Quantum paramagnetic systems

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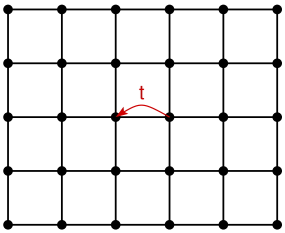
Menu du Jour

- Magnetic ordered phases
- Quantum paramagnetic phases
- Frustrated magnetic systems
- Theoretical tools for spin systems
- Triangular–lattice AFM Heisenberg model

From Hubbard to Heisenberg

- electron spin: $\sigma = \uparrow\downarrow$
- half-filling:
  $n_{i\uparrow} + n_{i\downarrow} = 1$



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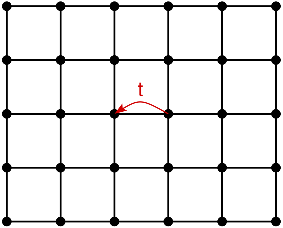

Hubbard model

$$H = -t \sum_{\langle i\,j \rangle\,\sigma} \left(c^{\dagger}_{i\,\sigma} c_{j\,\sigma} + \text{h.c.} \right) + U \sum_{i} \hat{n}_{i\uparrow} \hat{n}_{i\downarrow} \qquad U > 0$$

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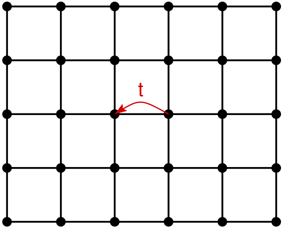

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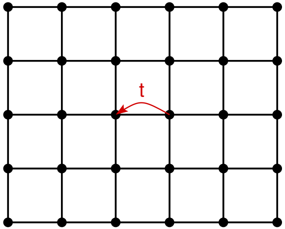

Heisenberg model

$$H = J \sum_{\langle i\, j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j \qquad J = 4t^2/U \quad \text{exchange coupling}$$

$$S_i^k = \frac{1}{2} c_{i\,\alpha}^\dagger \sigma_{\alpha\beta}^k c_{i\,\beta} \qquad k = x, y, z$$

Antiferromagnet (AFM) Heisenberg model

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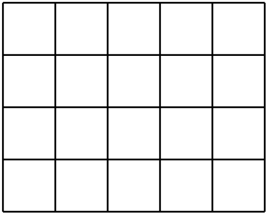

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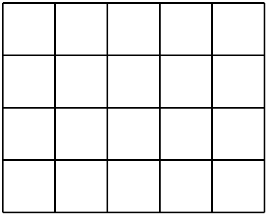
Classical spins: $S \gg 1/2$

Ground state ($T = 0$):

Néel order $\quad \xi_{\mathrm{mag}} \to \infty$

$M = 0$ and $M_{stagg} = M_0$



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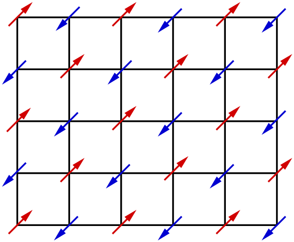

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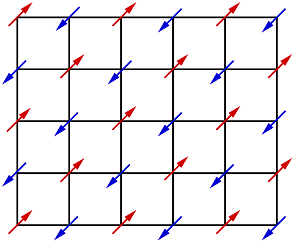

Quantum case, $S = 1/2$: Néel + quantum fluctuations

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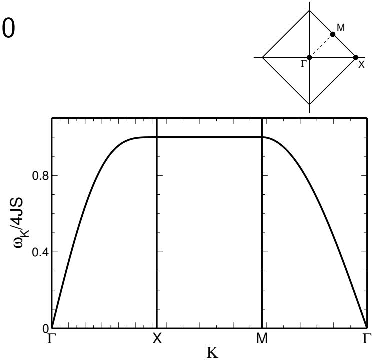

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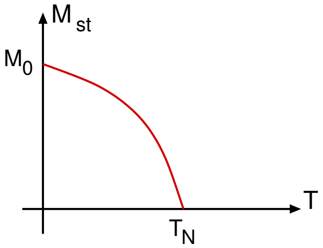


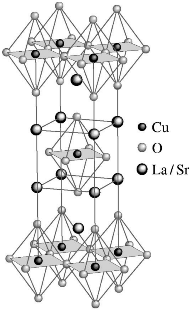
Quantum case, $S = 1/2$: Néel + quantum fluctuations

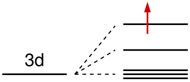
2-D: paramagnetic (disorder) phase ($T \neq 0$)

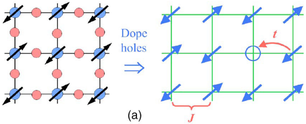
3-D: phase transition at finite temperature T_N (Néel)

Heisenberg model - antiferromagnetic (AFM)

Ex.: La_2CuO_4 $T_N \approx 300\ K$ Cu^{+2}: $[Ar]3d^9$







Coldea et al., 2001; Damascelli et al., 2003; Lee et al., 2006; Vojta, 2009

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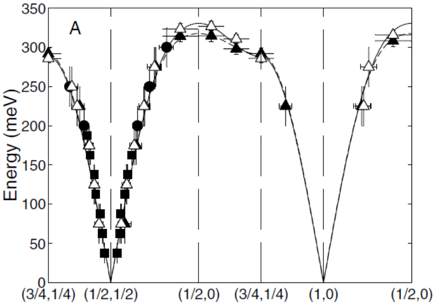

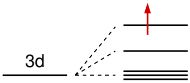

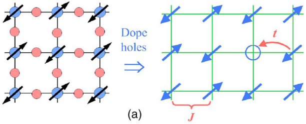

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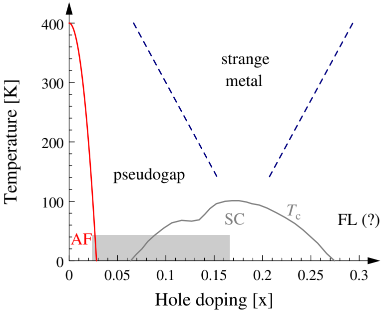
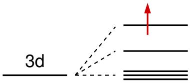
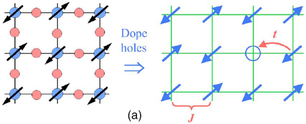






$La_{2-x}Sr_xCuO_4$: high-temperature superconductor

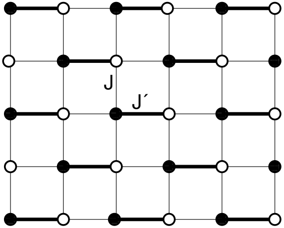
Coldea et al., 2001; Damascelli et al., 2003; Lee et al., 2006; Vojta, 2009

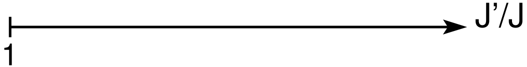
Staggered dimerized AFM Heisenberg model

$$H = \sum_{\langle i\, j \rangle} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

$J_{ij} = J' \geq J > 0$

$S = 1/2$ $T = 0$





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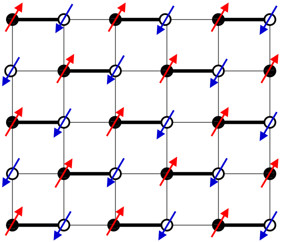

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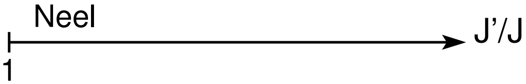

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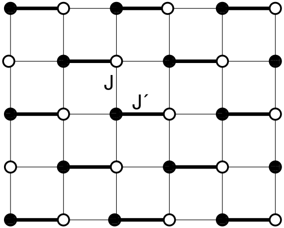

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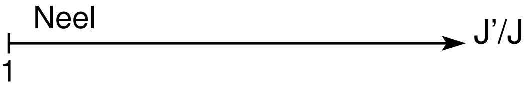

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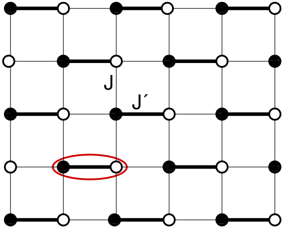

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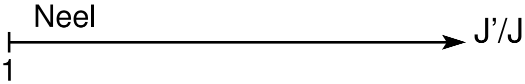

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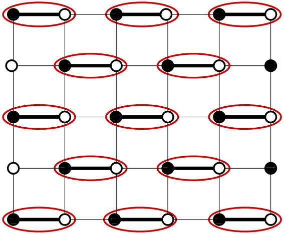

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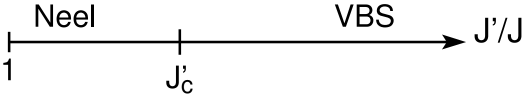

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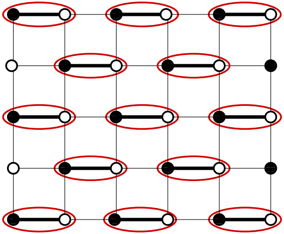

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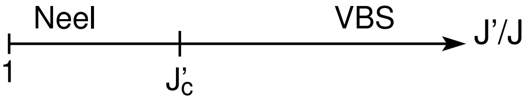

VBS: Valence bond solid (quantum paramagnetic phase)

$\xi_{\rm mag} \sim 1$

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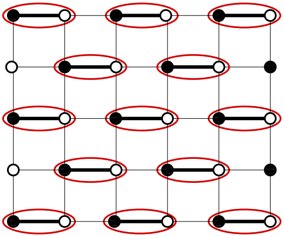

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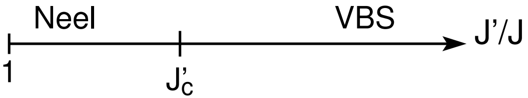


VBS: Valence bond solid $\qquad$ (quantum paramagnetic phase)

$$\xi_{\mathrm{mag}} \sim 1$$

Quantum Monte Carlo: $J'_c = 2.5186\,J$ $\qquad$ Wenzel et al., 2008

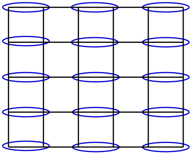
About quantum phase transition: $\qquad$ Fritz et al., 2011

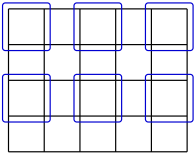
Quantum paramagnets: valence bond solid (VBS)

- VBS $\sim$ crystal of singlets (valence bonds)

  $\xi_{\mathrm{mag}} \sim 1$

- Spin-rotation symmetry preserved
- Columnar: rotational and translational symmetries spontaneously broken
- Plaquette: translational symmetries spontaneously broken

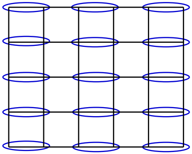




Sachdev, 2008

Quantum paramagnets: valence bond solid (VBS)

- Gapped excitations: triplons (spin-1) **confined phase**



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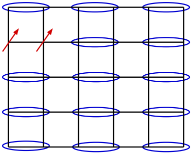

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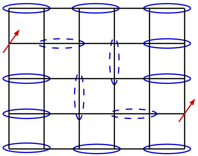

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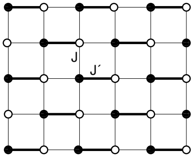

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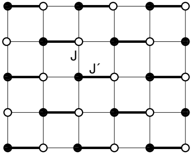
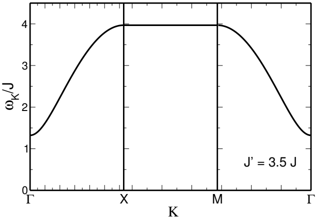
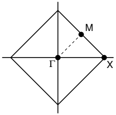
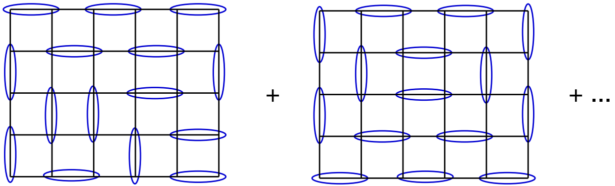






Quantum paramagnets: spin liquids

- Spin liquid $\sim$ "liquid of singlets" (no long range order)
- Resonating Valence Bond (RVB)
- Spin-rotation and translational symmetries preserved
- Gapped/gapless excitations: spinons (spin-1/2, fractionalized)
- **Deconfined phase**



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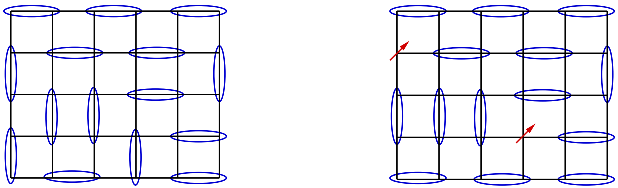


Sachdev, 2008; Balents, 2010

Spinon

Excitations:

- particle-like
  spin-1/2 and charge: $\pm e$
  Ex.: metal (Fermi liquid).

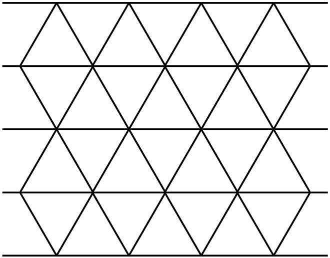
- magnon-like
  spin-1 and charge: 0
  Ex.: spin-waves (FM and AFM), triplons (VBS),

- spinons
  spin-1/2 and charge: 0 fractionalized excitations

Frustrated magnetic systems

AFM Heisenberg model on the triangular lattice

$$H = J \sum_{\langle i\, j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j \qquad J > 0$$



Chernyshev and Zhitomirsky, 2009

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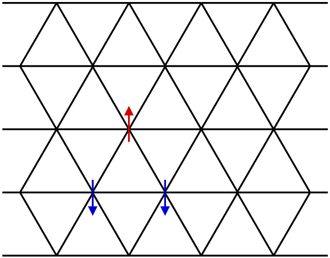

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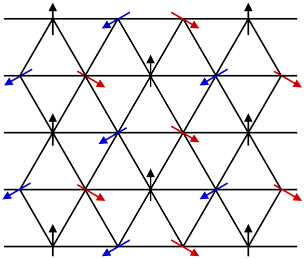

non-collinear 120° Néel order

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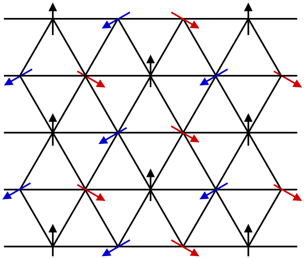

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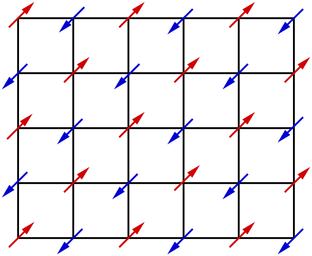

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AFM Heisenberg model on the triangular lattice

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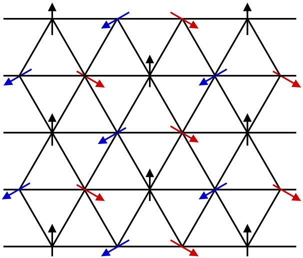
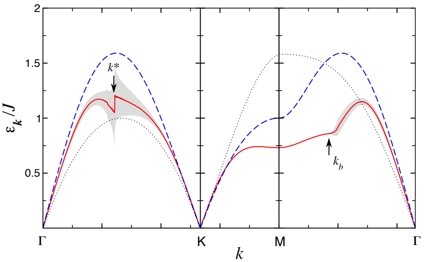


non-collinear 120° Néel order



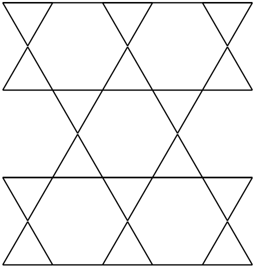
spin wave spectrum

Chernyshev and Zhitomirsky, 2009

Frustrated magnetic systems

AFM Heisenberg model on the kagome lattice

$$H = J \sum_{\langle i j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j \qquad J > 0$$



Ground state: spin liquid, VBS with 36-site unit cell

Yang et al., 2008; Han et al., 2012

Frustrated magnetic systems

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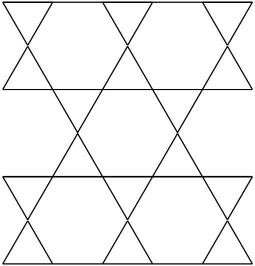

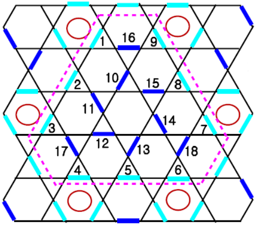

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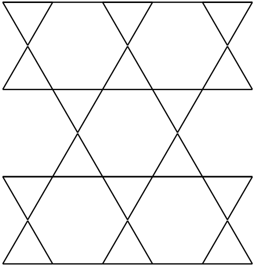
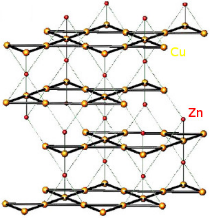




Ground state: spin liquid, VBS with 36-site unit cell

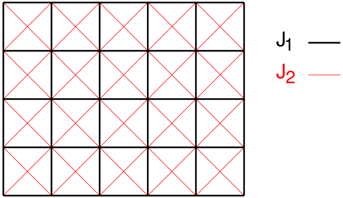
$ZnCu_3(OH)_6Cl_2$

Yang et al., 2008; Han et al., 2012

Frustrated magnetic systems

J_1–J_2 AFM Heisenberg model

$$H = J_1 \sum_{\langle i\,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J_2 \sum_{\langle\langle i\,j \rangle\rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$



$J_1, J_2 > 0$

$S = 1/2 \qquad T = 0$

Misguisch and Lhuillier, 2003

Frustrated magnetic systems

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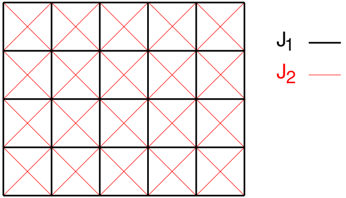

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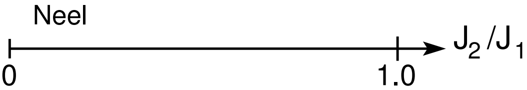
Classical phase diagram



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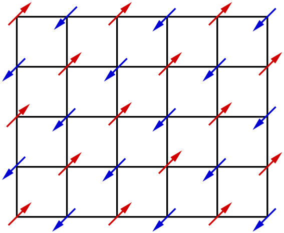

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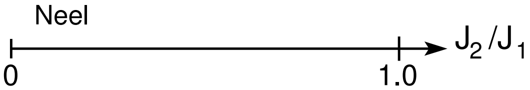

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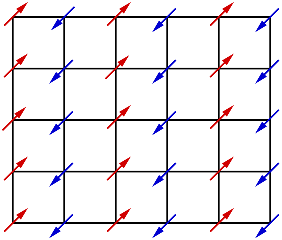

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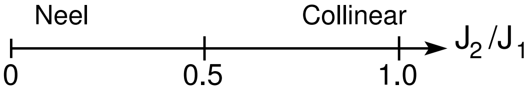

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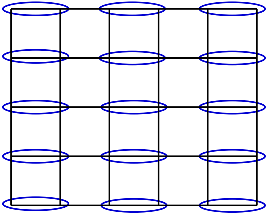

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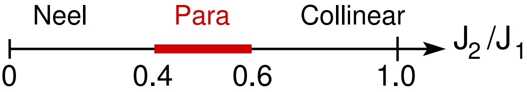
Quantum phase diagram



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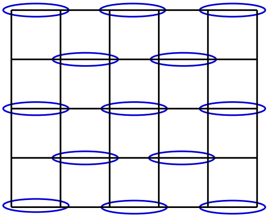

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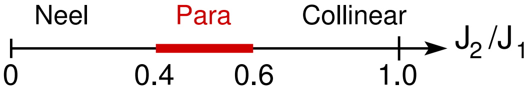
Quantum phase diagram



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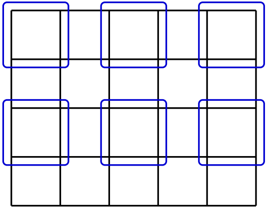

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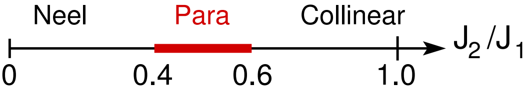

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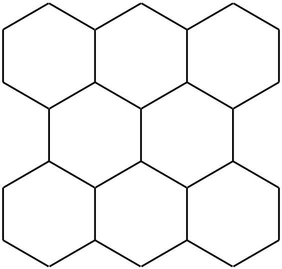

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Meng et al., 2010; Ganesh et al., 2013; Zhu et al., 2013

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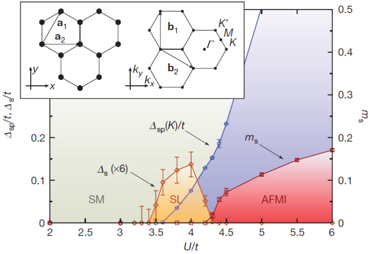

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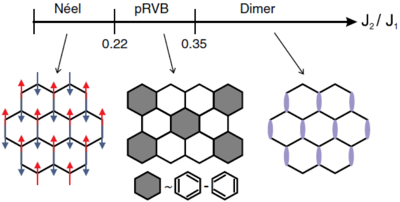

$J_1, J_2 > 0$

$S = 1/2$ $T = 0$

Phase diagram:

density matrix renormalization group (DMRG)

Meng et al., 2010; Ganesh et al., 2013; Zhu et al., 2013

Spin–wave theory

Holstein–Primakoff representation

Auerbach, 1994

$$S^- = a^\dagger \left(2S - a^\dagger a\right)^{1/2}$$

$$S^+ = \left(2S - a^\dagger a\right)^{1/2} a$$

$$S^z = S - a^\dagger a$$

$a_i^\dagger$: boson

$$S_i^z |\uparrow\uparrow\uparrow \ \ldots \uparrow\rangle = S|\uparrow\uparrow\uparrow \ \ldots \uparrow\rangle$$

Spin–wave theory

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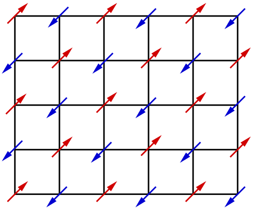

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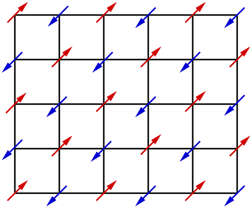

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Spin–wave theory

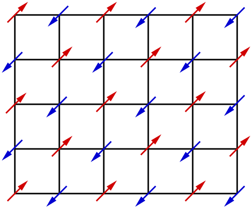
Holstein–Primakoff representation

Auerbach, 1994

$$S^- \approx \sqrt{2S} a^\dagger \left(1 - \frac{a^\dagger a}{4S}\right)$$

$$S^+ \approx \sqrt{2S} \left(1 - \frac{a^\dagger a}{4S}\right) a$$

$$S^z = S - a^\dagger a$$



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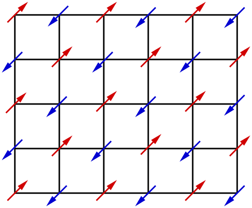

$a_i^\dagger$: boson

$$S_i^z | \uparrow \downarrow \uparrow \ldots \uparrow \rangle = (S-1) | \uparrow \downarrow \uparrow \ldots \uparrow \rangle$$

$$\mathcal{H} = \sum_{ij} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j \qquad \rightarrow \qquad \mathcal{H}_{\text{boson}} = \mathcal{H}_2 + \mathcal{H}_4$$

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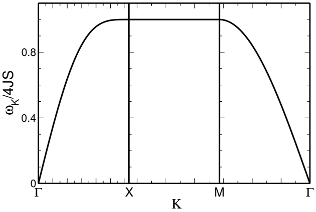

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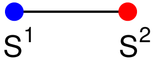
Bond operator representation

Hilbert space: 2 spins-1/2

$$|s\rangle \equiv s^\dagger|0\rangle$$

$$|\alpha\rangle \equiv t_\alpha^\dagger|0\rangle \qquad \alpha = x, y, z$$



$s^\dagger$ and $t_\alpha^\dagger$: bosons

$$S_\alpha^{1/2} = \pm\frac{1}{2}\left(s^\dagger t_\alpha + t_\alpha^\dagger s - i\epsilon_{\alpha\beta\gamma}t_\beta^\dagger t_\gamma\right)$$

constraint: $s^\dagger s + \sum_\alpha t_\alpha^\dagger t_\alpha = 1$

generalization lattice → description VBS

Sachdev and Bhatt, 1990

Slave bosons/fermions

Spin operator

$$\mathbf{S} = \frac{1}{2} b^{\dagger}_{\alpha} \hat{\sigma}_{\alpha,\beta} b_{\beta} \qquad\qquad b^{\dagger}_{\alpha} : \text{spinon}$$

- spinon: fermion
- spinon: boson (Schwinger bosons)

Wen, 2004; Wang et al., 2006; Ran et al., 2007

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Mean–field approximation: Z_2, U(1) spin liquids

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Mean–field approximation: Z_2, U(1) spin liquids

Several labels: $b_{j\sigma} \rightarrow e^{i\phi_j} b_{j\sigma}$

Fluctuations: phase mean–field ansatz $\sim$ gauge fields

Wen, 2004; Wang et al., 2006; Ran et al., 2007

Valence bond solid in frustrated antiferromagnets

Idea: Frustration effects on valence bond solid

- triplon (spin) gap at incommensurate momentum
- finite lifetime high energy triplon excitations

Collaborator: Matthias Vojta (TU Dresden)

Details: Phys. Rev. B **85**, 104416 (2012)

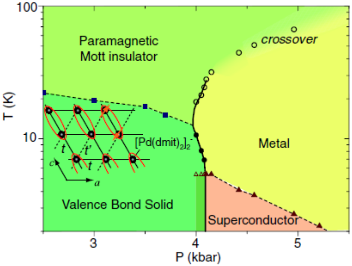
Financial support: Fapesp and DFG - SFB 608 (Köln)

Experimental data: $EtMe_3P[Pd(dmit)_2]_2$

Organic material

Triangular lattice, $J = 250\,K$ (mag. susceptibility, high-T)

P–T phase diagram (resistivity measurements)



VBS borders SC, contrast high-T_c: Néel borders SC

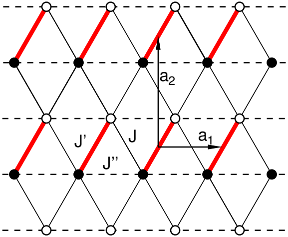
Shimizu et al., 2007

Anisotropic and dimerized AFM Heisenberg model

$$\mathcal{H} = \sum_{\langle i\,j \rangle} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

spin–1/2

$J_{ij} = J = 1,\ J',\ J''$



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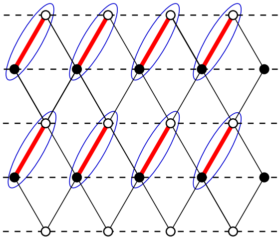

spin–1/2

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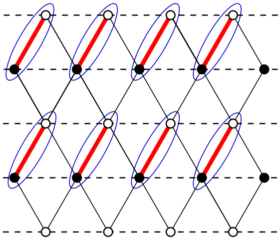
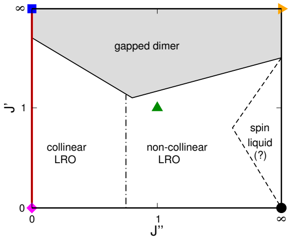


spin–$1/2$

$J_{ij} = J = 1,\ J',\ J''$

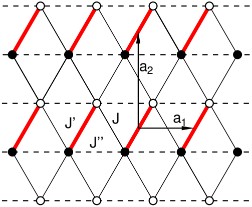
Phase diagram $\quad T = 0$:

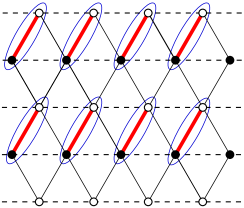


Effective triplon model

Bond operator formalism: $S_\alpha^{1/2} \sim \pm \left(s^\dagger t_\alpha + t_\alpha^\dagger s - i\epsilon_{\alpha\beta\gamma} t_\beta^\dagger t_\gamma \right)$

$$\mathcal{H} = \sum_{\langle i\,j \rangle} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j \qquad \rightarrow \qquad \mathcal{H} = \mathcal{H}_0 + \mathcal{H}_2 + \mathcal{H}_3 + \mathcal{H}_4$$





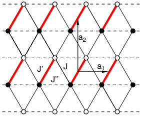
Reference state: columnar VBS $\qquad s_i^\dagger = s_i = \langle s_i^\dagger \rangle = \langle s_i \rangle \rightarrow \sqrt{N_0}$

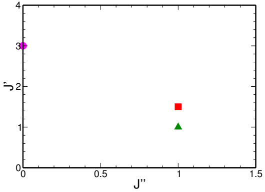
Triplon dispersion relation – harmonic approximation

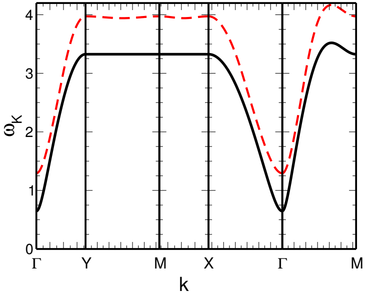
$J' = 3.0 \quad J'' = 0.0$

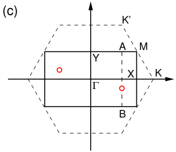
nonfrustrated

commensurate (C)









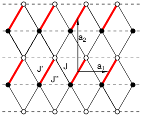
$E_{\mathbf{p}} = min(\epsilon_{\mathbf{p}-\mathbf{k}} + \epsilon_{\mathbf{k}})$

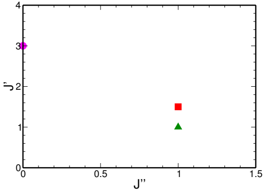
Triplon dispersion relation – harmonic approximation

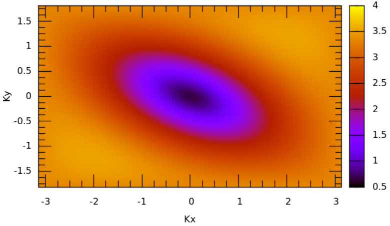
$J' = 3.0 \quad J'' = 0.0$

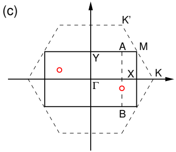
nonfrustrated

commensurate (C)









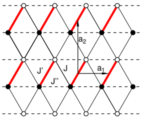
$$E_{\mathbf{p}} = min(\epsilon_{\mathbf{p}-\mathbf{k}} + \epsilon_{\mathbf{k}})$$

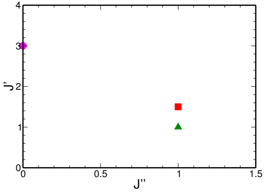
Triplon dispersion relation – harmonic approximation

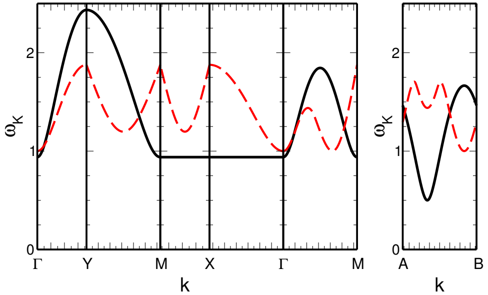
$J' = 1.5 \quad J'' = 1.0$

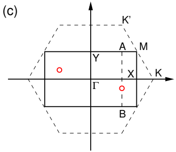
frustrated

incommensurate (INC)









$$E_{\mathbf{p}} = min(\epsilon_{\mathbf{p}-\mathbf{k}} + \epsilon_{\mathbf{k}})$$

Triplon dispersion relation – harmonic approximation

$J' = 1.5 \quad J'' = 1.0$

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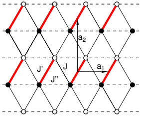
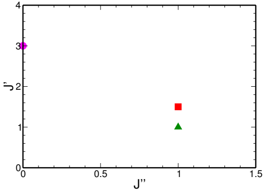
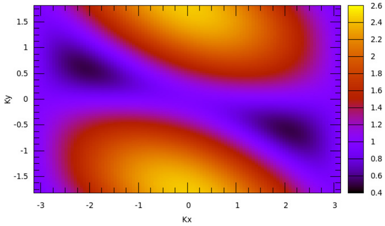
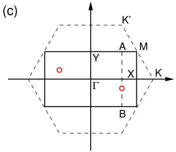




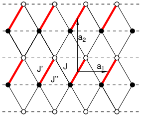


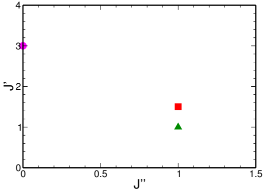


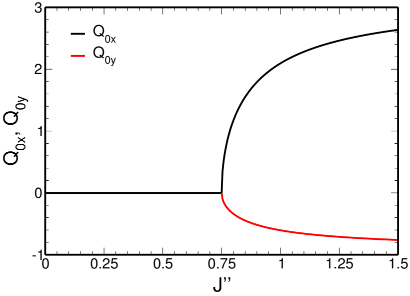
$$E_{\mathbf{p}} = min(\epsilon_{\mathbf{p}-\mathbf{k}} + \epsilon_{\mathbf{k}})$$

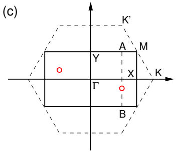
Triplon dispersion relation – harmonic approximation

$\mathbf{Q}_0$: momentum $\sim$ gap Δ









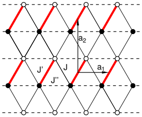
Triplon dispersion relation – harmonic approximation

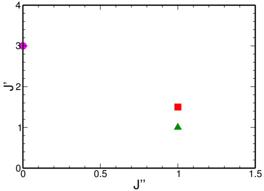
Gap $\Delta \times J'$

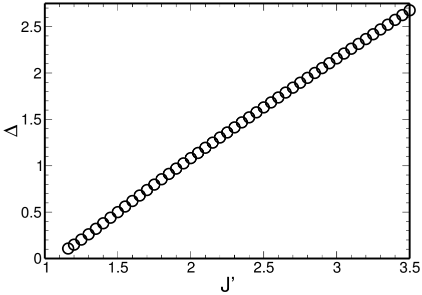
line $J'' = 1.0$

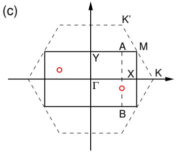
$\Delta = 0$:

continuous phase transition









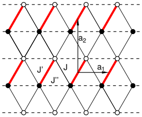
Triplon dispersion relation – harmonic approximation

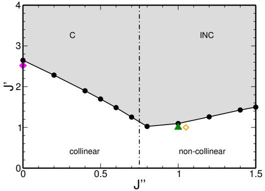
Gap $\Delta \times J'$

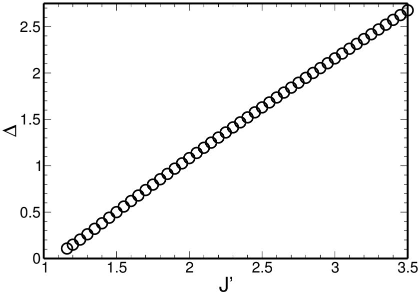
line $J'' = 1.0$

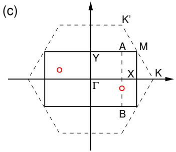
$\Delta = 0$:

continuous phase transition

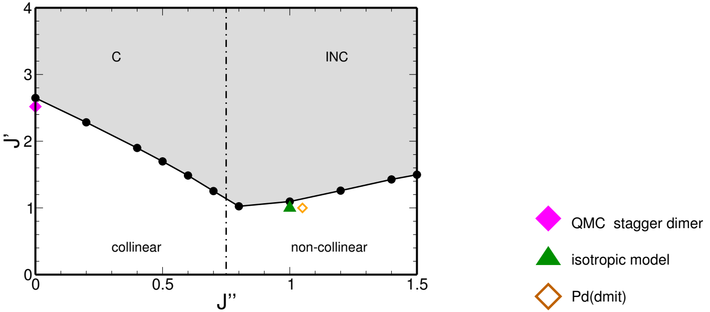


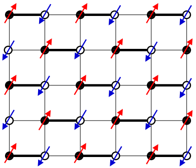


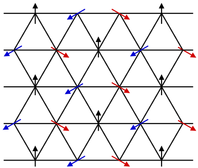




Harmonic approximation – phase diagram







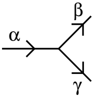
Beyond harmonic approximation

Quartic term $\mathcal{H}_4$:

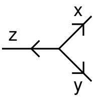
self-consistent Hartree–Fock approximation

Cubic term $\mathcal{H}_3$:

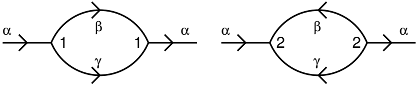
lowest–order perturbation theory



(1)

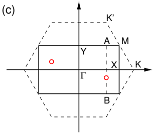


(2)



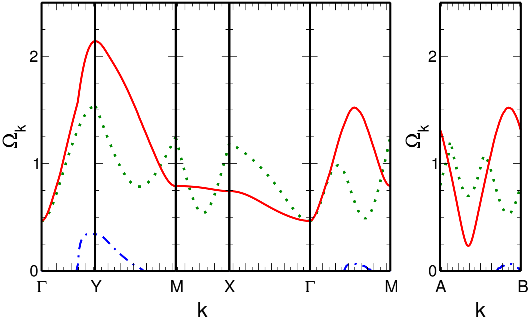
Beyond harmonic approximation

Triplon dispersion relation



$$\omega = \Omega_{\mathbf{k}} - i\tilde{\Gamma}_{\mathbf{k}}$$

$\tilde{\Gamma}_{\mathbf{k}}^{-1}$: lifetime

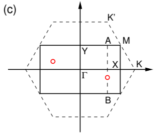


$J' = 1.5 \quad J'' = 1.0$

$E_{\mathbf{p}} = min(\epsilon_{\mathbf{p-k}} + \epsilon_{\mathbf{k}})$

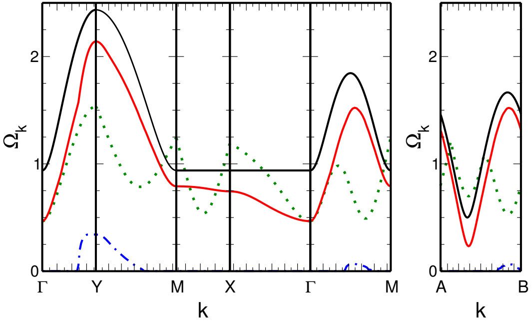
Beyond harmonic approximation

Triplon dispersion relation



$$\omega = \Omega_{\mathbf{k}} - i\tilde{\Gamma}_{\mathbf{k}}$$

$\tilde{\Gamma}_{\mathbf{k}}^{-1}$: lifetime



$J' = 1.5 \quad J'' = 1.0$

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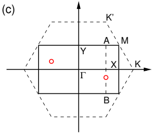
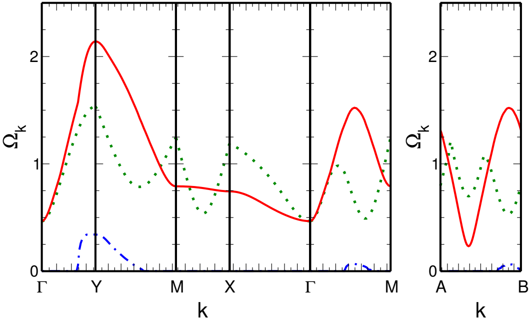
Beyond harmonic approximation

Triplon dispersion relation

$$\omega = \Omega_{\mathbf{k}} - i\tilde{\Gamma}_{\mathbf{k}}$$

$\tilde{\Gamma}_{\mathbf{k}}^{-1}$: lifetime

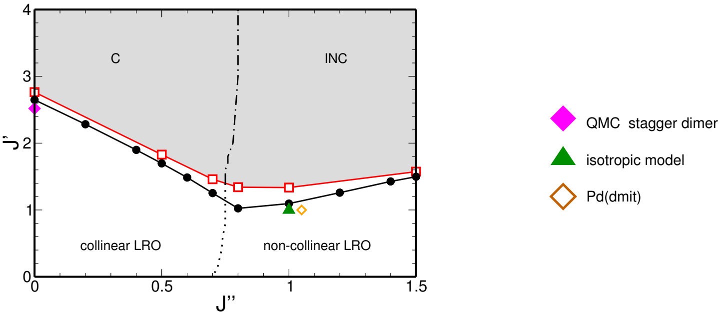




$J' = 1.5 \quad J'' = 1.0$

$E_{\mathbf{p}} = min(\epsilon_{\mathbf{p-k}} + \epsilon_{\mathbf{k}})$

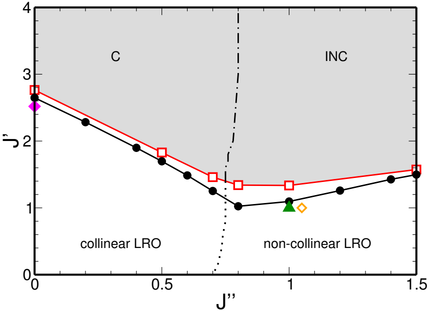
Beyond harmonic approximation – phase diagram

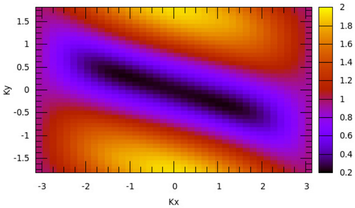


neutron scattering measurements $P[Pd(dmit)_2]_2$:

- triplon (spin) gap at incommensurate momentum
- finite lifetime high energy triplon excitations

Beyond harmonic approximation – phase diagram



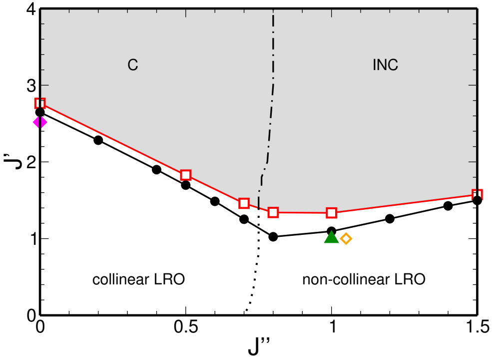


- Commensurate–incommensurate transition
- multicritical (Lifshitz) point: 3 phases

$$G^{-1}(\mathbf{k}, \omega) \sim \omega^2 - \Delta^2 + c k_\perp^2 + d k_\parallel^4$$

Summary

- Valence bond phase triangular lattice AFM Heisenberg model: bond operator formalism



- Details: Phys. Rev. B **85**, 104416 (2012)

Effective triplon model

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_2 + \mathcal{H}_3 + \mathcal{H}_4$$

where

$$\mathcal{H}_0 = -3J'N/8 - \mu N(N_0 - 1)/2$$

$$\mathcal{H}_2 = \sum_{\mathbf{k}} \left[A_{\mathbf{k}} t^{\dagger}_{\mathbf{k}\alpha} t_{\mathbf{k}\alpha} + \frac{1}{2} B_{\mathbf{k}} \left(t^{\dagger}_{\mathbf{k}\alpha} t^{\dagger}_{-\mathbf{k}\alpha} + \mathrm{H.c.} \right) \right]$$

Effective triplon model

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_2 + \mathcal{H}_3 + \mathcal{H}_4$$

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$$\mathcal{H}_3 = \frac{1}{2\sqrt{N'}} \epsilon_{\alpha\beta\lambda} \sum_{\mathbf{p},\mathbf{k}} \xi_{\mathbf{k}-\mathbf{p}} \; t^{\dagger}_{\mathbf{k}-\mathbf{p}\alpha} t^{\dagger}_{\mathbf{p}\beta} t_{\mathbf{k}\lambda} + \mathrm{H.c.}$$

Effective triplon model

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_2 + \mathcal{H}_3 + \mathcal{H}_4$$

where

$$\mathcal{H}_0 = -3J'N/8 - \mu N(N_0 - 1)/2$$

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$$\mathcal{H}_3 = \frac{1}{2\sqrt{N'}} \epsilon_{\alpha\beta\lambda} \sum_{\mathbf{p},\mathbf{k}} \xi_{\mathbf{k}-\mathbf{p}} \; t^{\dagger}_{\mathbf{k}-\mathbf{p}\alpha} t^{\dagger}_{\mathbf{p}\beta} t_{\mathbf{k}\lambda} + \mathrm{H.c.}$$

$$\mathcal{H}_4 = \frac{1}{2N'} \epsilon_{\alpha\beta\lambda} \epsilon_{\alpha\mu\nu} \sum_{\mathbf{q},\mathbf{p},\mathbf{k}} \gamma_{\mathbf{k}} \; t^{\dagger}_{\mathbf{p}+\mathbf{k}\beta} t^{\dagger}_{\mathbf{q}-\mathbf{k}\mu} t_{\mathbf{q}\nu} t_{\mathbf{p}\lambda}$$

Effective triplon model

Harmonic approximation:

$$\mathcal{H} \approx \mathcal{H}_0 + \sum_{\mathbf{k}} \left[A_{\mathbf{k}} t^\dagger_{\mathbf{k}\alpha} t_{\mathbf{k}\alpha} + \frac{1}{2} B_{\mathbf{k}} \left(t^\dagger_{\mathbf{k}\alpha} t^\dagger_{-\mathbf{k}\alpha} + \mathrm{H.c.} \right) \right]$$

Effective triplon model

Harmonic approximation:

$$\mathcal{H} \approx \mathcal{H}_0 + \sum_{\mathbf{k}} \left[A_{\mathbf{k}} t^{\dagger}_{\mathbf{k}\alpha} t_{\mathbf{k}\alpha} + \frac{1}{2} B_{\mathbf{k}} \left(t^{\dagger}_{\mathbf{k}\alpha} t^{\dagger}_{-\mathbf{k}\alpha} + \mathrm{H.c.} \right) \right]$$

Bogoliubov transformation: $t^{\dagger}_{\mathbf{k}\alpha} = u_{\mathbf{k}} b^{\dagger}_{\mathbf{k}\alpha} - v_{\mathbf{k}} b_{-\mathbf{k}\alpha}$

Effective triplon model

Harmonic approximation:

$$\mathcal{H} \approx \mathcal{H}_0 + \sum_{\mathbf{k}} \left[A_{\mathbf{k}} t^{\dagger}_{\mathbf{k}\alpha} t_{\mathbf{k}\alpha} + \frac{1}{2} B_{\mathbf{k}} \left(t^{\dagger}_{\mathbf{k}\alpha} t^{\dagger}_{-\mathbf{k}\alpha} + \mathrm{H.c.} \right) \right]$$

Bogoliubov transformation: $t^{\dagger}_{\mathbf{k}\alpha} = u_{\mathbf{k}} b^{\dagger}_{\mathbf{k}\alpha} - v_{\mathbf{k}} b_{-\mathbf{k}\alpha}$

$$\mathcal{H} = \bar{E}_0 + \sum_{\mathbf{k}\,\alpha} \omega_{\mathbf{k}} b^{\dagger}_{\mathbf{k}\alpha} b_{\mathbf{k}\alpha} \qquad \omega_{\mathbf{k}}(N_0, \mu) = \sqrt{A^2_{\mathbf{k}} - B^2_{\mathbf{k}}}$$

Effective triplon model

Harmonic approximation:

$$\mathcal{H} \approx \mathcal{H}_0 + \sum_{\mathbf{k}} \left[A_{\mathbf{k}} t^\dagger_{\mathbf{k}\alpha} t_{\mathbf{k}\alpha} + \frac{1}{2} B_{\mathbf{k}} \left(t^\dagger_{\mathbf{k}\alpha} t^\dagger_{-\mathbf{k}\alpha} + \mathrm{H.c.} \right) \right]$$

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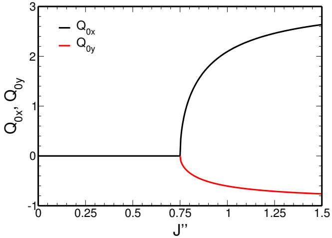
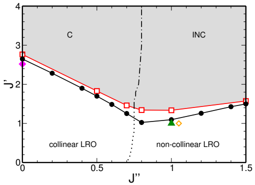
Saddle point conditions: $\partial \bar{E}_0 / \partial N_0 = 0$ and $\partial \bar{E}_0 / \partial \mu = 0$
$\Rightarrow$ self-consistent equations

Commensurate–incommensurate transition

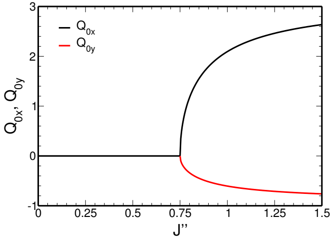
bond operator formalism

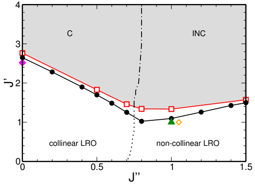




Commensurate–incommensurate transition

bond operator formalism

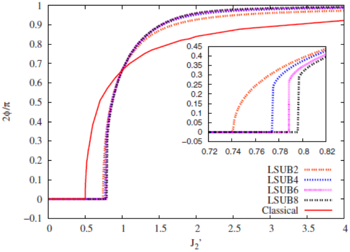




Coupled cluster method

(numerical)

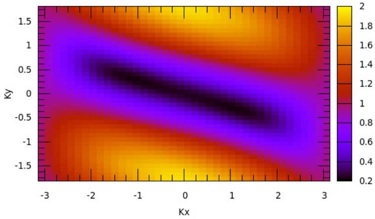
line $J' = 1.0$



Bishop et al., 2009

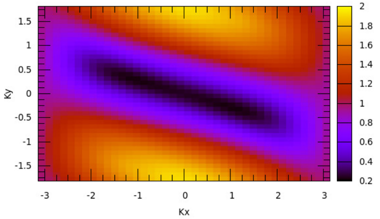
Commensurate–incommensurate transition

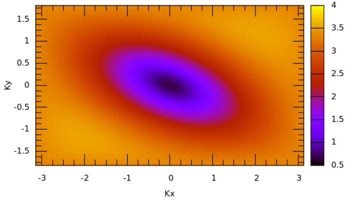
Triplon dispersion relation $J' = 1.5 \quad J'' = 0.8$



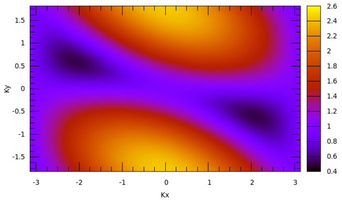
Commensurate–incommensurate transition

Triplon dispersion relation $J' = 1.5 \quad J'' = 0.8$





$J' = 3.0 \quad J'' = 0.0$
commensurate (C)



$J' = 1.5 \quad J'' = 1.0$
incommensurate (INC)

Effective triplon model – coefficients

$$
\begin{aligned}
A_{\mathbf{k}} &= \frac{J'}{4} - \mu + B_{\mathbf{k}} \\
B_{\mathbf{k}} &= \frac{1}{2} N_0 \left[(2J'' - 1)\cos(k_x) - \cos(\sqrt{3}k_y) - \cos(k_x + \sqrt{3}k_y) \right] \\
\xi_{\mathbf{k}} &= -\sqrt{N_0} \left[\sin(k_x) + \sin(\sqrt{3}k_y) + \sin(k_x + \sqrt{3}k_y) \right] \\
\gamma_{\mathbf{k}} &= -\frac{1}{2} \left[(2J'' + 1)\cos(k_x) + \cos(\sqrt{3}k_y) + \cos(k_x + \sqrt{3}k_y) \right]
\end{aligned}
$$