbance should still enable physicists to make multiple measurements and get the same result twice, most real measurements cause a greater disturbance than this ideal minimum, and prohibit physicists from making repeated measurements.

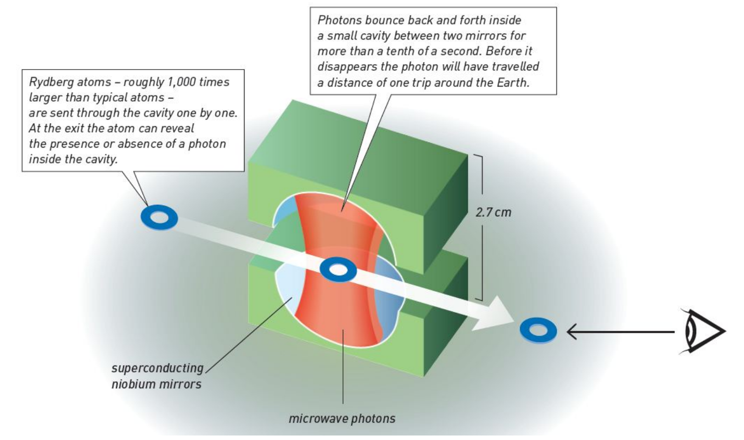

Figure 2: In the Serge Haroche laboratory in Paris, in vacuum and at a temperature of almost absolute zero, the microwave photons bounce back and forth inside a small cavity between two mirrors. The mirrors are so reflective that a single photon stays for more than a tenth of a second before it's lost. During its long life time, many quantum manipulations can be performed with the trapped photon without destroying it.

In a recent study, physicists have demonstrated a new way to make one of the ideal measurements (QND measurements) allowing physicists to detect single particles repeatedly without destroying them. In the latest technique, developed by a team of physicists from Yale University, Princeton University, and the University of Waterloo, the scientists have shown how to measure the number of photons inside a microwave cavity in a way that preserves the photon state 90% of the time; in other words, the method is 90% QND. The physicists explain that, unlike previously reported QND methods, the new technique is strongly selective to chosen photon number states, which could make it useful for applications such as monitoring the state of a photon-based memory in a quantum computer.

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