Entanglement and decoherence: From Einstein and Schrödinger to quantum information and quantum metrology

Luiz Davidovich

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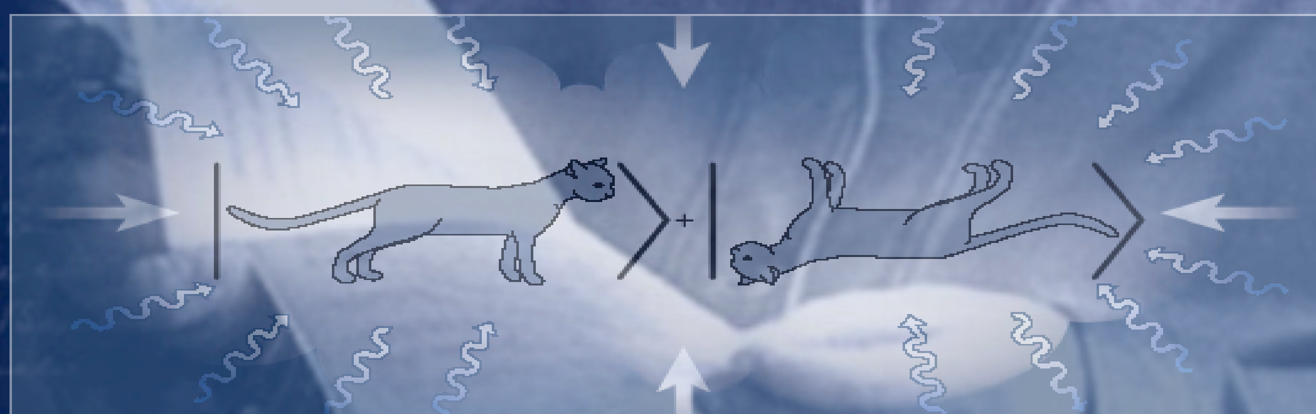
Universidade Federal do Rio de Janeiro

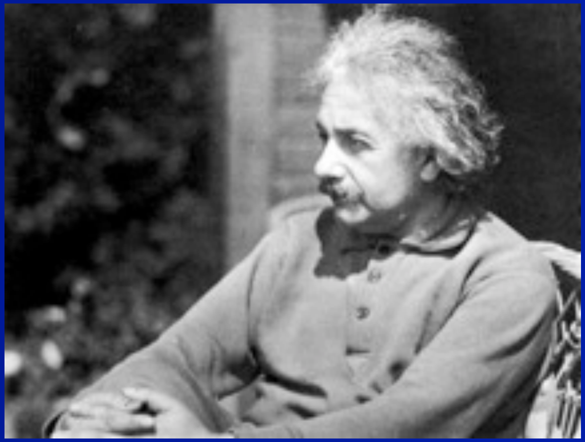
Rio de Janeiro, Brazil



Outline of the talk

- Decoherence and the classical limit of the quantum world
- Multiparticle systems and decoherence
- Quantum metrology and decoherence

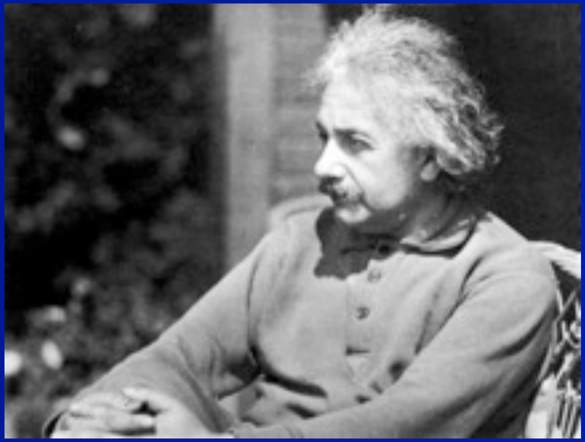
Quantum physics and localization



Letter from Einstein to Born, January 1, 1954

Quantum physics and localization

- "Let Ψ_1 and Ψ_2 be two solutions of the same Schrödinger equation. Then $\Psi = \Psi_1 + \Psi_2$ also represents a solution of the Schrödinger equation, with equal claim to describe a possible real state. When the system is a macrosystem, and when Ψ_1 and Ψ_2 are `narrow' with respect to the macro-coordinates, then in by far the greater number of cases, this is no longer true for Ψ. Narrowness in regard to macro-coordinates is a requirement which is not only independent of the principles of quantum mechanics, but, moreover, incompatible with them."



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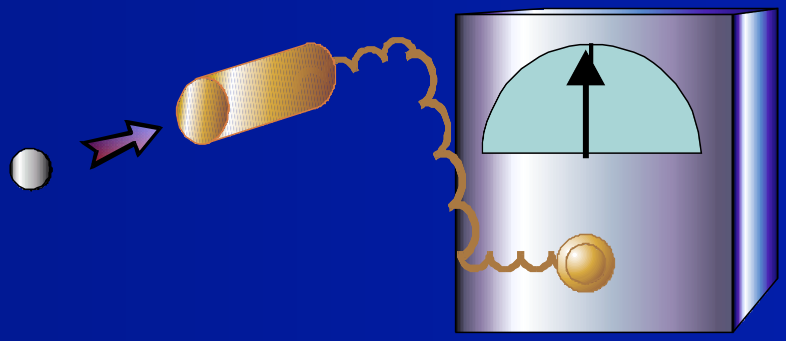
Quantum measurement

Quantum measurement



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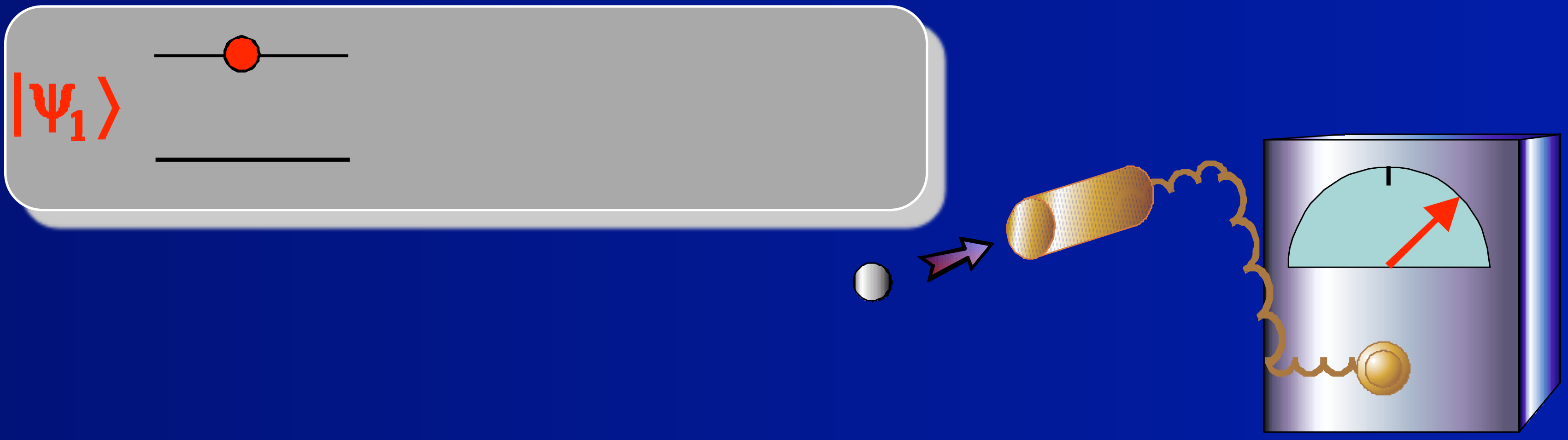

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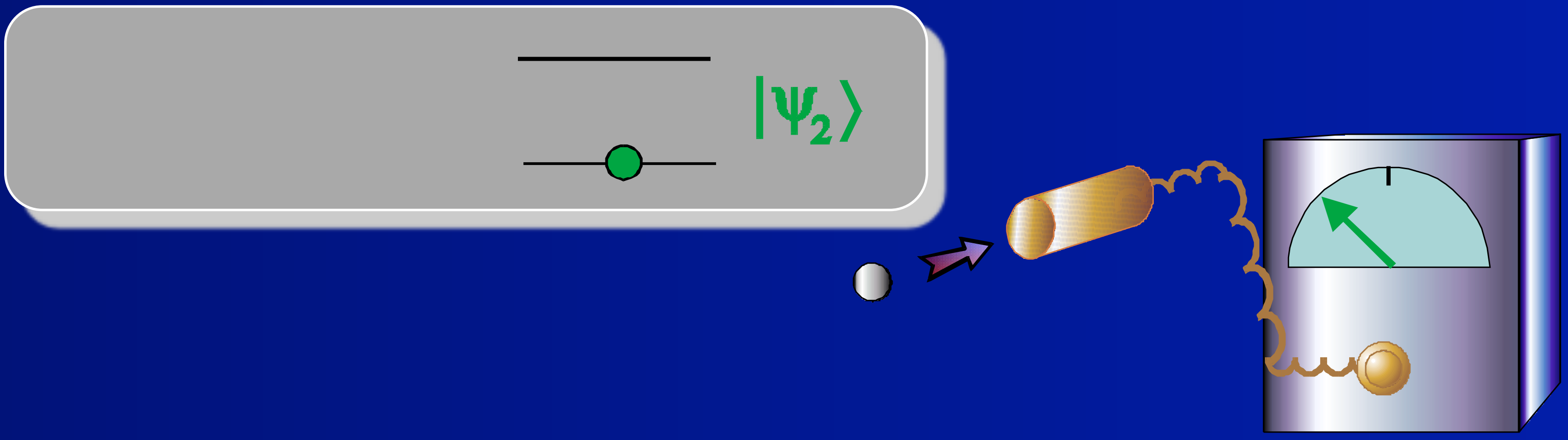
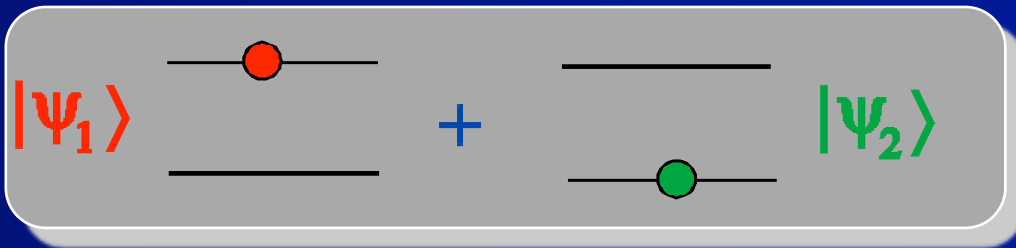
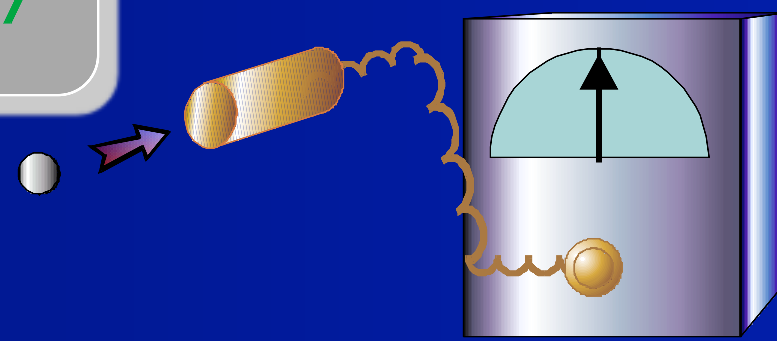


Quantum measurement

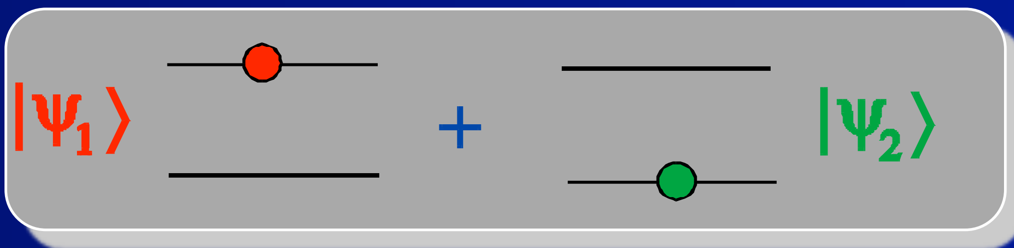


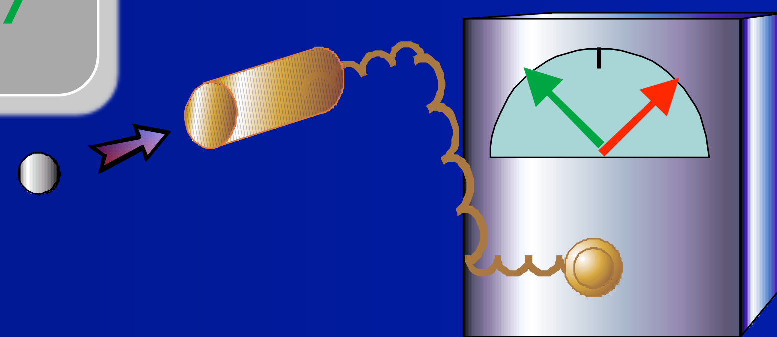


Linear evolution:

$$|\text{BEFORE}\rangle = \left(|\Psi_1\rangle + |\Psi_2\rangle\right)|\uparrow\rangle/\sqrt{2}$$

Quantum measurement





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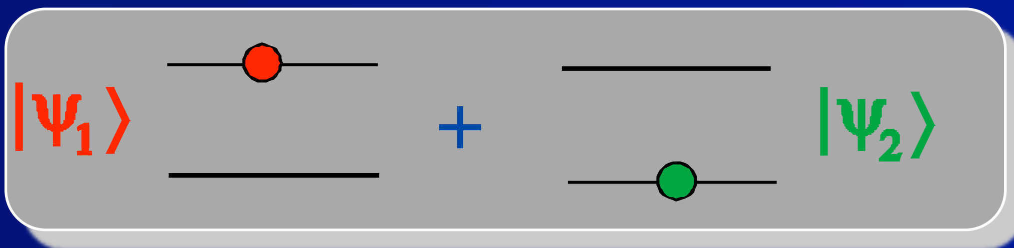

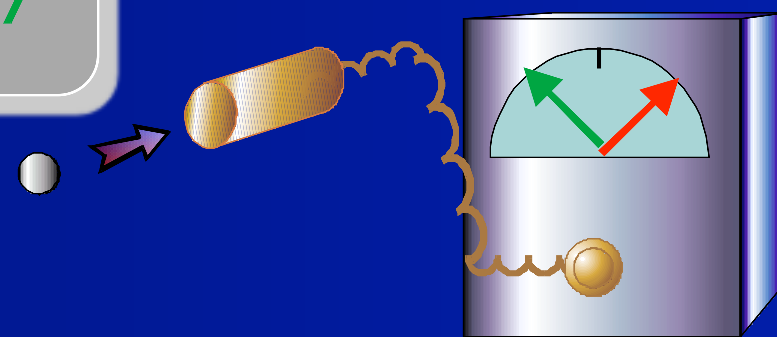

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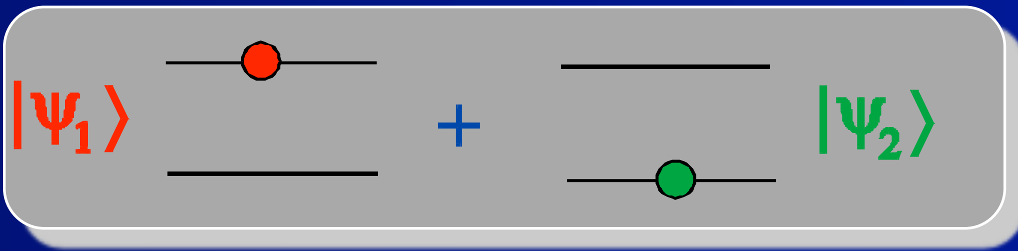

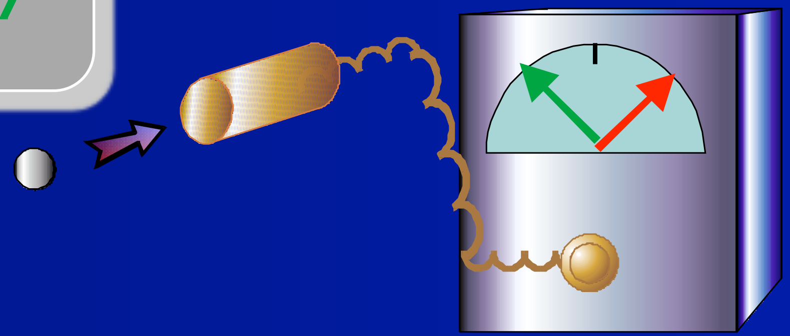

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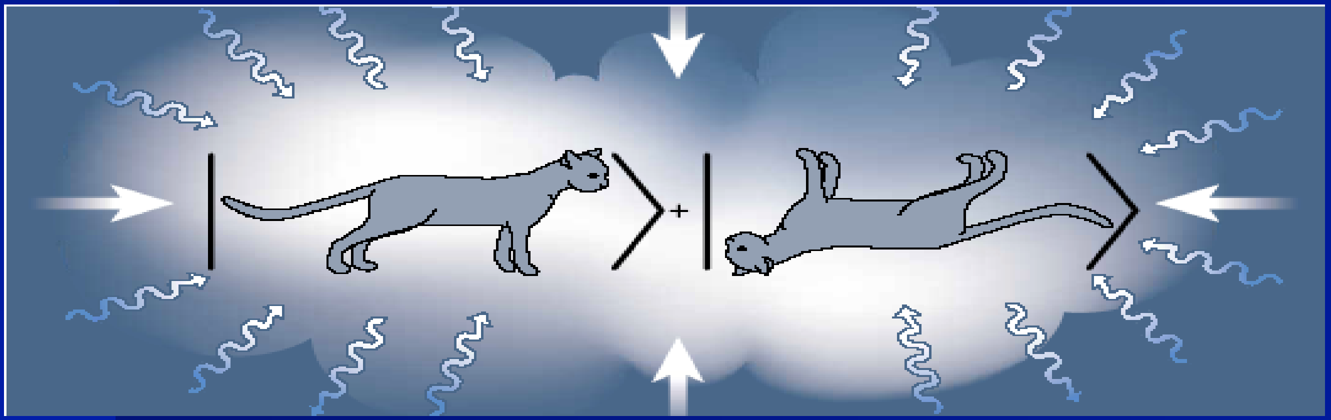

Why interference cannot be seen?

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- Decoherence: entanglement with the environment - same process by which quantum computers become classical computers!

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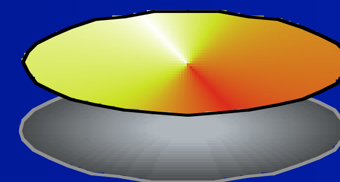
- Decoherence: entanglement with the environment - same process by which quantum computers become classical computers!
- Dynamics of decoherence: related to elusive boundary between quantum and classical world

Decoherence dynamics

$$\frac{1}{\mathcal{N}}(|\alpha\rangle + |-\alpha\rangle)$$

$$\rightarrow \frac{1}{2}(|\alpha\rangle\langle\alpha| + |-\alpha\rangle\langle-\alpha|)$$



VOLUME 77, NUMBER 24 PHYSICAL REVIEW LETTERS 9 DECEMBER 1996

Observing the Progressive Decoherence of the "Meter" in a Quantum Measurement

M. Brune, E. Hagley, J. Dreyer, X. Maître, A. Maali, C. Wunderlich, J. M. Raimond, and S. Haroche
Laboratoire Kastler Brossel, Département de Physique de l'Ecole Normale Supérieure, 24 Rue Lhomond, F-75231 Paris Cedex 05, France*

(Received 10 September 1996)

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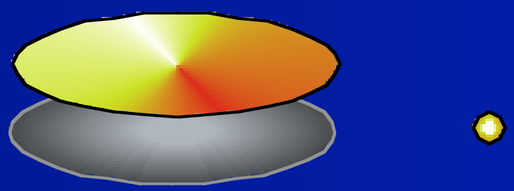

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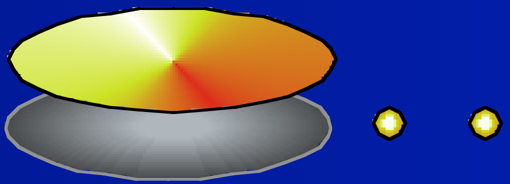

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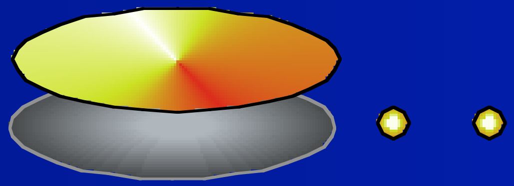

Exponential decay: $t_{\text{dec}} \approx t_{\text{cav}} / \langle n \rangle$

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Dynamics of entanglement

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- How is local dynamics related to nonlocal loss of entanglement?
- How does loss of entanglement scale with number of particles?

1935

The New York Times

EINSTEIN ATTACKS QUANTUM THEORY

Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

SEE FULLER ONE POSSIBLE

Believe a Whole Description of 'the Physical Reality' Can Be Provided Eventually.

PRINCETON, N. J., May 3.—Professor Albert Einstein will attack science's important theory of quantum mechanics, a theory of which he was a sort of grandfather. He concludes that while it is "correct" it is not "complete."

With two colleagues at the Institute for Advanced Study here, the noted scientist is about to report to the American Physical Society what is wrong with the theory of quantum mechanics, it has been learned exclusively by Science Service.

The quantum theory, with which science predicts with some success inter-atomic happenings, does not meet the requirements for a satisfactory physical theory, Professor Einstein will report in a joint paper with Dr. Boris Podolsky and Dr. N. Rosen.

Schrödinger on Entanglement

Naturwissenschaften **23**, 807 (1935)

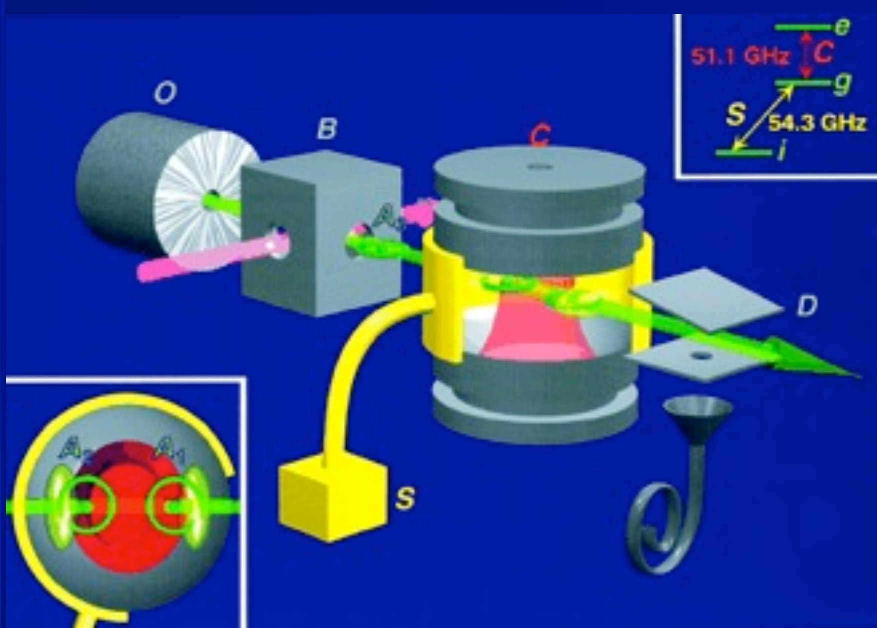
"This is the reason that knowledge of the individual systems can decline to the scantiest, even zero, while that of the combined system remains continually maximal. Best possible knowledge of a whole does not include best possible knowledge of its parts – and that is what keeps coming back to haunt us."

Emaranhamento átomo-fóton

16 JUNE 2000 VOL 288 SCIENCE

Step-by-Step Engineered Multiparticle Entanglement

Arno Rauschenbeutel, Gilles Nogues, Stefano Osnaghi, Patrice Bertet, Michel Brune, Jean-Michel Raimond,* Serge Haroche



Blinov et al, C. *Nature* **428**, 153 (2004)

Emaranhamento de muitas partículas

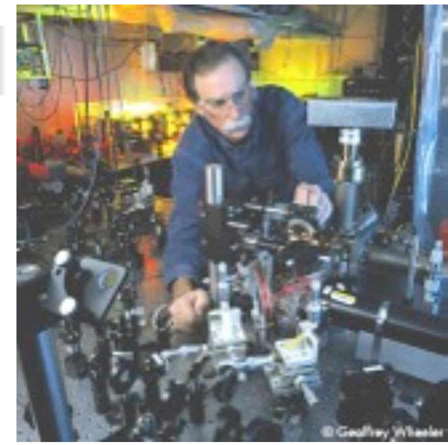
Emaranhamento de muitas partículas

Vol 438|1 December 2005|doi:10.1038/nature04251 nature

LETTERS

Creation of a six-atom 'Schrödinger cat' state

D. Leibfried[1], E. Knill[1], S. Seidelin[1], J. Britton[1], R. B. Blakestad[1], J. Chiaverini[1]†, D. B. Hume[1], W. M. Itano[1], J. D. Jost[1], C. Langer[1], R. Ozeri[1], R. Reichle[1] & D. J. Wineland[1]



Vol 438|1 December 2005|doi:10.1038/nature04279 nature

LETTERS

Scalable multiparticle entanglement of trapped ions

H. Häffner[1,3], W. Hänsel[1], C. F. Roos[1,3], J. Benhelm[1,3], D. Chek-al-kar[1], M. Chwalla[1], T. Körber[1,3], U. D. Rapol[1,3], M. Riebe[1], P. O. Schmidt[1], C. Becher[1]†, O. Gühne[3], W. Dür[2,3] & R. Blatt[1,3]

Emaranhamento multifotônico

letters to nature

NATURE | VOL 430 | 1 JULY 2004 | www.nature.com/nature

Experimental demonstration of five-photon entanglement and open-destination teleportation

Zhi Zhao[1], Yu-Ao Chen[1], An-Ning Zhang[1], Tao Yang[1], Hans J. Briegel[2] & Jian-Wei Pan[1,3]

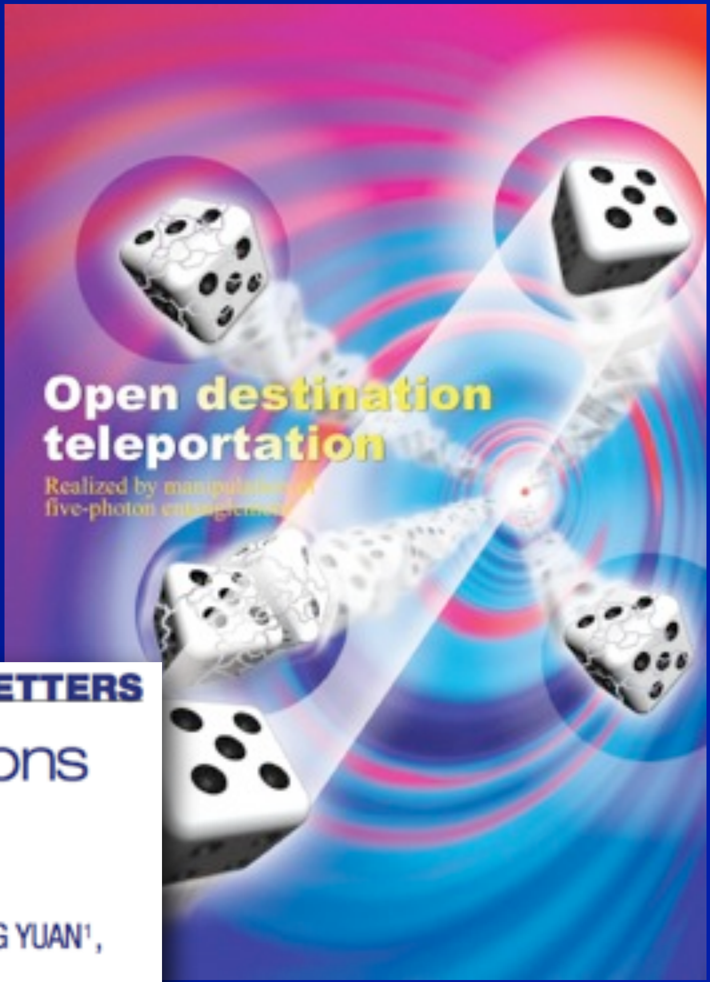
[1] *Department of Modern Physics and Hefei National Laboratory for Physical Sciences at Microscale, University of Science and Technology of China, Hefei,*

nature physics | VOL 3 | FEBRUARY 2007 | www.nature.com/naturephysics

LETTERS

Experimental entanglement of six photons in graph states

CHAO-YANG LU[1]*, XIAO-QI ZHOU[1], OTFRIED GÜHNE[2], WEI-BO GAO[1], JIN ZHANG[1], ZHEN-SHENG YUAN[1], ALEXANDER GOEBEL[3], TAO YANG[1] AND JIAN-WEI PAN[1,3]*



EMARANHAMENTO COMO UM RECURSO

- Emaranhamento é útil para comunicação, computação quântica e metrologia quântica!

Entangled and separable states

- Separable states:

– Pure states:

$$|\Psi_{12\ldots n}\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes \ldots |\psi_n\rangle$$

– Mixed states (R. F. Werner, PRA, 1989):

$$\rho_{12\ldots n} = \sum_{\mu} p_\mu \rho_1^\mu \otimes \rho_2^\mu \otimes \ldots \rho_n^\mu$$

$$0 \leq p_\mu \leq 1$$

- Entangled state: non-separable

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$$|\Psi_\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$$

$$|\Phi_\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$$

Bell states - Maximally entangled states: complete ignorance on each qubit

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Bell states - Maximally entangled states: complete ignorance on each qubit

$$\rho_{A,B} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Measures of entanglement for pure states

Von Neumann entropy

$$S_N(\rho_r) = -\mathrm{Tr}[\rho_r \log_2 \rho_r]$$

$\rho_r \longrightarrow$ reduced density matrix of A or B

Linear entropy

$$S_L(\rho_r) = 2\left(1 - \mathrm{Tr}\rho_r^2\right)$$

Separable state (two qubits):

$$S(\rho_r) = 0$$

Maximally entangled state:

$$\rho_A = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow S(\rho_A) = 1$$

Mixed states: Separability criterium

- If ρ is separable, then the partially transposed matrix is positive (Asher Peres, PRL, 1996):

$$\rho = \sum_i p_i \rho_i^A \otimes \rho_i^B \Rightarrow \rho^{T_B} = \sum_i p_i \rho_i^A \otimes (\rho_i^B)^T$$

- For 2X2 and 2X3 systems, ρ is separable iff it remains a density operator under the operation of partial transposition (Horodecki family 1996) - that is, it has a partial positive transpose (PPT)

Negativity as a measure of entanglement

K. Zyczkowski, P. Horodecki, A. Sampera, and M. Lewenstein, PRA, 1998

Vidal and Werner, PRA, 2002

$$\mathcal{N}(\rho_{AB}) \equiv 2\sum_i |\lambda_{i-}|$$

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Dimensions higher than 6: $\mathcal{N}$=0 does not imply separability!

Environment-Induced Sudden Death of Entanglement

M. P. Almeida, F. de Melo, M. Hor-Meyll, A. Salles, S. P. Walborn, P. H. Souto Ribeiro, L. Davidovich*

We demonstrate the difference between local, single-particle dynamics and global dynamics of entangled quantum systems coupled to independent environments. Using an all-optical experimental setup, we showed that, even when the environment-induced decay of each system is asymptotic, quantum entanglement may suddenly disappear. This "sudden death" constitutes yet another distinct and counterintuitive trait of entanglement.

The real-world success of quantum computation (*1*, *2*) and communication (*3*–*9*) relies on the longevity of entanglement in multiparticle quantum states. The presence of decoherence (*10*) in communication channels and computing devices, which stems from the unavoidable interaction between these systems and the environment, degrades the entanglement when the particles propagate or the computation evolves. Decoherence leads to local dynamics, associated with single-particle dissipation, diffusion, and decay, as well as to global dynamics, which may provoke the disappearance of entanglement at a finite time (*11*–*15*). This phenomenon, known as "entanglement sudden death" (*15*), is strikingly different from single-particle dynamics, which occurs asymptotically, and has thus stimulated much recent theoretical work (*11*–*15*). Here we demonstrate the sudden death of entanglement of a two-qubit system under the influence of independent environ-

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PHYSICAL REVIEW A **78**, 022322 (2008)

Experimental investigation of the dynamics of entanglement: Sudden death, complementarity, and continuous monitoring of the environment

A. Salles,[1,*] F. de Melo,[1,2] M. P. Almeida,[1,3] M. Hor-Meyll,[1] S. P. Walborn,[1] P. H. Souto Ribeiro,[1] and L. Davidovich[1]

[1]*Instituto de Física, Universidade Federal do Rio de Janeiro, Caixa Postal 68528, Rio de Janeiro, RJ 21941-972, Brazil*

[2]*Physikalisches Institut, Albert-Ludwigs-Universität Freiburg, Hermann-Herder-Strasse 3, D-79104 Freiburg, Germany*

[3]*Centre for Quantum Computer Technology, Department of Physics, University of Queensland, Brisbane, Queensland 4072, Australia*

(Received 30 April 2008; published 13 August 2008)

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A paradigmatic example: Atomic decay

- Qubit states: $|0\rangle \leftrightarrow |g\rangle, |1\rangle \leftrightarrow |e\rangle$
- "Amplitude channel":

$$|g\rangle_S \otimes |0\rangle_E \rightarrow |g\rangle_S \otimes |0\rangle_E$$

$$|e\rangle_S \otimes |0\rangle_E \rightarrow \sqrt{1-p}|e\rangle_S \otimes |0\rangle_E + \sqrt{p}|g\rangle_S \otimes |1\rangle_E$$

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→ Usual master equation for decay of two-level atom, upon tracing on environment (Markovian approximation)

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Weisskopf and Wigner (1930)!

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Our strategy: follow evolution as a function of *p*, not *t*

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$$|g\rangle_S \otimes |0\rangle_E \rightarrow |g\rangle_S \otimes |0\rangle_E$$

$$|e\rangle_S \otimes |0\rangle_E \rightarrow \sqrt{1-p}|e\rangle_S \otimes |0\rangle_E + \sqrt{p}|g\rangle_S \otimes |1\rangle_E$$

$$p = 1 - \exp(-\Gamma t)$$

→ Usual master equation for decay of two-level atom, upon tracing on environment (Markovian approximation)

Weisskopf and Wigner (1930)!

A paradigmatic example: Atomic decay

Our strategy: follow evolution as a function of *p*, not *t*

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→ Usual master equation for decay of two-level atom, upon tracing on environment (Markovian approximation)

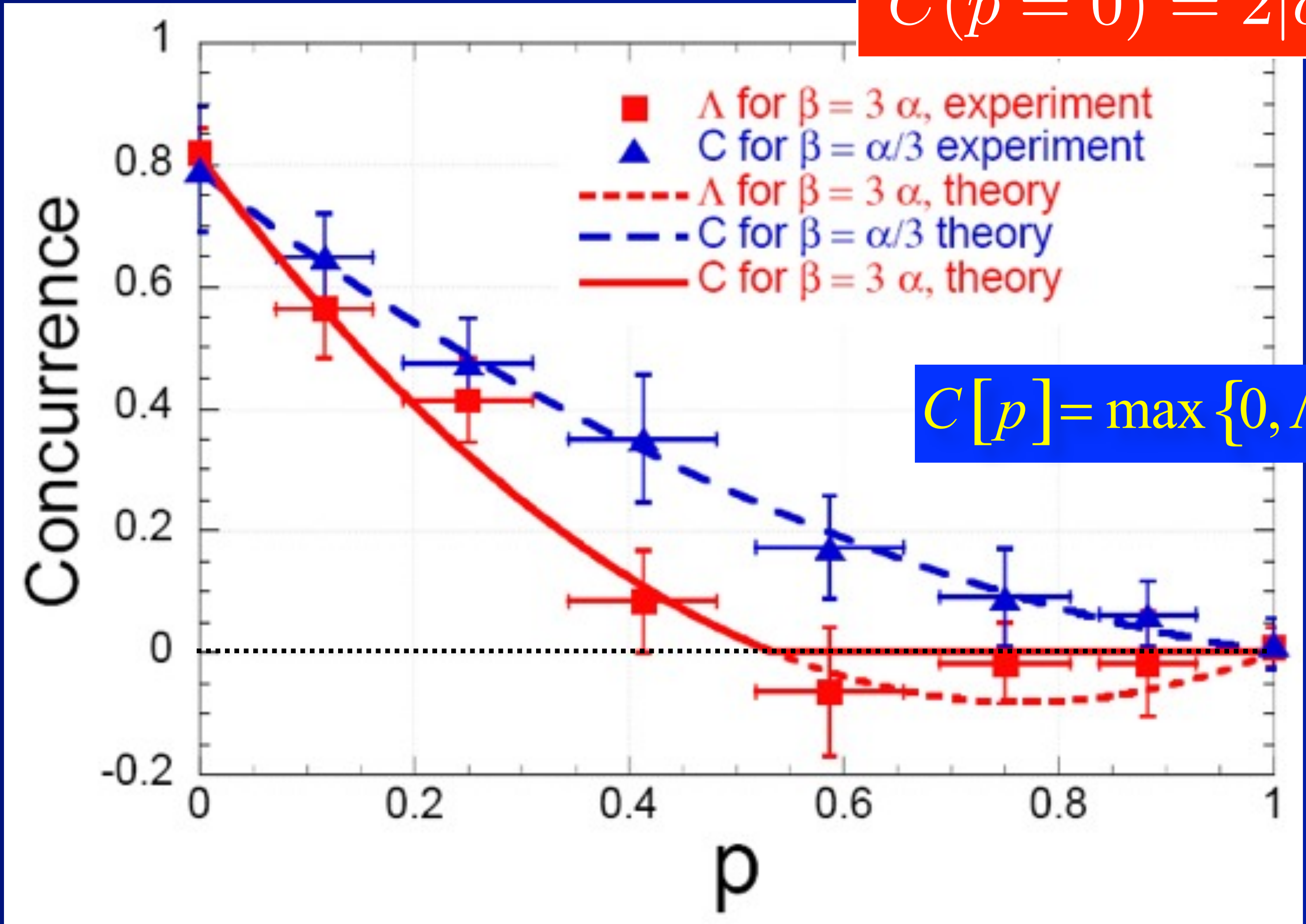
Weisskopf and Wigner (1930)!

Apply evolution to two qubits, take trace with respect to environment degrees of freedom, find evolution of two-qubit reduced density matrix, calculate entanglement

"Sudden death" of entanglement

$$|\Psi(0)\rangle = \alpha|gg\rangle + \beta|ee\rangle$$

$$C(p=0) = 2|\alpha\beta|$$



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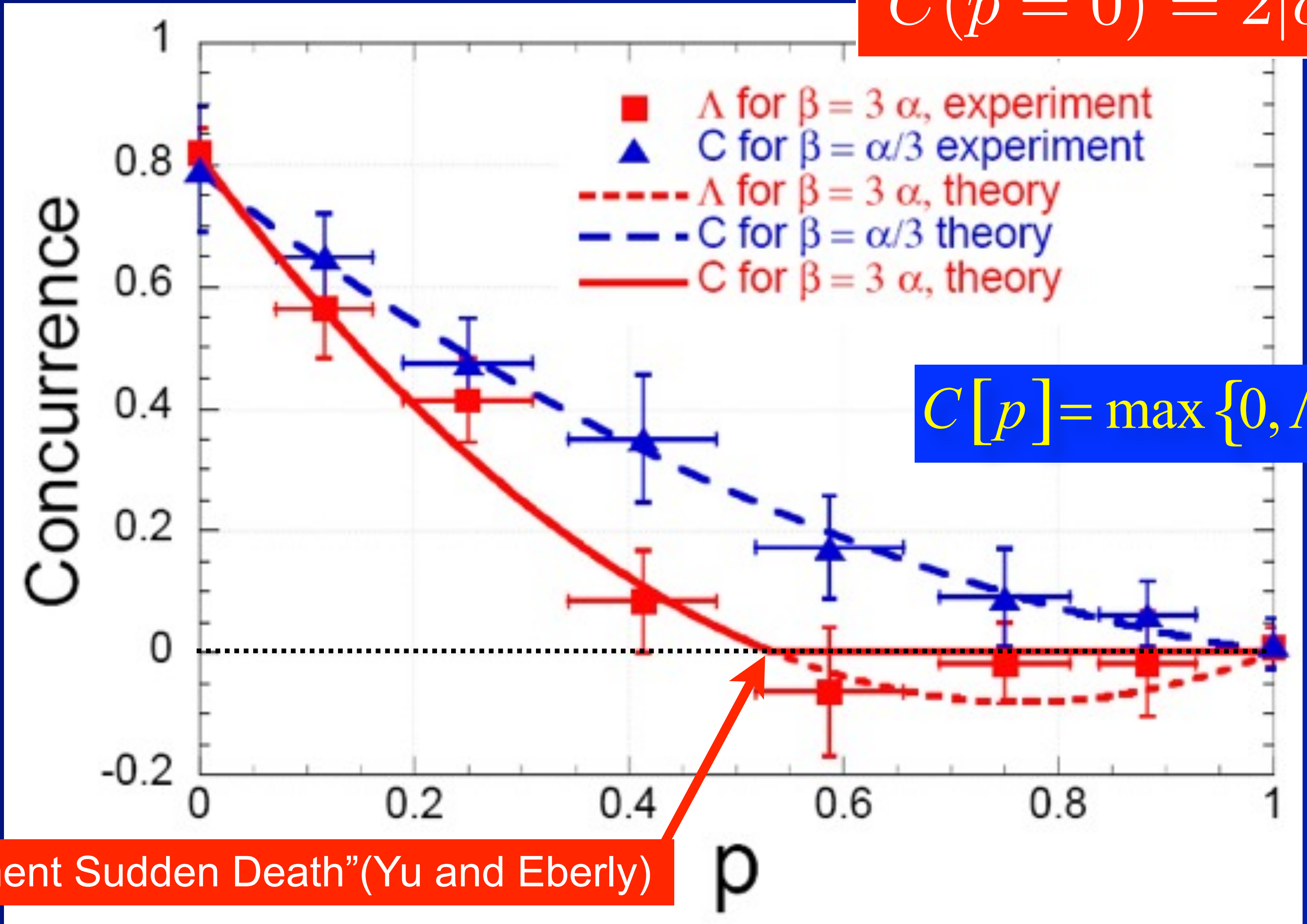

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"Entanglement Sudden Death"(Yu and Eberly)

Decay of entanglement for N qubits, other environments?

PRL **100,** 080501 (2008) PHYSICAL REVIEW LETTERS week ending 29 FEBRUARY 2008

Scaling Laws for the Decay of Multiqubit Entanglement

L. Aolita,[1] R. Chaves,[1] D. Cavalcanti,[2] A. Acín,[2,3] and L. Davidovich[1]
[1]*Instituto de Física, Universidade Federal do Rio de Janeiro. Caixa Postal 68528, 21941-972 Rio de Janeiro, RJ, Brasil*
[2]*ICFO-Institut de Ciencies Fotoniques, Mediterranean Technology Park, 08860 Castelldefels (Barcelona), Spain*
[3]*ICREA-Institució Catalana de Recerca i Estudis Avançats, Lluis Companys 23, 08010 Barcelona, Spain*
(Received 23 October 2007; published 27 February 2008)

$$|\Psi_0\rangle = \alpha|0\rangle^{\otimes N} + \beta|1\rangle^{\otimes N}$$

- Independent individual environments:

$$\mathcal{E}_i^{D}\rho_i = (1-p)\rho_i + (p)1/2$$ Depolarization

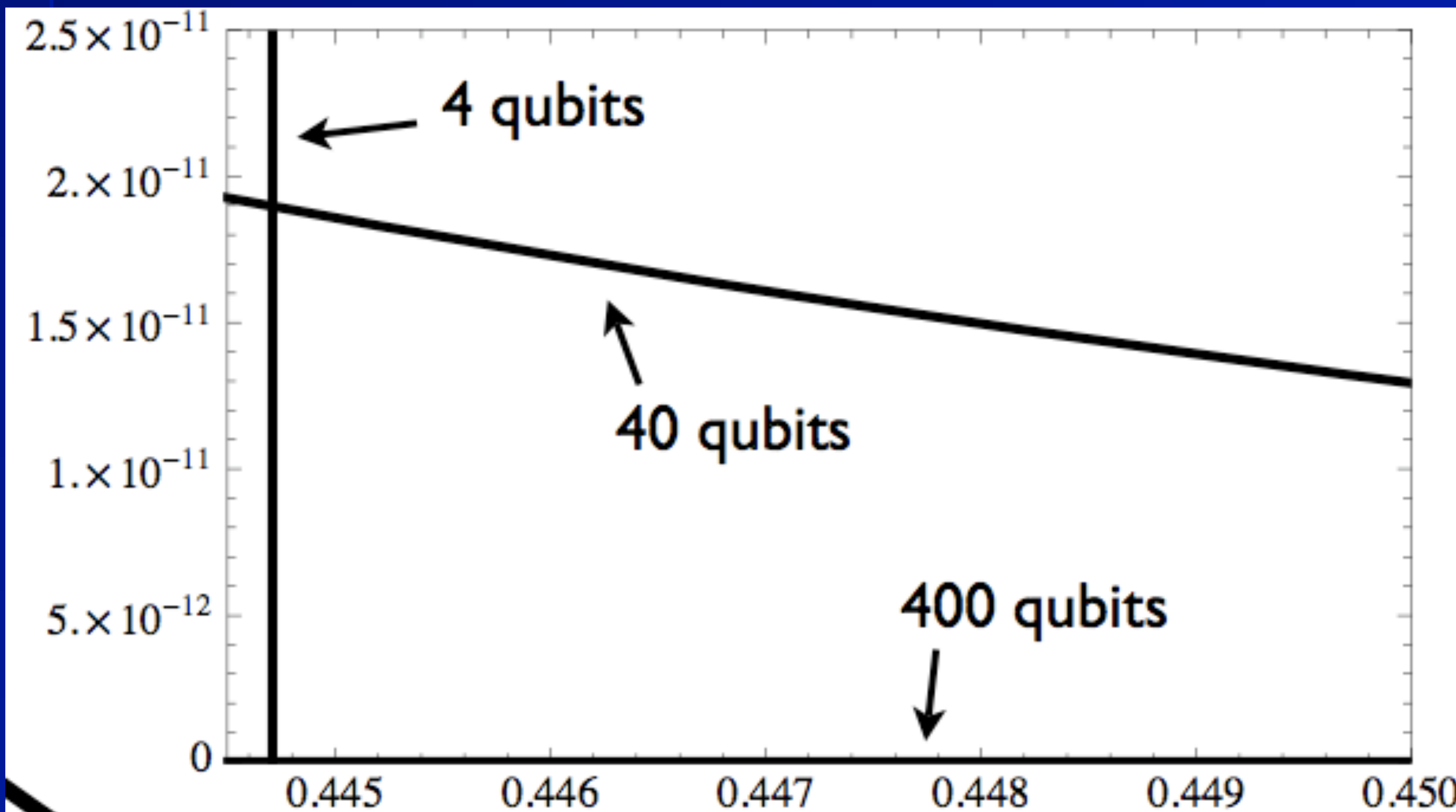
$$\mathcal{E}_i^{PD}\rho_i = (1-p)\rho_i + p\big(|0\rangle\langle 0|\rho_i|0\rangle\langle 0| + |1\rangle\langle 1|\rho_i|1\rangle\langle 1|\big)$$

+ Thermal

Dephasing

Does entanglement become more robust with increasing N?

$|\Psi_0\rangle = \alpha|0\rangle^{\otimes N} + \beta|1\rangle^{\otimes N}$ $\mathcal{E}_i^D \rho_i = (1-p)\rho_i + (p)1/2$



Is ESD relevant for many particles?

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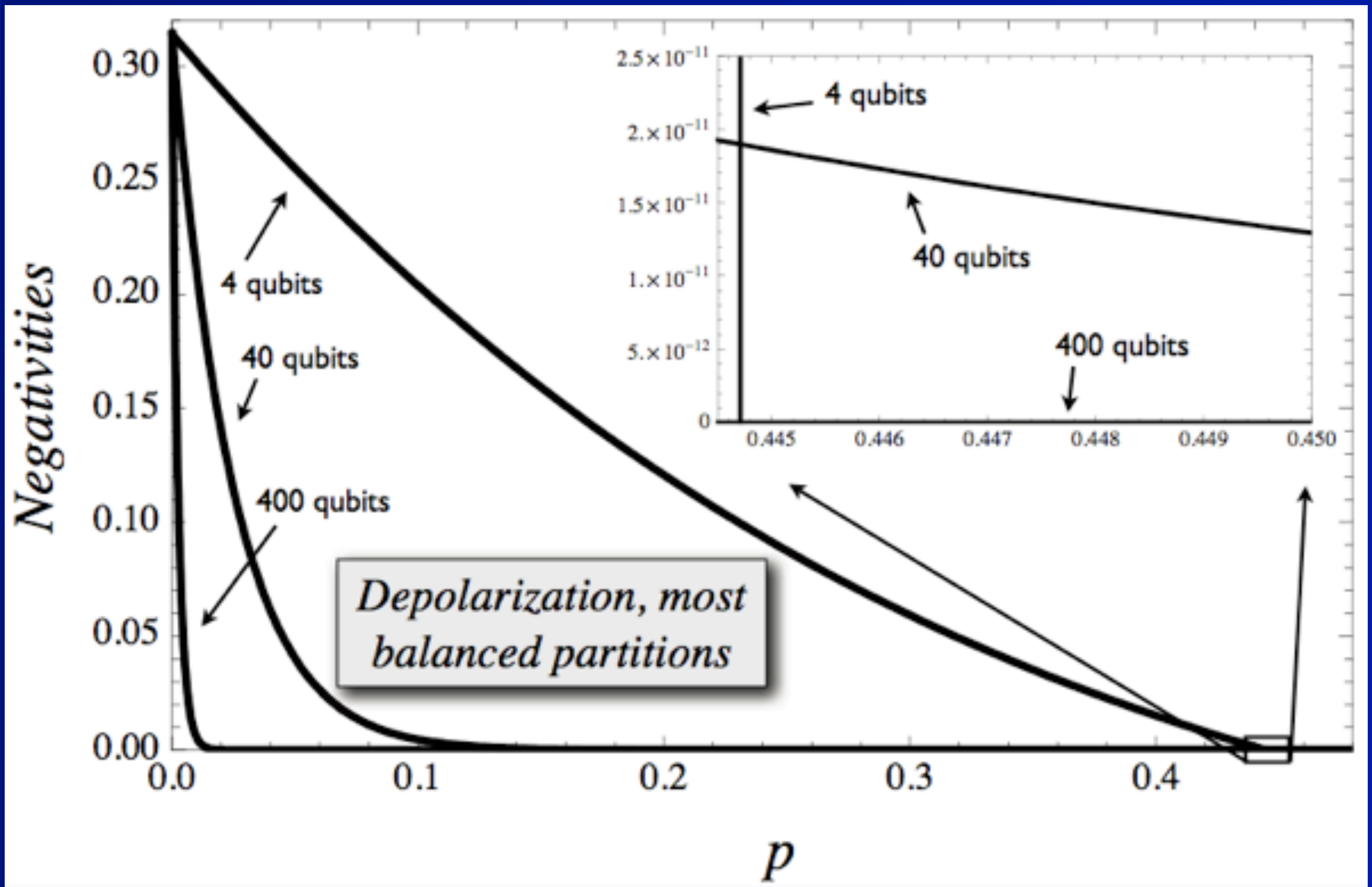

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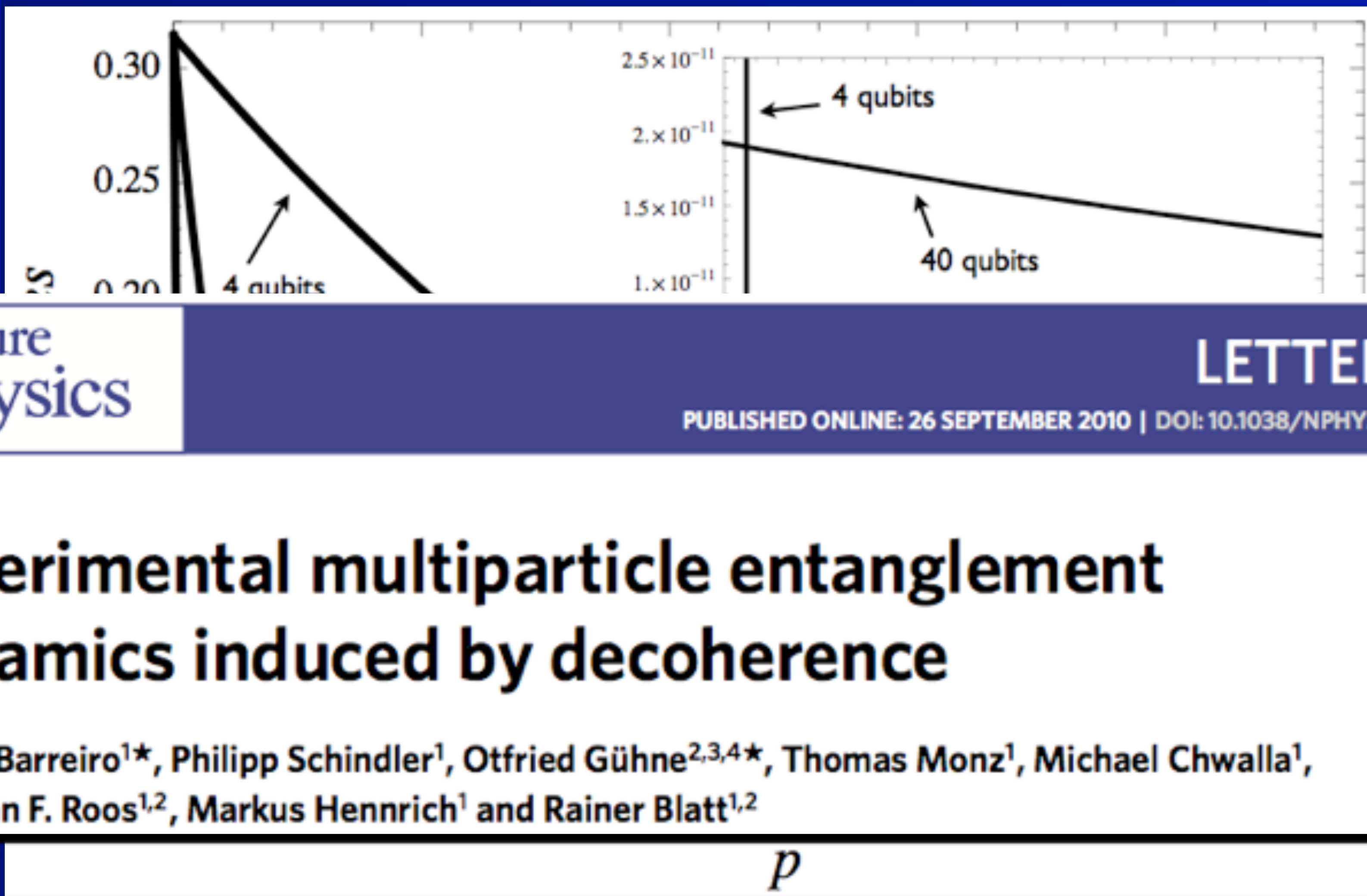


nature physics

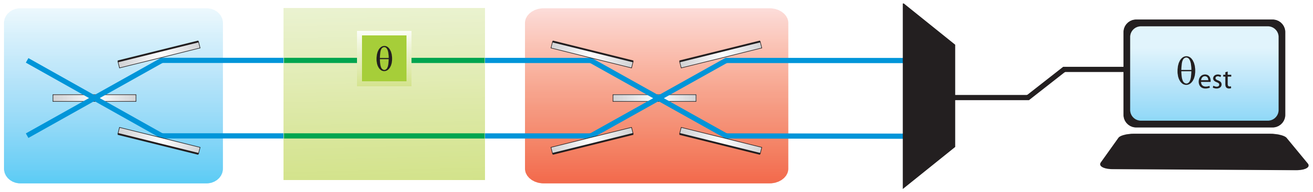
LETTERS

PUBLISHED ONLINE: 26 SEPTEMBER 2010 | DOI: 10.1038/NPHYS1781

Experimental multiparticle entanglement dynamics induced by decoherence

Julio T. Barreiro[1]★, Philipp Schindler[1], Otfried Gühne[2,3,4]★, Thomas Monz[1], Michael Chwalla[1], Christian F. Roos[1,2], Markus Hennrich[1] and Rainer Blatt[1,2]

Entanglement and quantum metrology



Standard limit: $\delta\theta \approx \frac{1}{\sqrt{\langle n \rangle}}$

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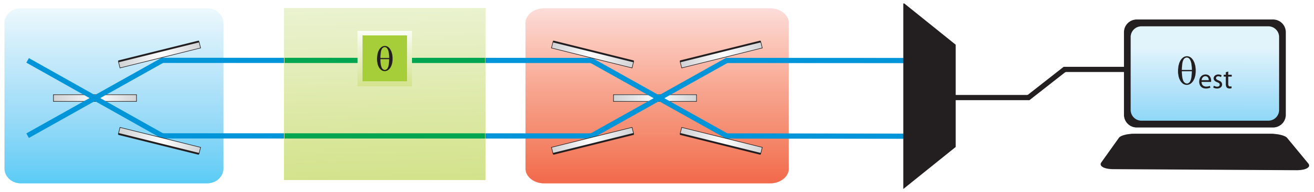

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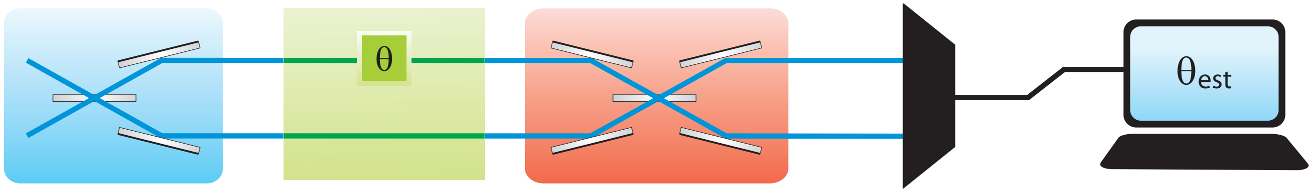

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Possible method to increase precision for the same average number of photons: Use NOON states [J. P. Dowling, PRA 57, 4736 (1998)]

$$|\psi(N)\rangle = \left(|N,0\rangle + |0,N\rangle\right)/\sqrt{2} \rightarrow |\psi(N,\theta)\rangle = \left(|N,0\rangle + e^{iN\theta}|0,N\rangle\right)/\sqrt{2}, \quad \left(\langle n \rangle = N\right)$$

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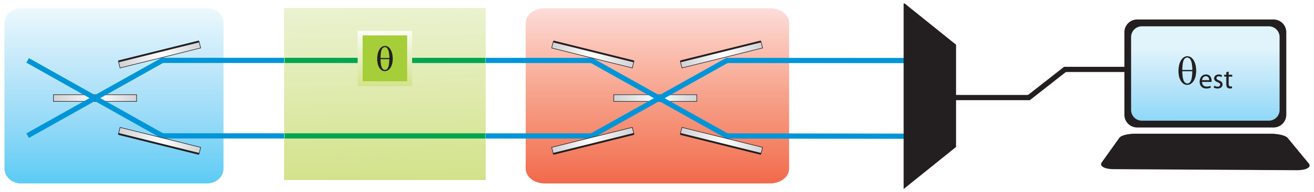

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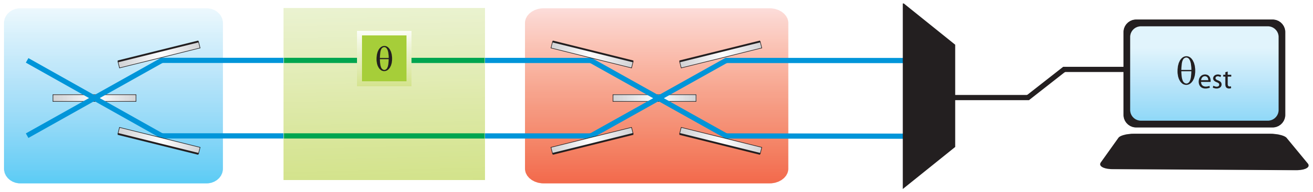

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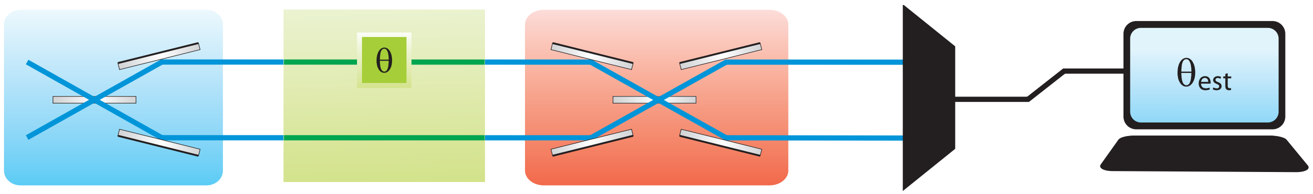
Entanglement and quantum metrology



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Heisenberg limit

Precision is better, for the same amount of resources (average number of photons)!

Example: Frequency measurements in ion traps

PHYSICAL REVIEW A VOLUME 54, NUMBER 6 DECEMBER 1996

Optimal frequency measurements with maximally correlated states

J. J . Bollinger, Wayne M. Itano, and D. J. Wineland
Time and Frequency Division, National Institute of Standards and Technology, Boulder, Colorado 80303

D. J. Heinzen
Physics Department, University of Texas, Austin, Texas 78712

(Received 16 August 1996)

Independent atoms:

$$\frac{1}{2^{N/2}}\underbrace{\left(|g\rangle+|e\rangle\right)\otimes\cdots\otimes\left(|g\rangle+|e\rangle\right)}_{N}\rightarrow\frac{1}{2^{N/2}}\underbrace{\left(|g\rangle+e^{iT\delta\omega}|e\rangle\right)\otimes\cdots\otimes\left(|g\rangle+e^{iT\delta\omega}|e\rangle\right)}_{N}$$

$(\delta\omega)T=\pi$ for orthogonality. Yields frequency uncertainty $1/(\sqrt{N}T)$

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Correlated atoms:

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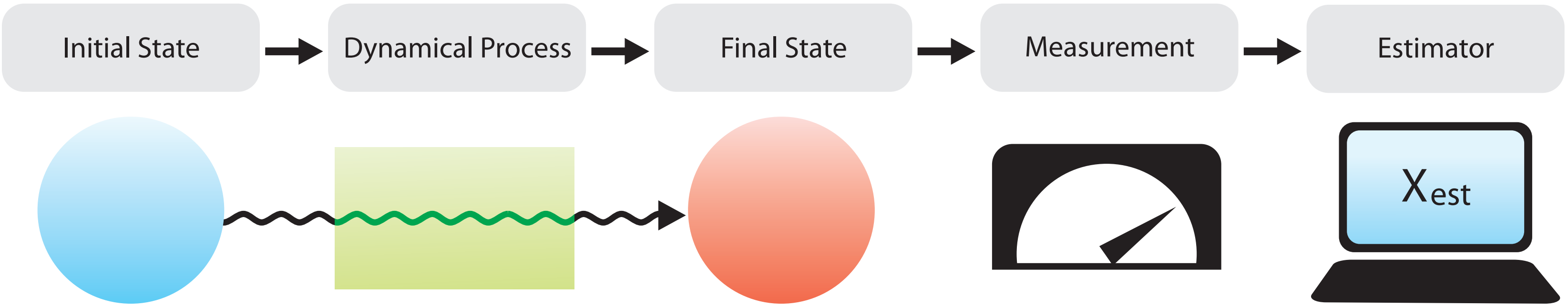
$(\delta\omega)T=\pi/N$ for orthogonality. Yields frequency uncertainty 1/NT, T time for single measurement

Steps in parameter estimation



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2. Send probe through process to be investigated
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4. Associate each experimental result *j* with estimation

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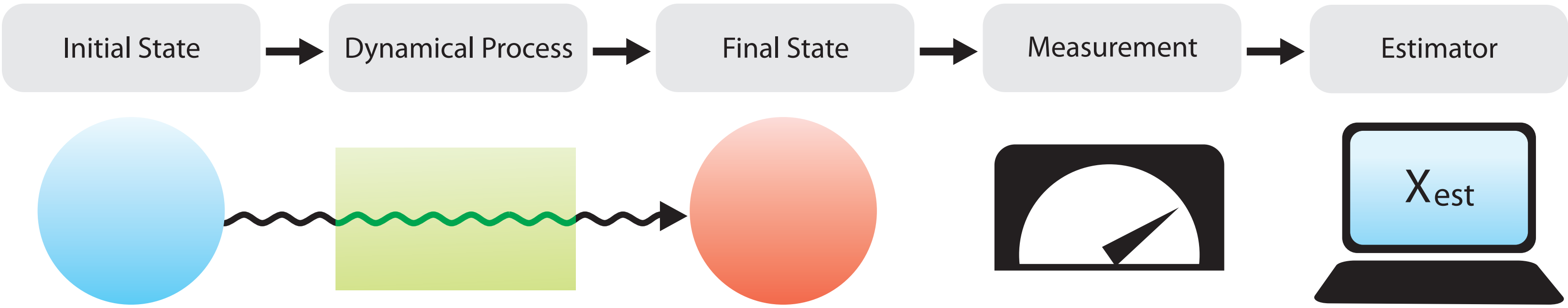

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$$\delta X \equiv \sqrt{\langle [X_{\text{est}}(j) - X]^2 \rangle_j \Big|_{X = X_{\text{real}}}} \rightarrow \text{Merit quantifier}$$

$$\langle X_{\text{est}} \rangle = X_{\text{real}},\ d\langle X_{\text{est}} \rangle / dX = 1 \rightarrow \text{Unbiased estimator}$$

Classical parameter estimation

H. Cramér

C. R. Rao

R. A. Fisher

Cramér-Rao bound for unbiased estimators:

$$\delta X \geq 1/\sqrt{\nu F(X_{\text{real}})}, \quad F(X) \equiv \sum_j p_j(X) \left(\frac{d \ln\left[p_j(X) \right]}{dx} \right)^2$$

$\nu \rightarrow$ Number of repetitions of the experiment

$p_j(X) \rightarrow$ probability of getting an experimental result j

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Fisher information

$\nu \rightarrow$ Number of repetitions of the experiment

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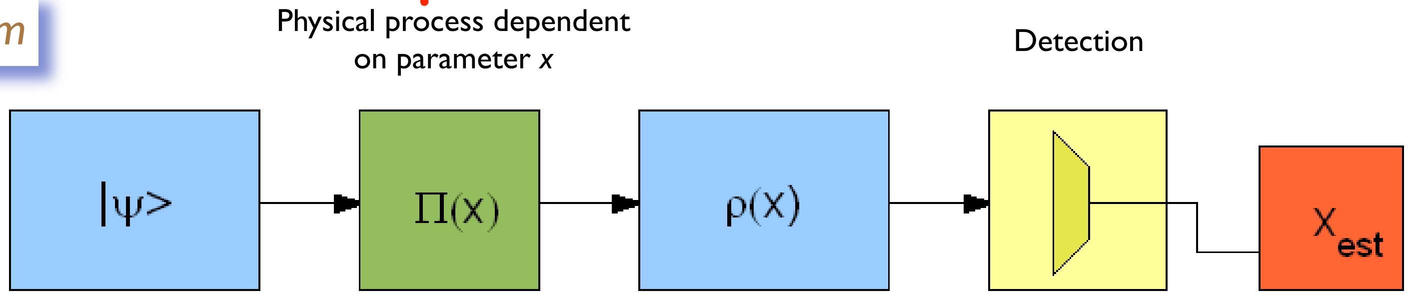
Fisher's theorem: Inequality can be saturated (i.e., it is possible to make it an equality) when $\nu \rightarrow \infty$, by choosing an appropriate estimator X_{est}.

Quantum parameter estimation

Holevo, Helstrom



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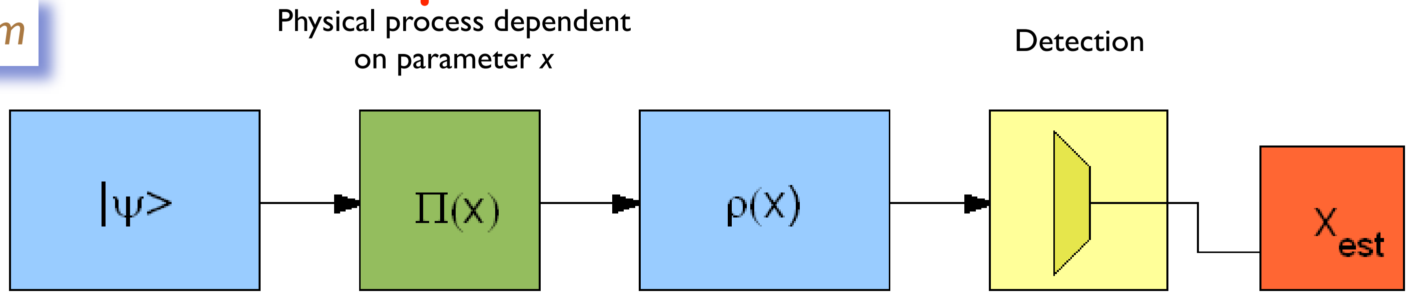

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Closed systems: $\hat{\rho}(X) = \hat{U}(X)\hat{\rho}\hat{U}^{\dagger}(X)$

Quantum parameter estimation

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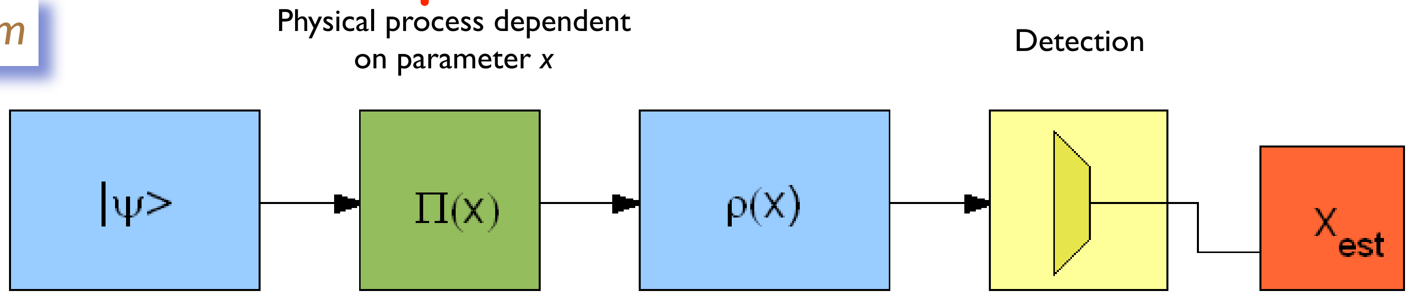

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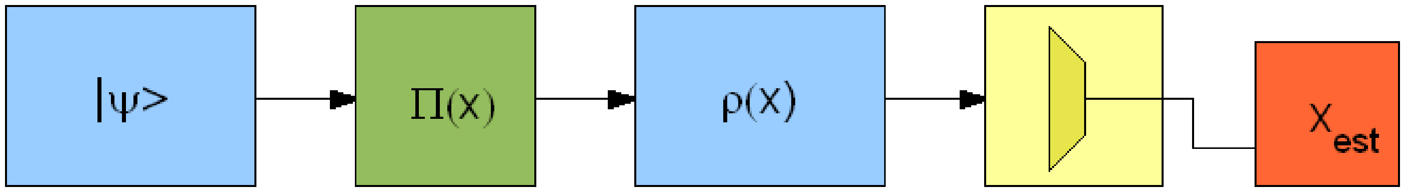
Quantum parameter estimation

Holevo, Helstrom

Physical process dependent on parameter *x*

Detection



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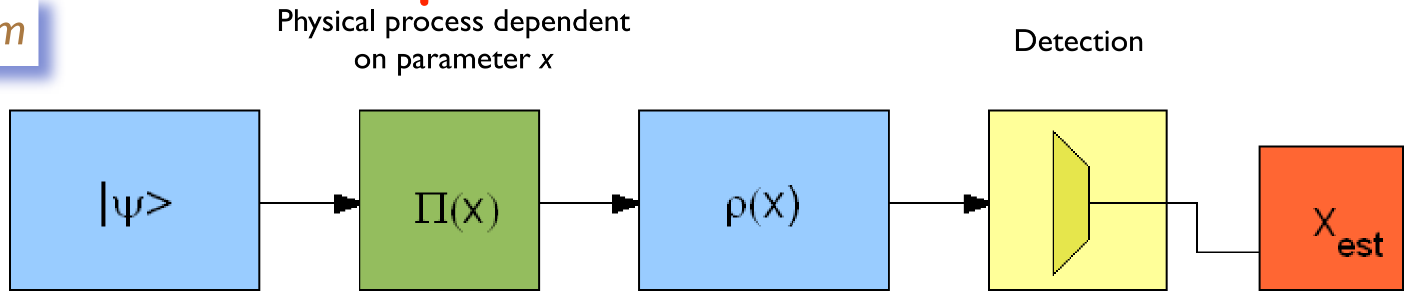
Quantum parameter estimation

Holevo, Helstrom



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Quantum parameter estimation

Holevo, Helstrom

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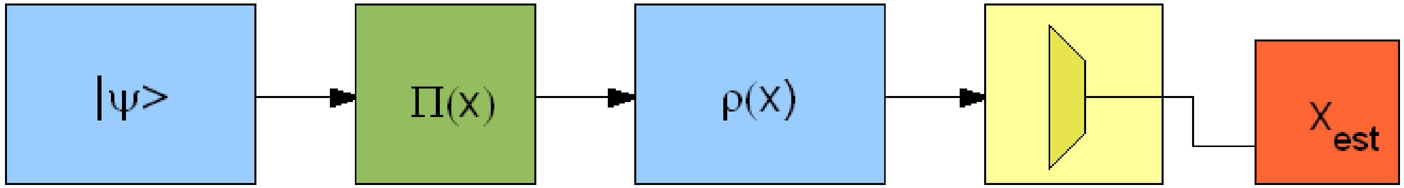

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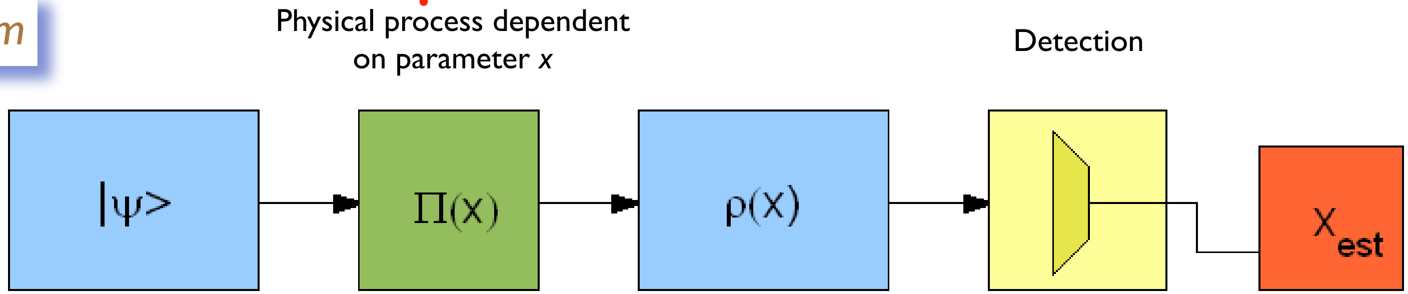
Quantum parameter estimation

Holevo, Helstrom



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Quantum Fisher information for pure states

Initial state of the probe: $|\psi(0)\rangle$
Final X-dependent state: $|\psi(X)\rangle = \hat{U}(X)|\psi(0)\rangle$, $\hat{U}(x)$unitary operator.

Then (Helstrom 1976):

$$\mathcal{F}_Q(X) = 4\langle(\Delta\hat{H})^2\rangle_0\,, \quad \langle(\Delta\hat{H})^2\rangle_0 \equiv \langle\psi(0)|\left[\hat{H}(X) - \langle\hat{H}(X)\rangle_0\right]^2|\psi(0)\rangle$$

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$$\hat{H}(X) \equiv i\frac{d\hat{U}^\dagger(X)}{dX}\hat{U}(X)$$

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If $\hat{U}(X) = \exp(i\hat{O}X)$, $\hat{O}$ independent of X, then $\hat{H} = \hat{O}$

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$$\mathcal{F}_Q(X) = 4\langle(\Delta\hat{H})^2\rangle_0\,, \quad \langle(\Delta\hat{H})^2\rangle_0 \equiv \langle\psi(0)|\left[\hat{H}(X) - \langle\hat{H}(X)\rangle_0\right]^2|\psi(0)\rangle$$

where

$$\hat{H}(X) \equiv i\frac{d\hat{U}^\dagger(X)}{dX}\hat{U}(X)$$

If $\hat{U}(X) = \exp(i\hat{O}X)$, $\hat{O}$ independent of X, then $\hat{H} = \hat{O}$

$$\delta x \geq 1/2\sqrt{\nu\left\langle\Delta\hat{H}^2\right\rangle}$$

$\Rightarrow$ Generalized uncertainty relation: Should maximize the variance to get better precision!

Example of Generalized Uncertainty Relations: Spatial displacement and momentum

[For more details, see Braunstein, Caves, and Milburn, Annals of Physics **247**, 135 (1996)]

$$|\psi(X)\rangle = e^{iX\hat{P}}|\psi(0)\rangle \Rightarrow \hat{H} = i\frac{d\hat{U}^\dagger(X)}{dX}\hat{U}(X) = \hat{P}$$

$$\mathcal{F}_Q(X) = \langle(\Delta\hat{P})^2\rangle_0 \Rightarrow \langle(\delta X)^2\rangle \geq \frac{1}{\nu\langle(\Delta\hat{P})^2\rangle_0}$$

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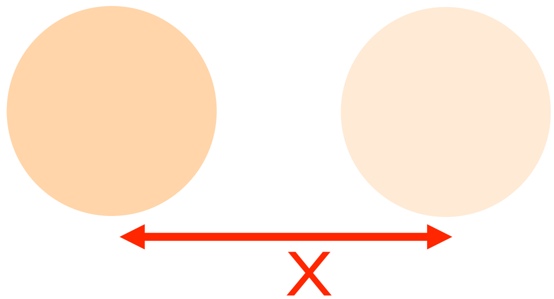

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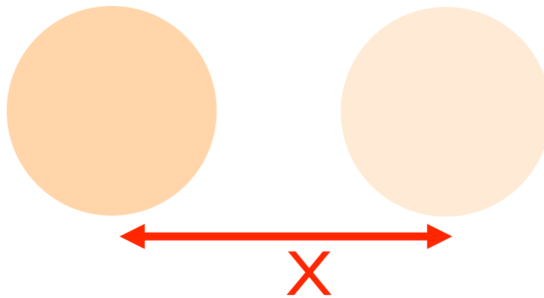

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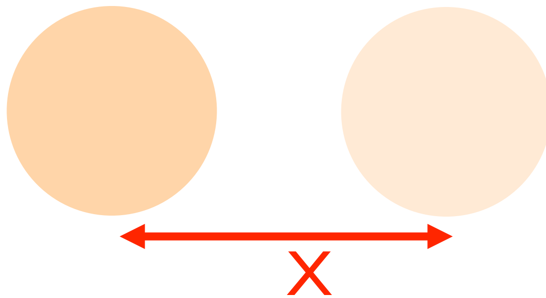

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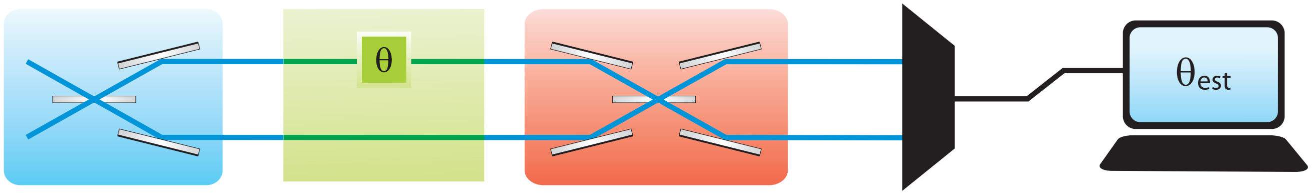
Maximizing variance of P for better precision: squeezed states or superpositions of coherent states

Example of Generalized Uncertainty Relations (2): Revisiting optical interferometry



$\mathcal{F}_Q(X) = 4\langle(\Delta\hat{n})^2\rangle_0$ where $\langle(\Delta\hat{n})^2\rangle_0$ is the photon-number variance in the upper arm.

Standard limit: coherent states

(Ignoring repetitions of the experiment)

$$\mathcal{F}_Q(X) = 4\langle(\Delta\hat{n})^2\rangle_0 = 4\langle\hat{n}\rangle \Rightarrow \delta\theta \geq \frac{1}{2\sqrt{\langle n\rangle}}$$

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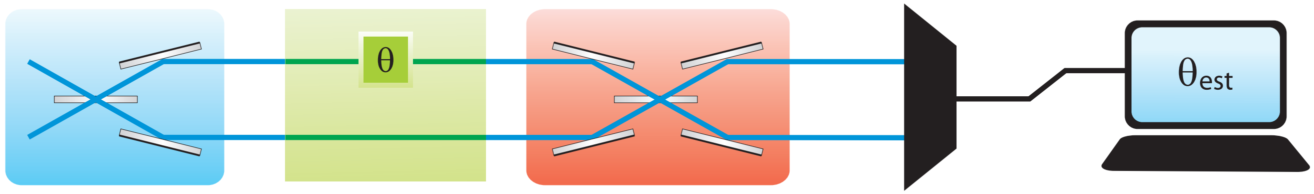

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Increasing the precision: maximize variance with NOON states:

$$|\psi(N)\rangle = (|N,0\rangle + |0,N\rangle)/\sqrt{2} \rightarrow |\psi(N,\theta)\rangle = (|N,0\rangle + e^{iN\theta}|0,N\rangle)/\sqrt{2}$$

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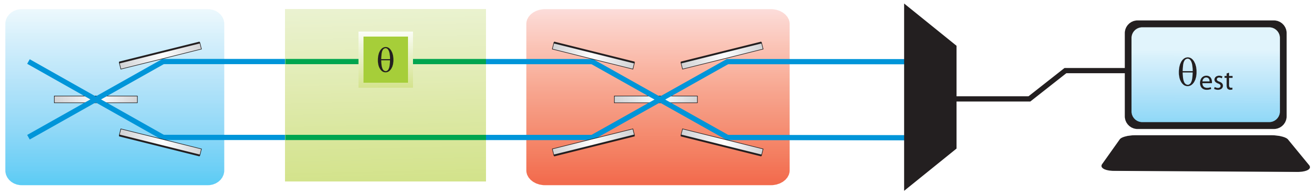

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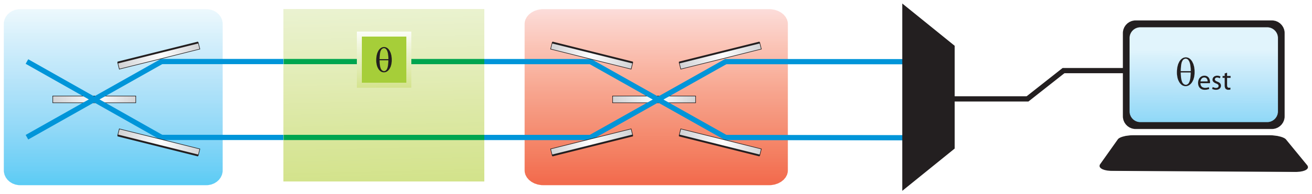

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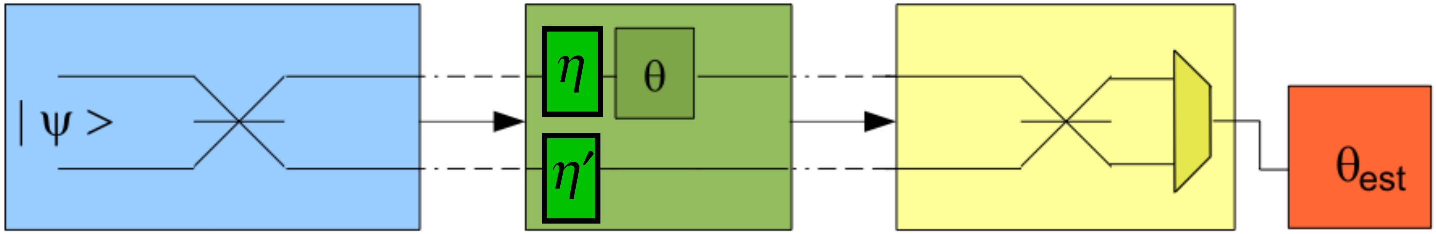
Precision is better, for the same amount of resources.

Parameter estimation with decoherence



Loss of a single photon transforms NOON state into a separable state!

$$|\psi(N)\rangle = \frac{|N,0\rangle + |0,N\rangle}{\sqrt{2}} \rightarrow |N-1,0\rangle \text{ or } |0,N-1\rangle$$

No simple analytical expression for Fisher information!

For small N, more robust states can be numerically calculated

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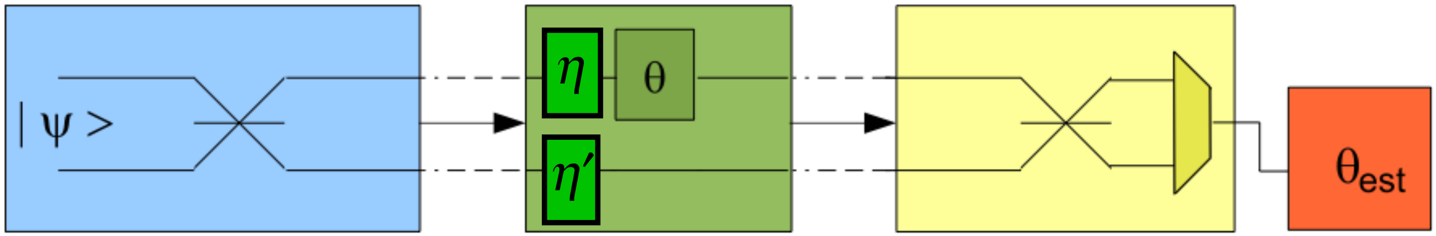

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Experimental test with more robust states:

nature photonics

LETTERS

PUBLISHED ONLINE: 4 APRIL 2010 | DOI: 10.1038/NPHOTON.2010.39

Experimental quantum-enhanced estimation of a lossy phase shift

M. Kacprowicz[1], R. Demkowicz-Dobrzański[1,2]*, W. Wasilewski[2], K. Banaszek[1,2] and I. A. Walmsley[3]

nature physics

ARTICLES

PUBLISHED ONLINE: 27 MARCH 2011 | DOI: 10.1038/NPHYS1958

General framework for estimating the ultimate precision limit in noisy quantum-enhanced metrology

B. M. Escher*, R. L. de Matos Filho and L. Davidovich

Braz J Phys
DOI 10.1007/s13538-011-0037-y

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GENERAL AND APPLIED PHYSICS

Quantum Metrology for Noisy Systems

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news & views

QUANTUM METROLOGY

Beauty and the noisy beast

Elegant but extremely delicate quantum procedures can increase the precision of measurements. Characterizing how they cope with the detrimental effects of noise is essential for deployment to the real world.

Lorenzo Maccone and Vittorio Giovannetti

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GENERAL AND APPLIED PHYSICS

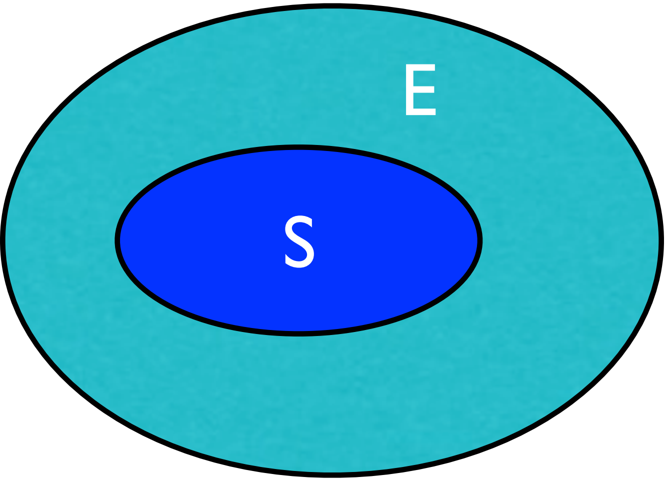
Quantum Metrology for Noisy Systems

B. M. Escher · R. L. de Matos Filho · L. Davidovich

Parameter estimation with losses: Extended space approach

B. M. Escher, R. L. Matos Filho, and L. D., Nature Physics

Given $\hat{\rho}_0 = |\psi\rangle\langle\psi|$ so that $\hat{\rho}(x) = \sum_\ell \hat{\Pi}_\ell(X)\hat{\rho}_0\hat{\Pi}_\ell^\dagger(X)$, define in S+E



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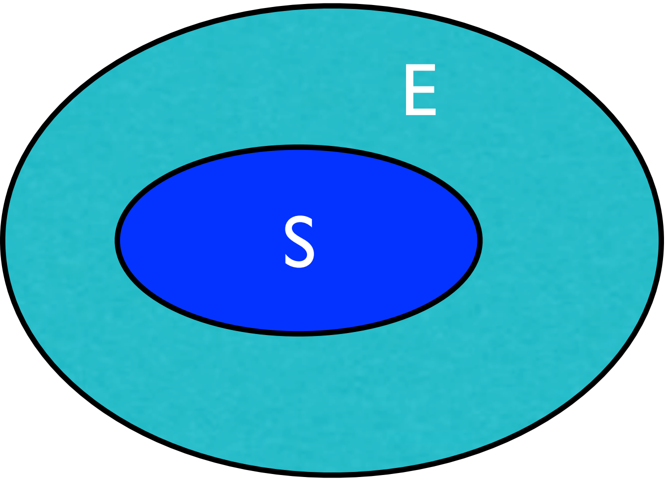

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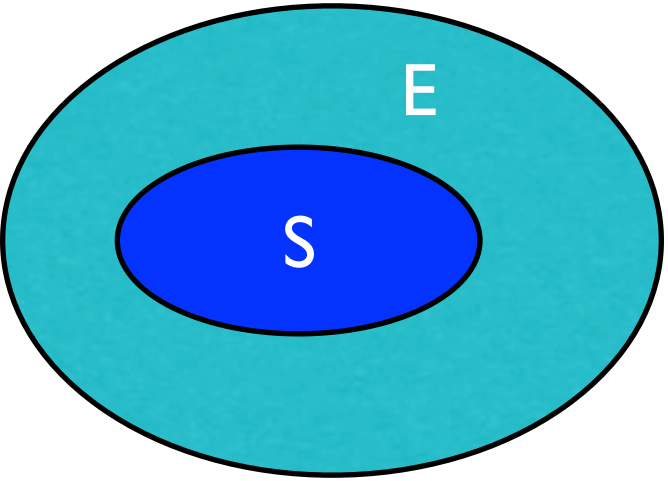

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Physical meaning of this bound: information obtained about parameter when S+E is monitored

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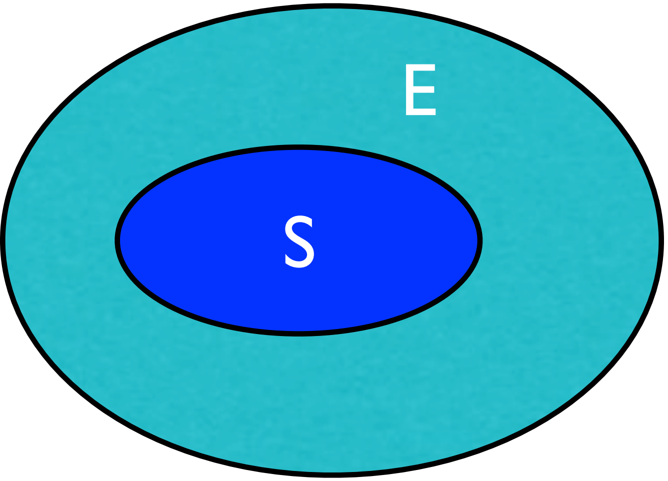

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Least upper bound: Minimization over all Kraus operators - difficult problem

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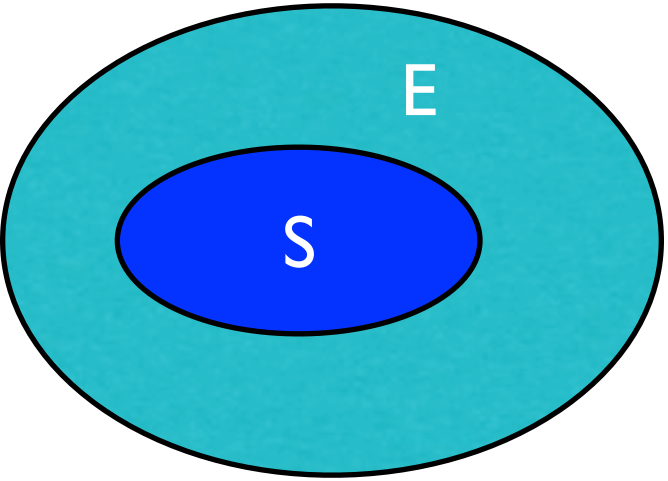

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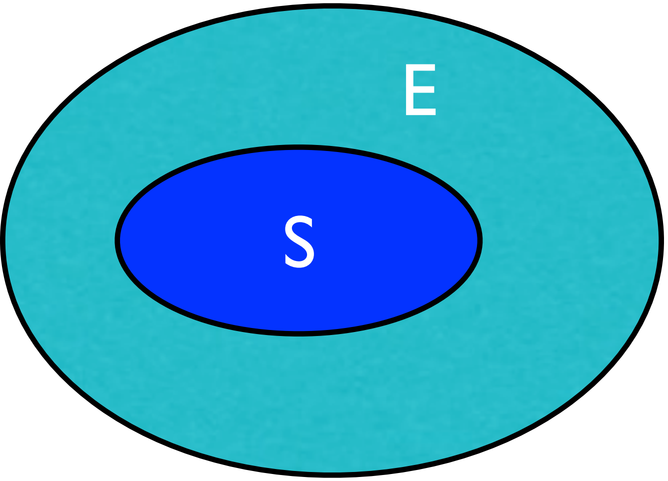

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Then, monitoring S+E gives same information as monitoring S

Parameter estimation with losses: Extended space approach

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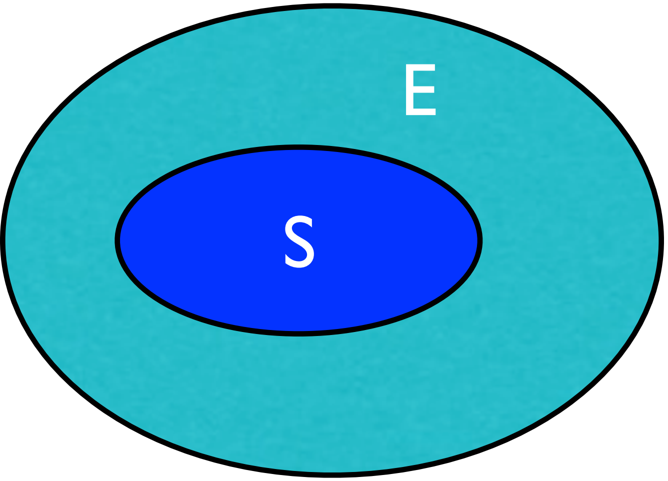

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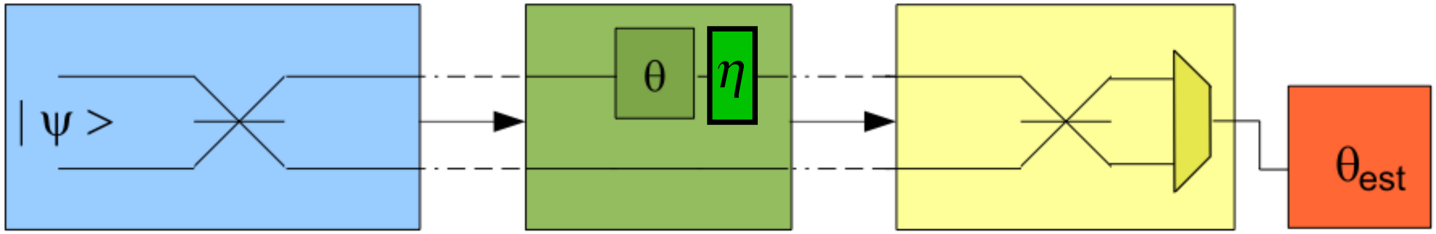
Quantum limits for lossy optical interferometry



With losses (upper arm): $$\frac{d\hat{\rho}(t)}{dt} = -i\omega\left[\hat{n},\hat{\rho}(t)\right] + \gamma\left[\hat{a}\hat{\rho}(t)\hat{a}^{\dagger} - \frac{1}{2}\left(\hat{n}\hat{\rho}(t) - \hat{\rho}(t)\hat{n}\right)\right], \quad \hat{n} = \hat{a}^{\dagger}\hat{a}$$

$\langle \hat{n} \rangle \longrightarrow$ Average number of photons in the upper arm

Equivalent description in terms of the Kraus operators:

$$\hat{\rho}(t) = \sum_{\ell} \Pi_{\ell}(t)\hat{\rho}(0)\Pi_{\ell}^{\dagger}(t)$$

Upon deriving this equation with respect to t, one should find the master equation - there are many possible choices of Kraus operators that lead to the above master equation.

Quantum limits for lossy optical interferometry

States with well-defined total photon number: $|\psi_0\rangle = \sum_{n=0}^{N} \beta_n |n, N-n\rangle$

$$2\sqrt{\nu}\delta\theta \geq \left[1 + \sqrt{1 + \frac{1-\eta}{\eta} N}\right] / N, \quad \eta = e^{-\gamma t}$$

$\eta = 1 \rightarrow$ no absorption

$\eta = 0 \rightarrow$ complete absorption

$\nu \longrightarrow$ Number of repetitions

Quantum limits for lossy optical interferometry

States with well-defined total photon number: $|\psi_0\rangle = \sum_{n=0}^{N} \beta_n |n, N-n\rangle$

$$2\sqrt{\nu}\delta\theta \geq \left[1 + \sqrt{1 + \frac{1-\eta}{\eta} N}\right] / N, \quad \eta = e^{-\gamma t}$$

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$$\eta \rightarrow 1 \text{ or } N \ll \frac{\eta}{1-\eta} \Rightarrow \sqrt{\nu}\delta\theta \geq 1/N \rightarrow \text{ Heisenberg limit}$$

$$N \gg \frac{\eta}{1-\eta} \Rightarrow \delta\theta \geq \frac{\sqrt{1-\eta}}{2\sqrt{\nu\eta N}}$$

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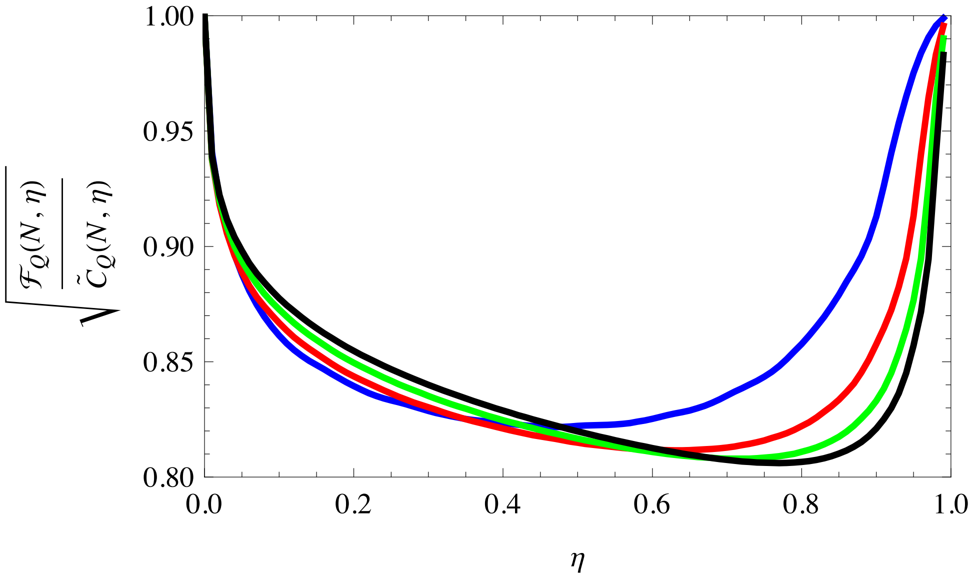
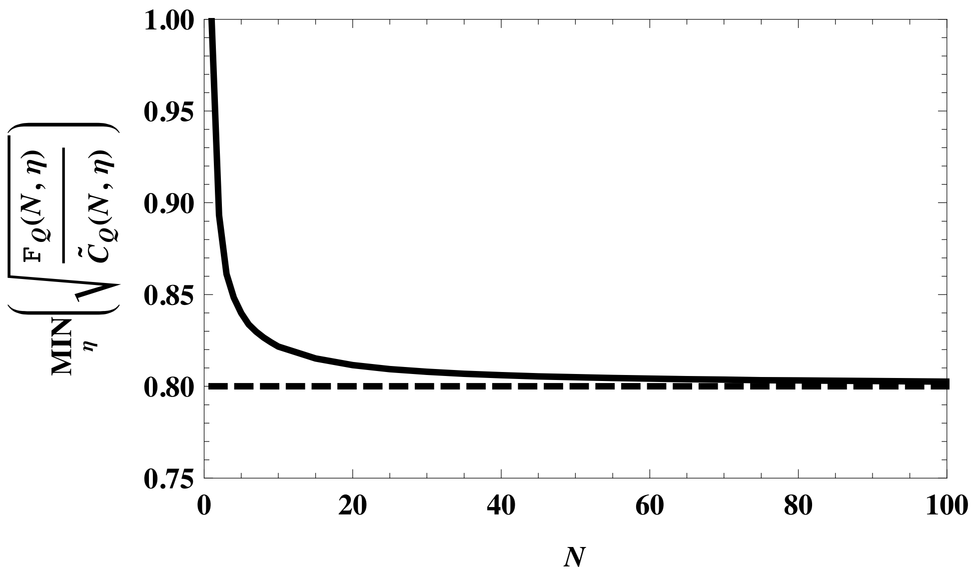
For N sufficiently large, $1/\sqrt{N}$ behavior is always reached!

How good is this bound?



Comparison between numerical maximum value of $\mathscr{F}_Q$ and upper bound $\mathscr{C}_Q$ as a function of η, for $N = 10$ (blue), $N = 20$ (red), $N = 30$ (green), and $N = 40$ (black).

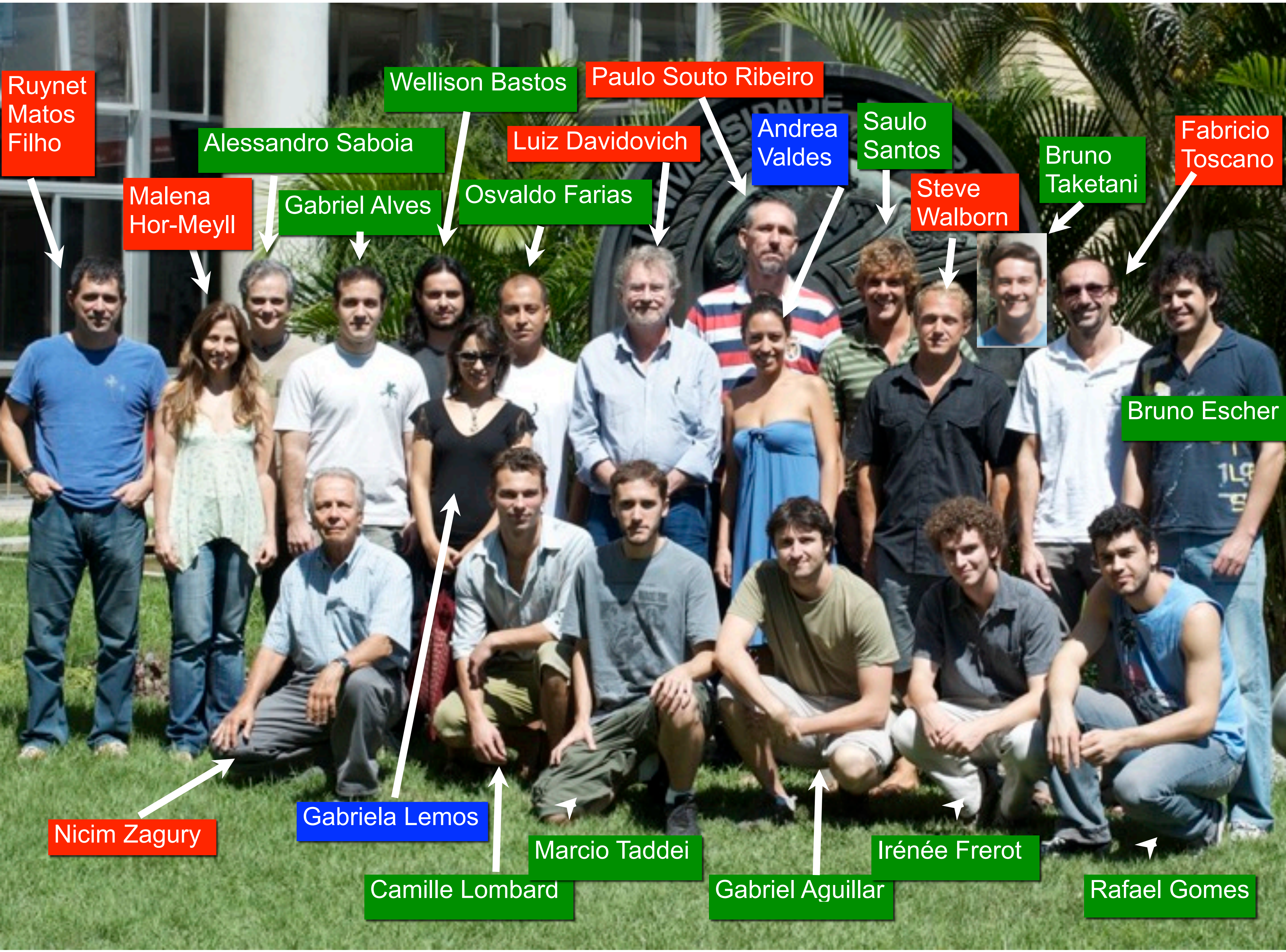


Behavior of the minimum for all values of η, as a function of N

$$1/\sqrt{\nu\tilde{C}_Q} \leq \delta\theta \leq 1.25/\sqrt{\nu\tilde{C}_Q}$$

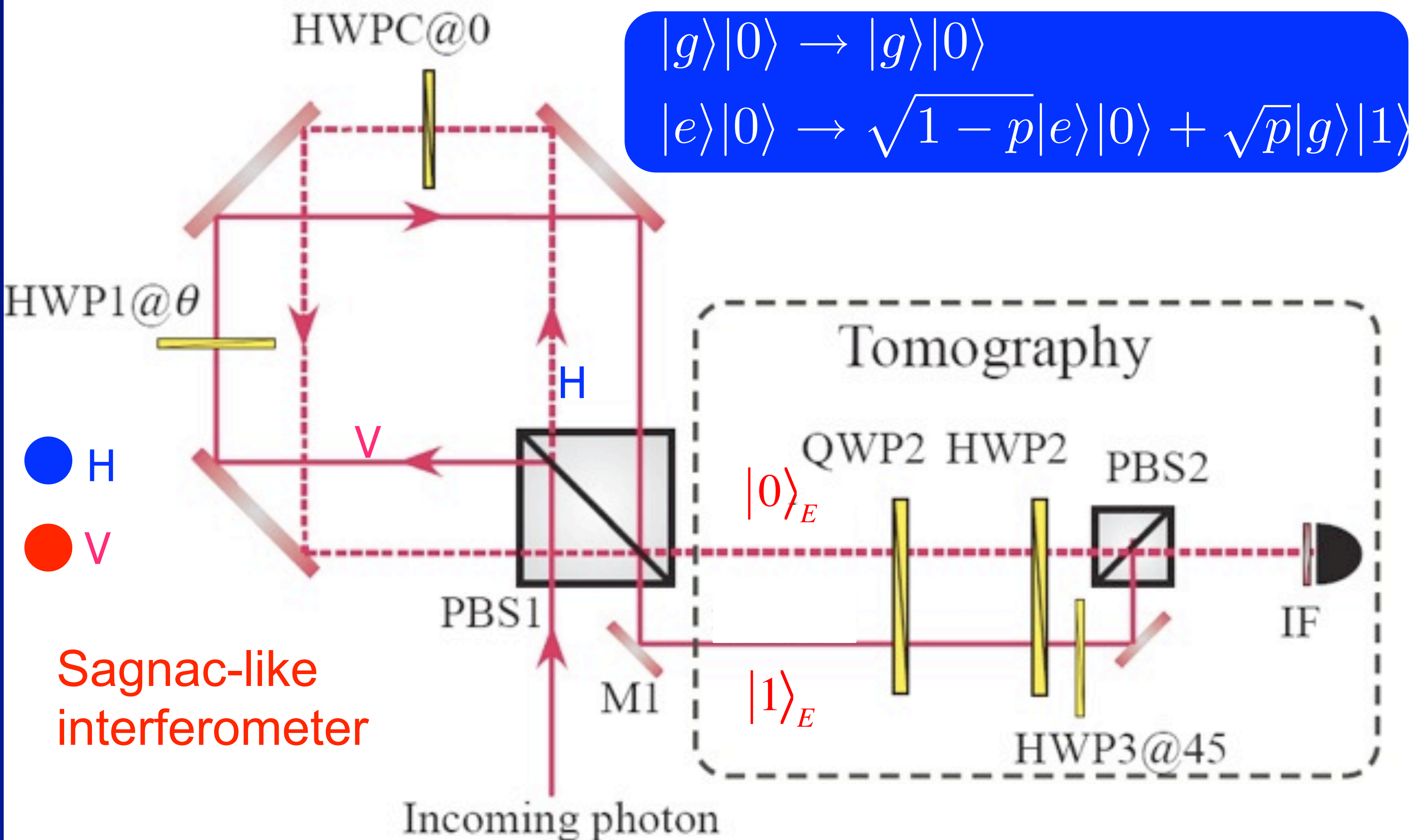
Conclusions

- Entanglement: from a puzzling quantum-mechanical effect to a useful tool: quantum communications, quantum computation, quantum metrology
- Open problems: characterization of multiparticle entanglement, physical interpretation of entanglement measures, effect of decoherence on multi-particle entanglement
- Twin-photon beams: useful for studying decoherence and disentanglement → local X global behavior of entangled states
- Quantum metrology: intense activity today


Ruynet Matos Filho
Wellison Bastos
Paulo Souto Ribeiro
Alessandro Saboia
Luiz Davidovich
Andrea Valdes
Saulo Santos
Fabricio Toscano
Bruno Taketani
Malena Hor-Meyll
Gabriel Alves
Osvaldo Farias
Steve Walborn
Bruno Escher
Nicim Zagury
Gabriela Lemos
Marcio Taddei
Irénée Frerot
Camille Lombard
Gabriel Aguillar
Rafael Gomes

Realization of amplitude map with photons

$$|g\rangle|0\rangle \rightarrow |g\rangle|0\rangle$$
$$|e\rangle|0\rangle \rightarrow \sqrt{1-p}|e\rangle|0\rangle + \sqrt{p}|g\rangle|1\rangle$$



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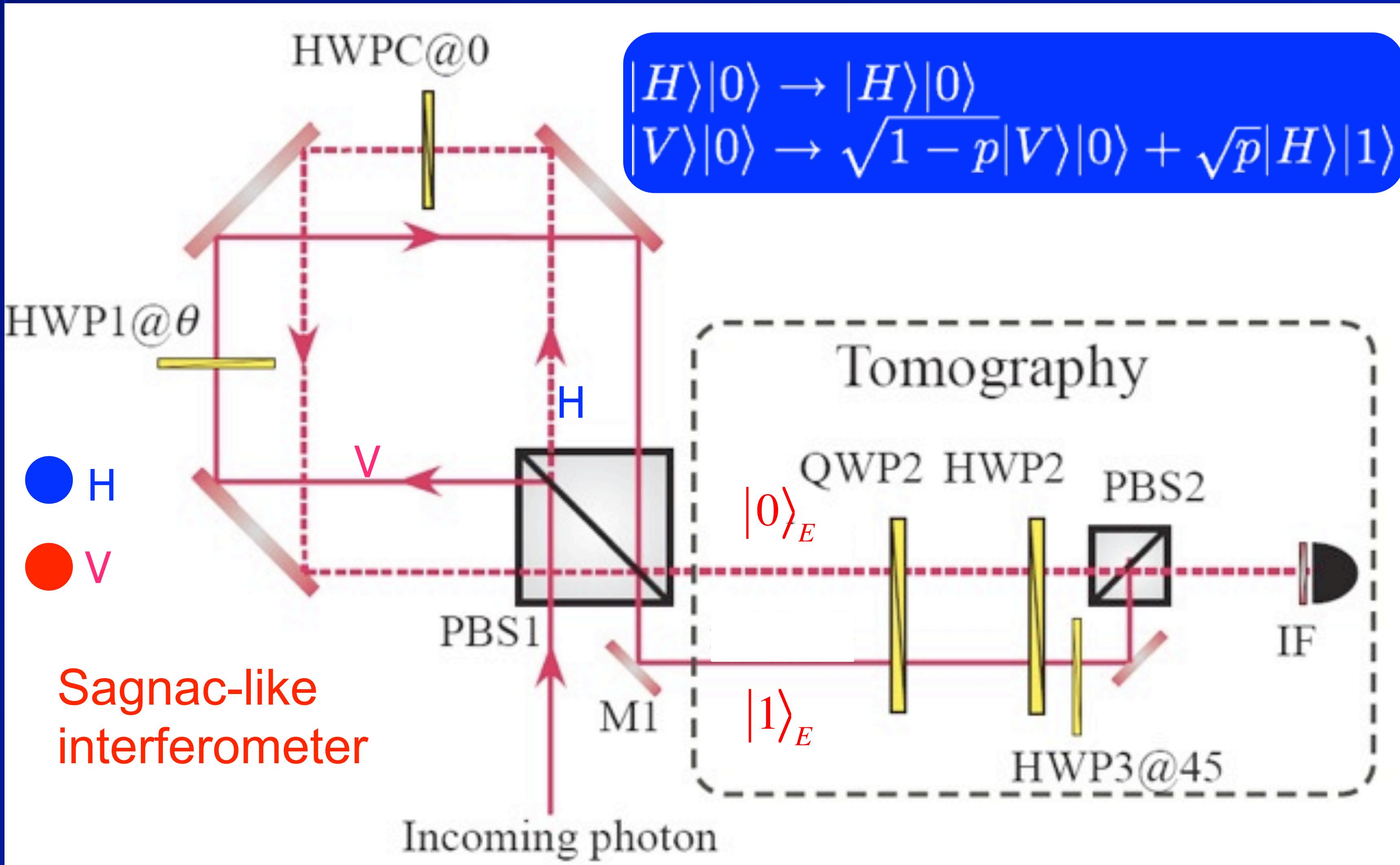

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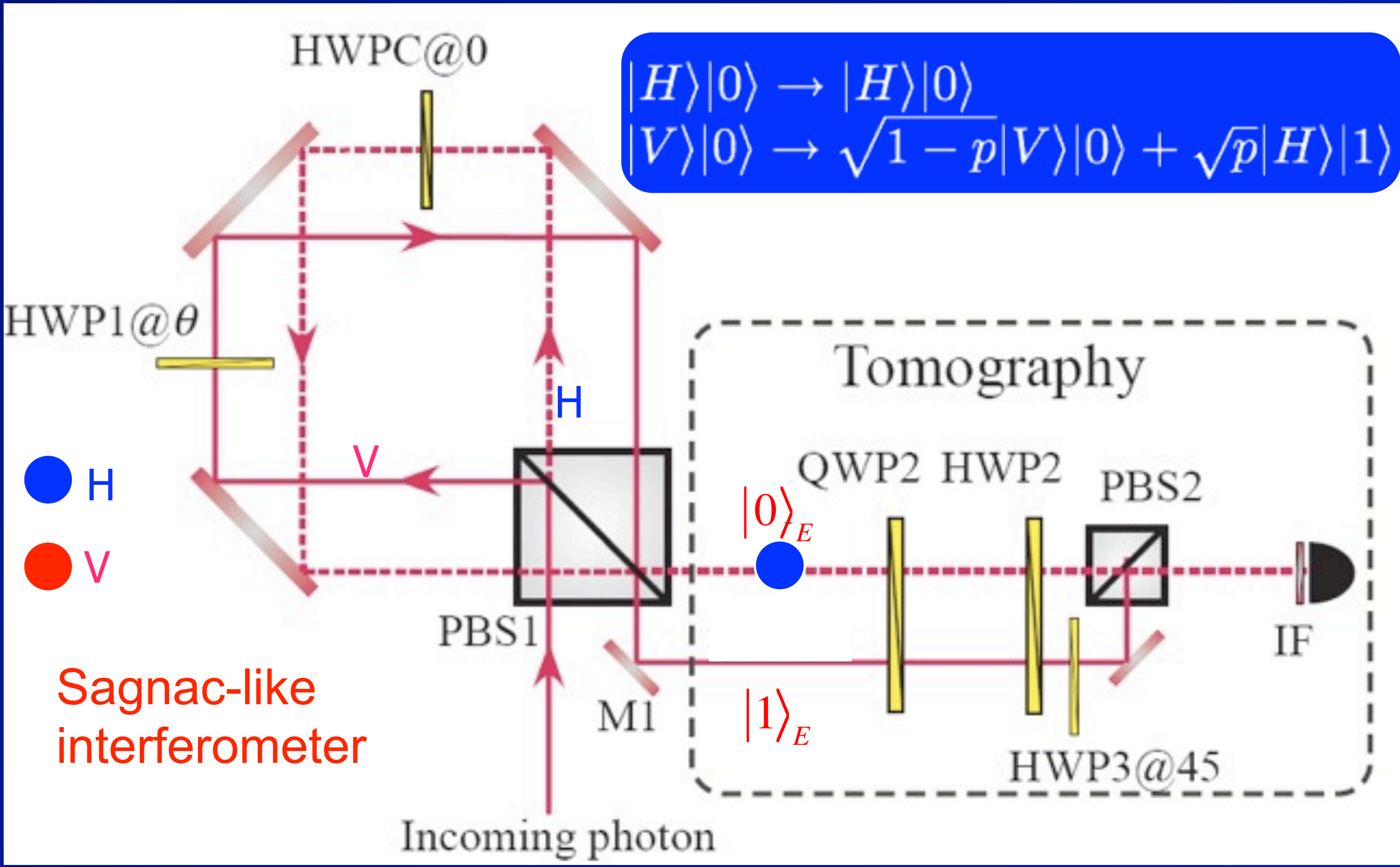

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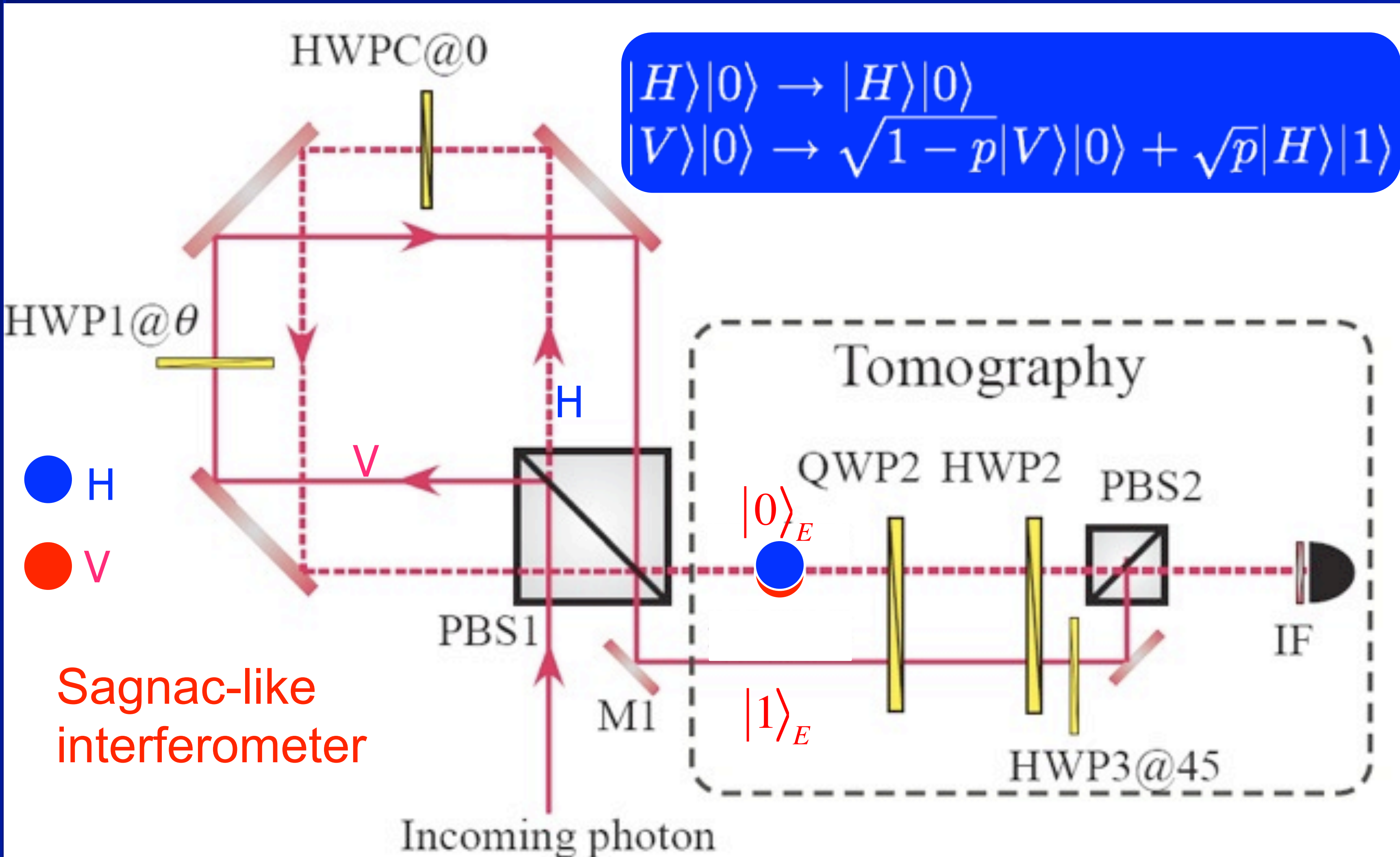

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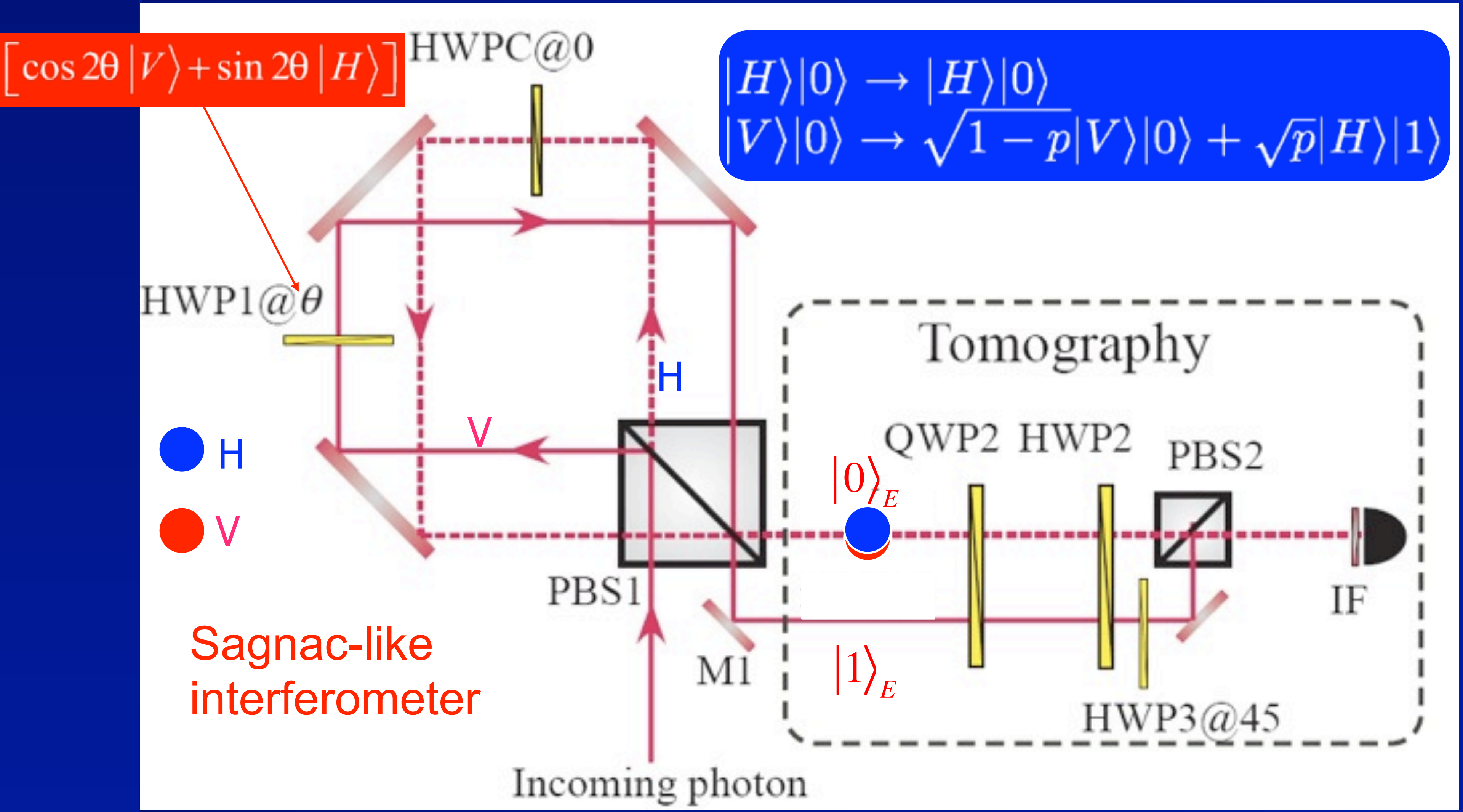

Realization of amplitude map with photons

$$[\cos 2\theta \,|V\rangle + \sin 2\theta \,|H\rangle]$$

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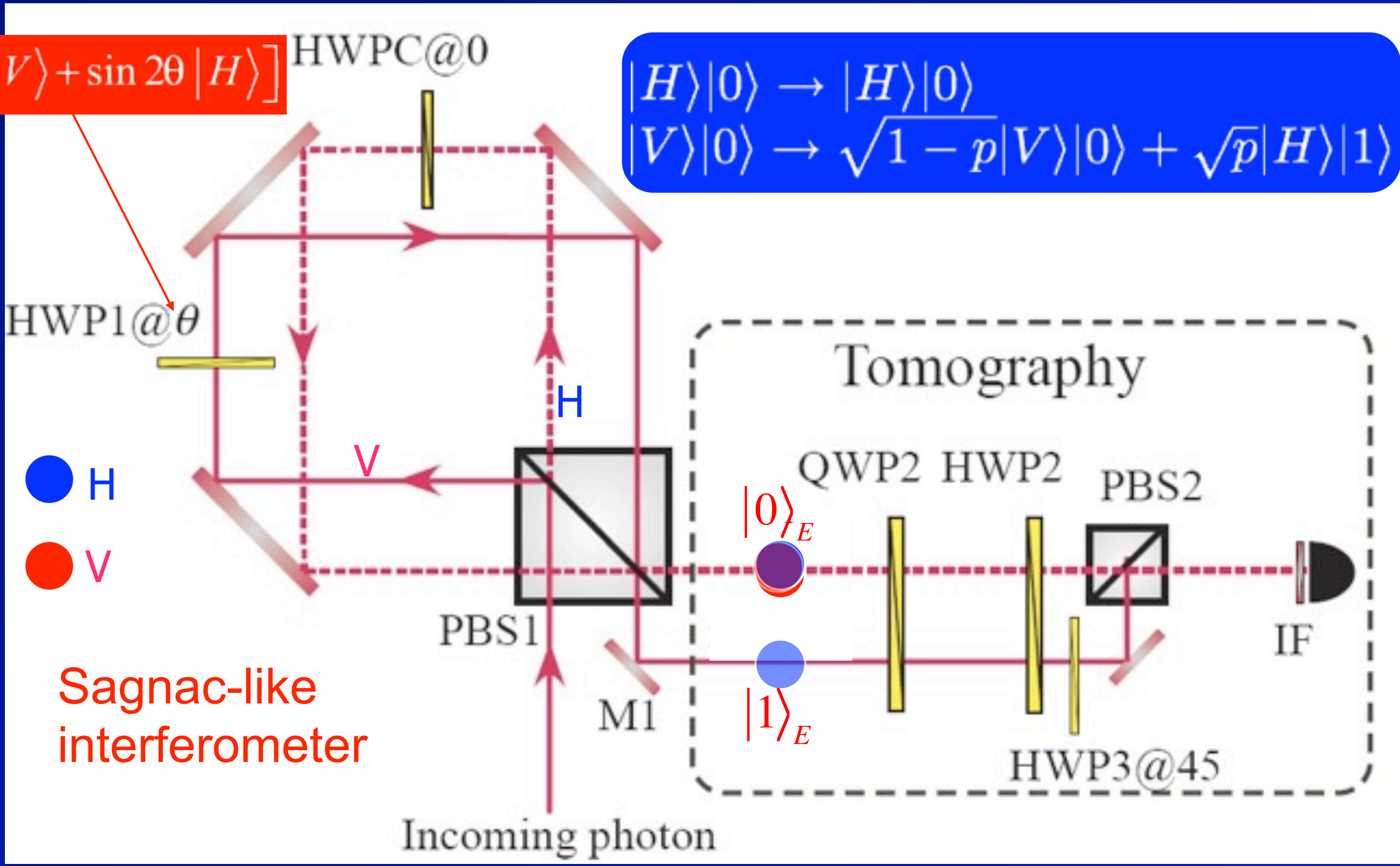
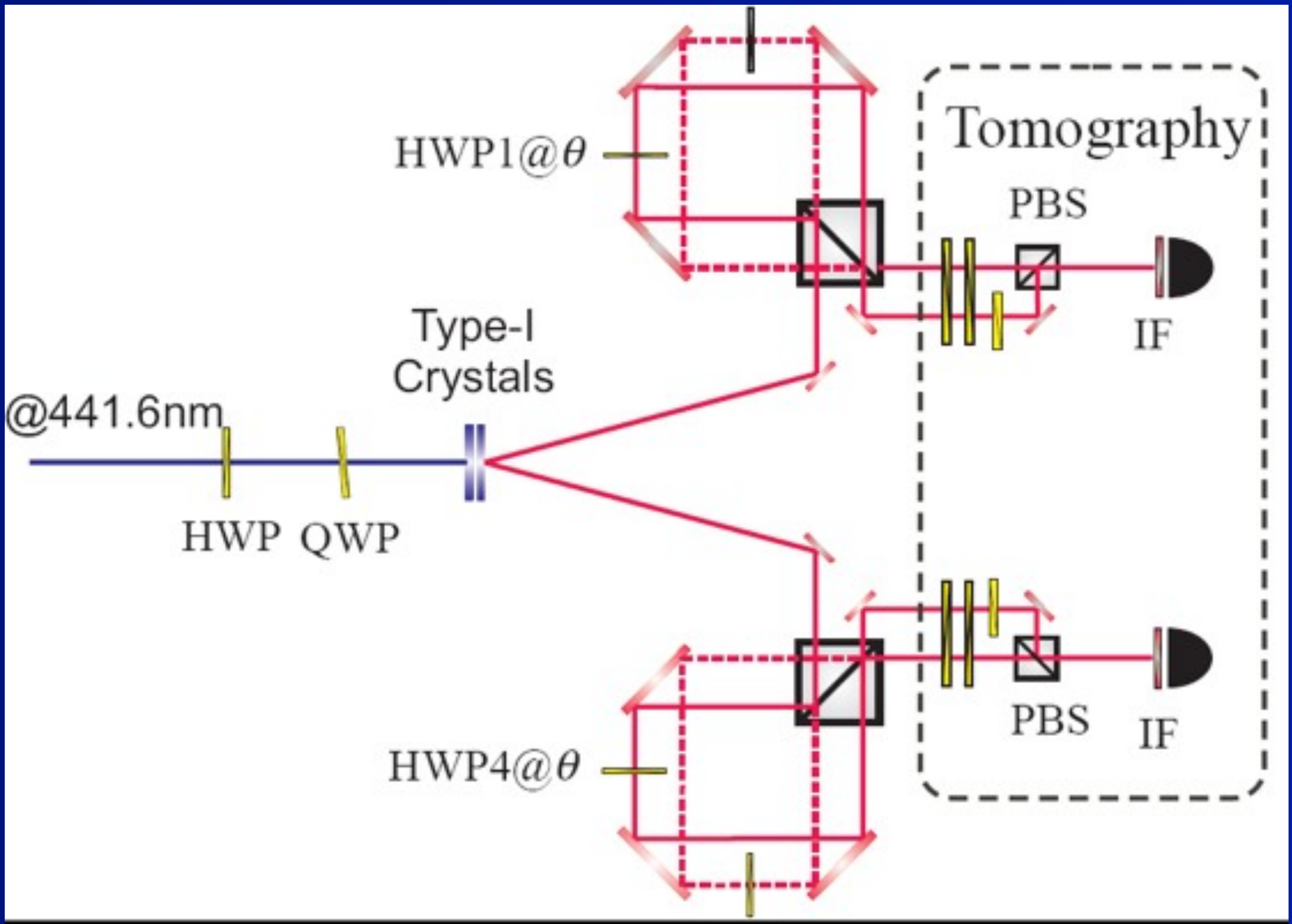
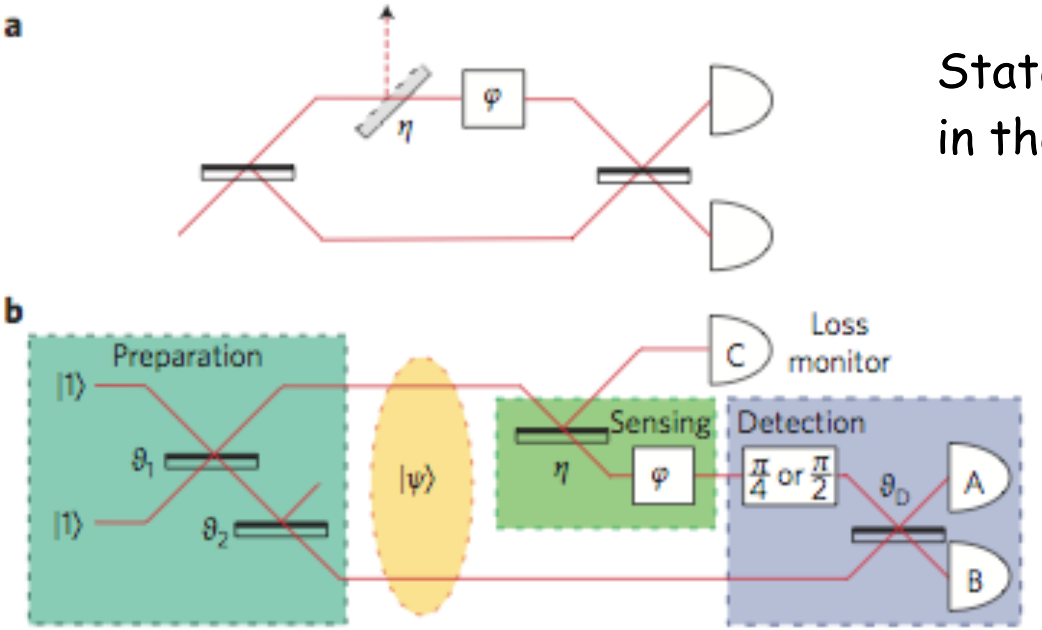


Investigating the dynamics of entanglement



Parameter estimation with losses - experiments



States leading to minimum uncertainty in the presence of noise:

$$|\psi\rangle = \sqrt{x_2}\,|20\rangle + \sqrt{x_1}\,|11\rangle - \sqrt{x_0}\,|02\rangle$$

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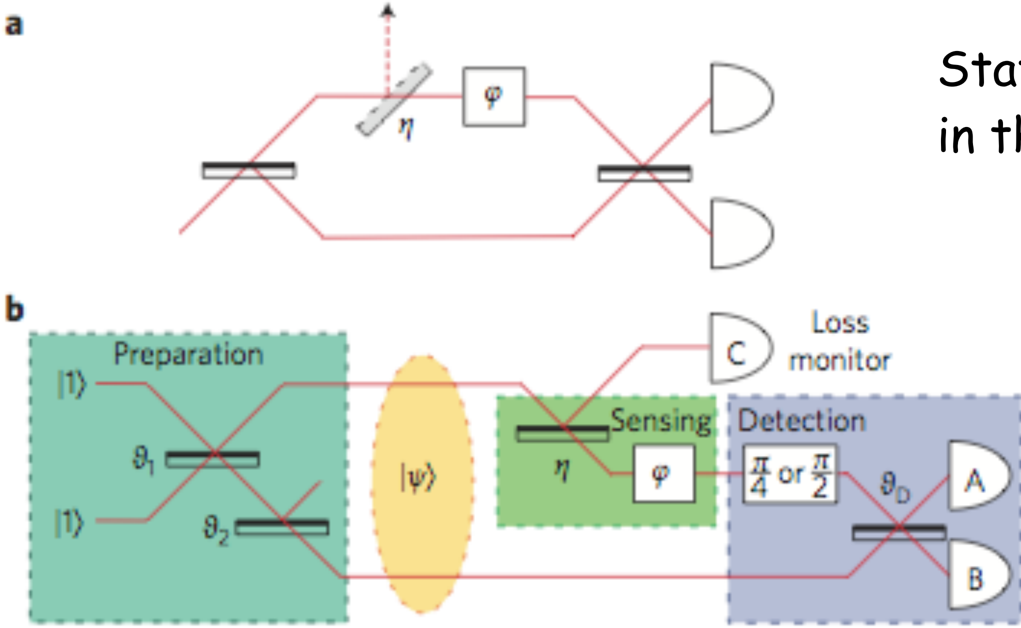

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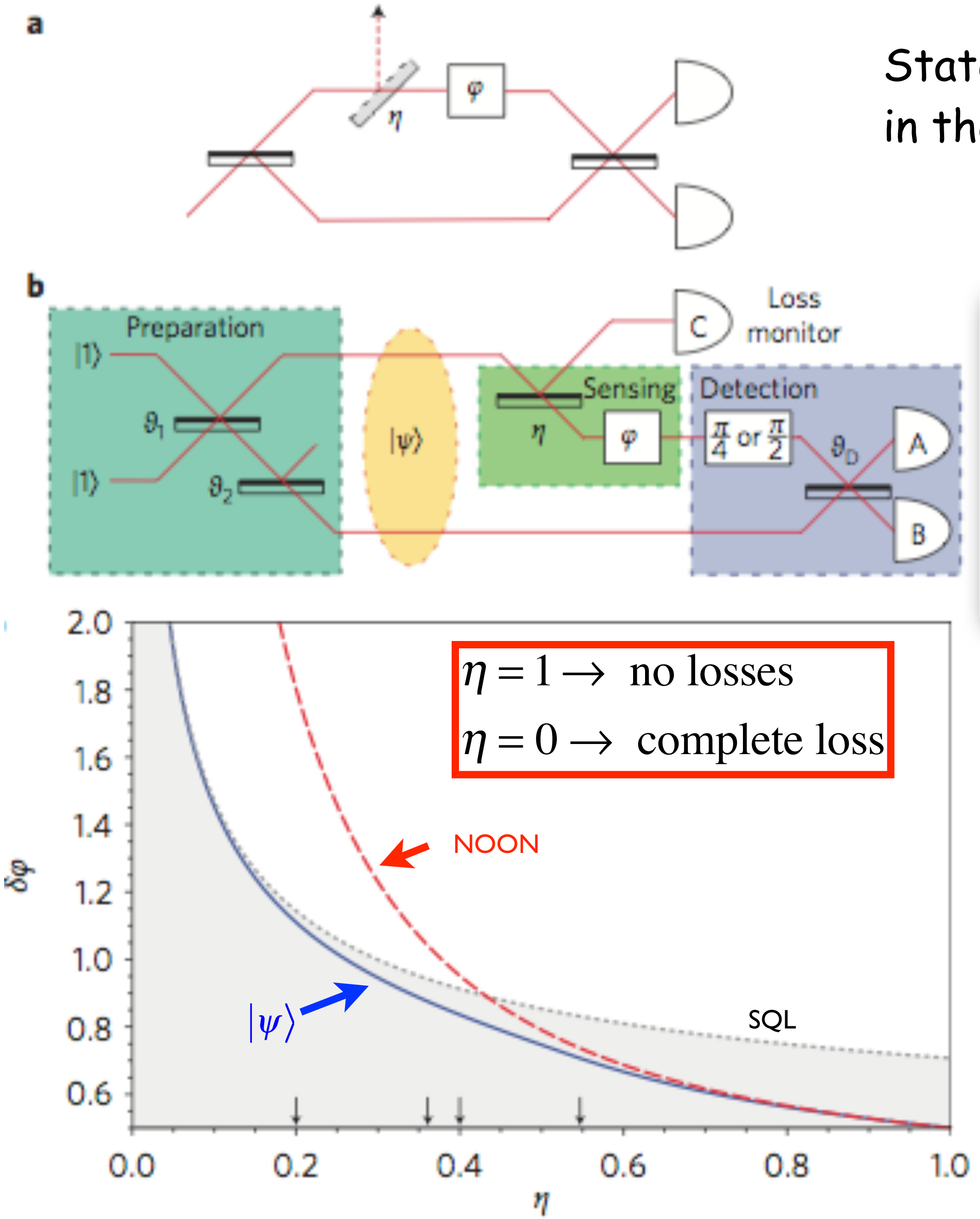

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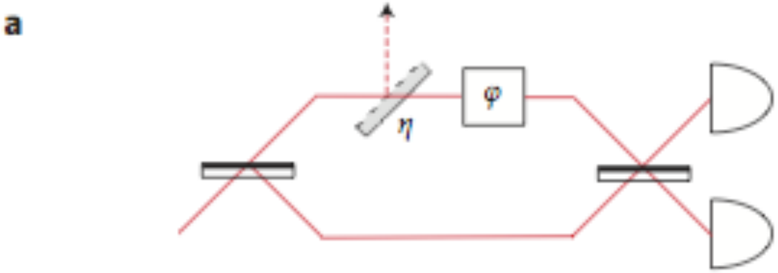

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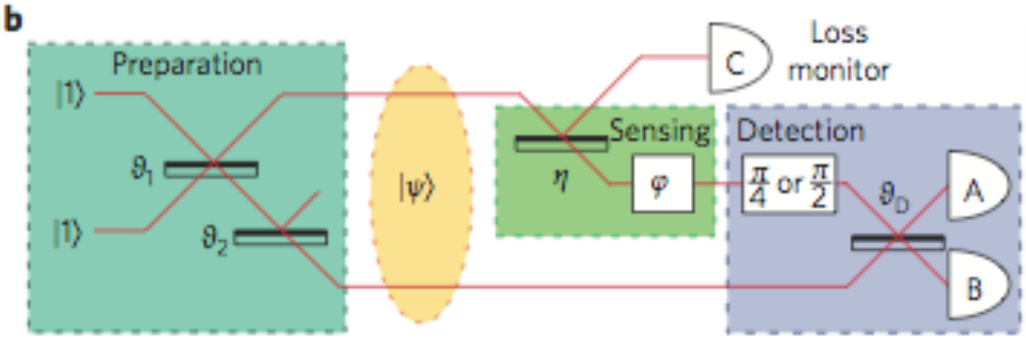

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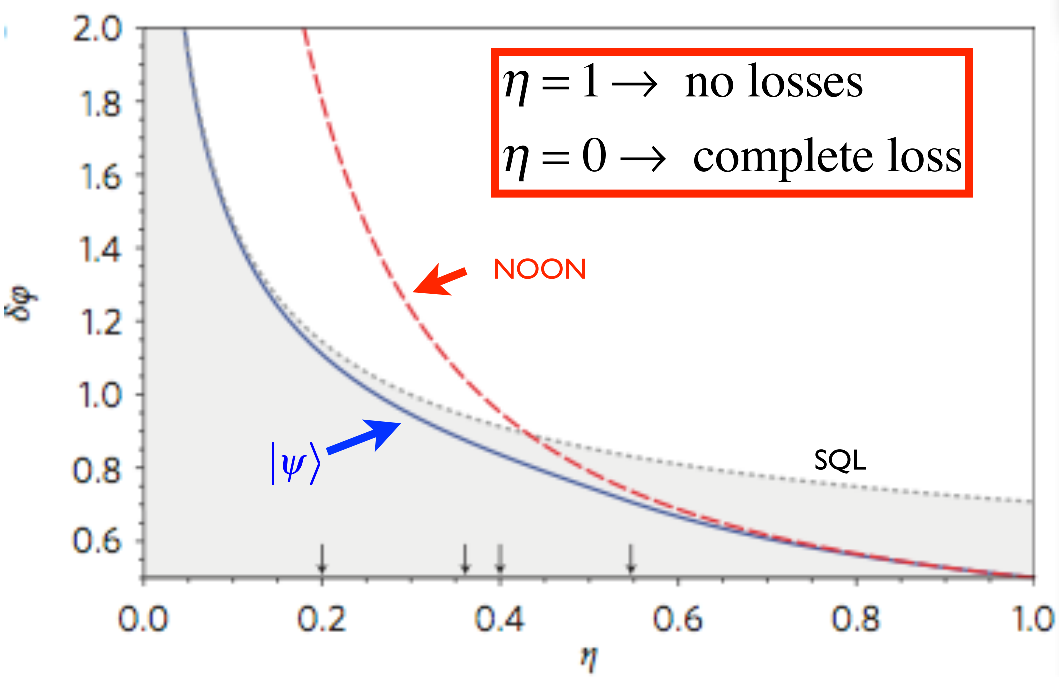

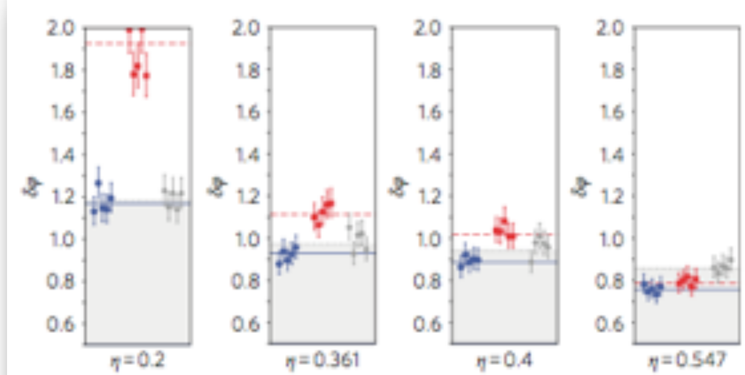

Figure 5 | Uncertainty of phase estimates. Uncertainties obtained using two-photon optimal (circles) and NOON (squares) states, as well as attenuated laser pulses in the SIL regime (diamonds), rescaled by the square root of the number of coincidences. For each transmission η, data are shown for five phases $\varphi = 0, \pm 0.2, \pm 0.4$ rad. Horizontal lines represent the theoretical Cramér-Rao bounds for given classes of input states, taking into account imperfections of the interferometer.

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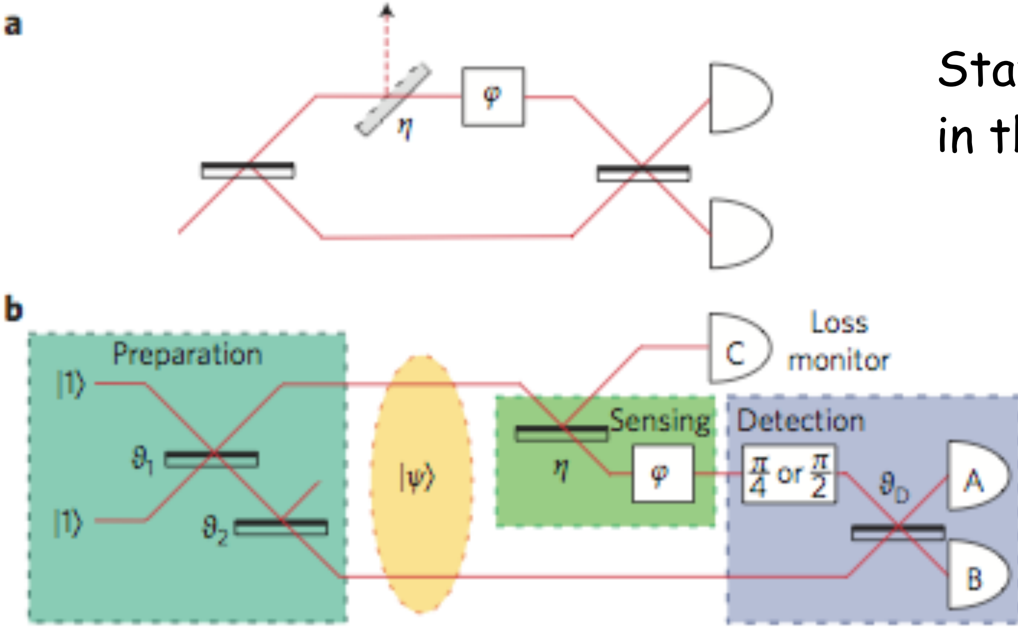

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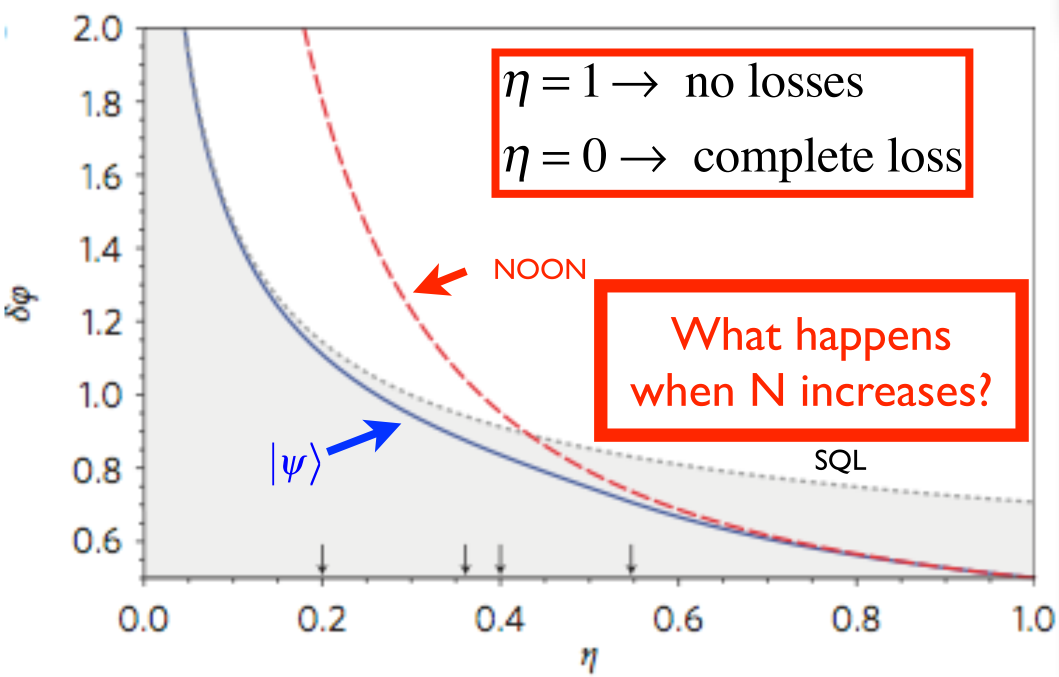

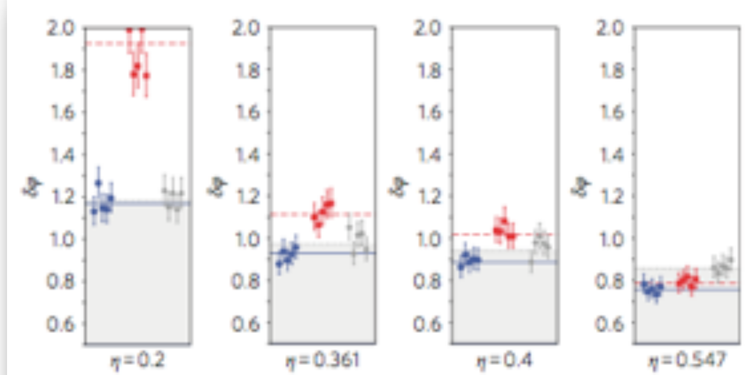

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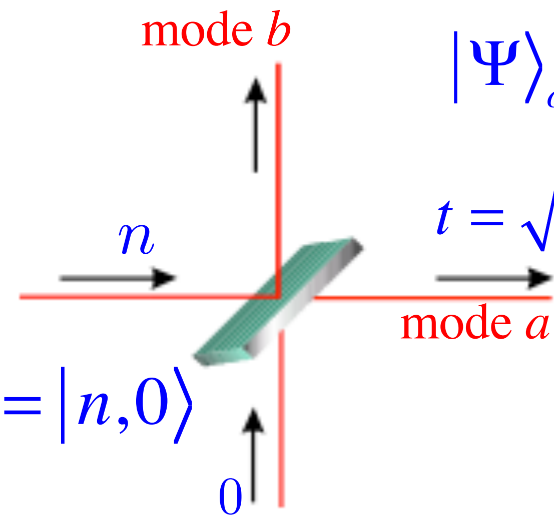
Possible model for photon loss: beam splitter

$$|\Psi\rangle_{out} = \sum_{\ell=0}^{n} \binom{n}{\ell}^{1/2} r^{\ell} t^{n-\ell} |n-\ell, \ell\rangle$$

$t = \sqrt{\eta} \rightarrow$ transmissivity, $r = \sqrt{1-t^2} \rightarrow$ reflectivity



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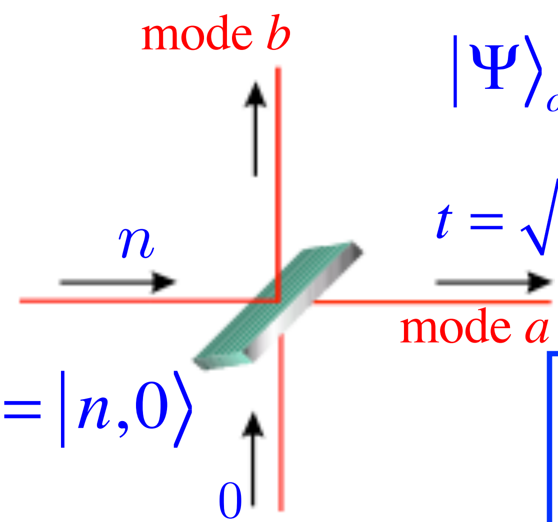

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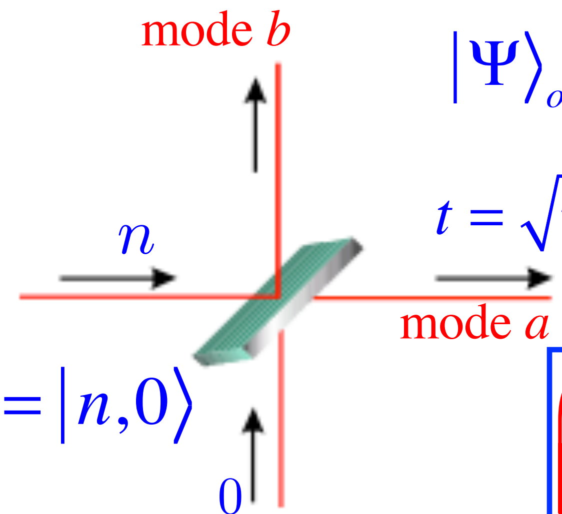

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If $|\Psi\rangle_{in} = \left(\sum_{n=0}^{\infty} a_n |n\rangle\right)_a \otimes |0\rangle_b \Rightarrow \rho_{out}^{(a)}(\eta) = \sum_{\ell=0}^{\infty} \hat{\Pi}_\ell(\eta) |\Psi^{(a)}\rangle_{in} \, {}_{in}\langle\Psi^{(a)}| \hat{\Pi}_\ell^\dagger(\eta)$ **(A)**

where $\hat{\Pi}_\ell(\eta) = \sqrt{\dfrac{(1-\eta)^\ell}{\ell!}} \eta^{\hat{n}/2} \hat{a}^\ell \Rightarrow \sum_\ell \hat{\Pi}_\ell^\dagger(\eta) \hat{\Pi}_\ell(\eta) = 1$

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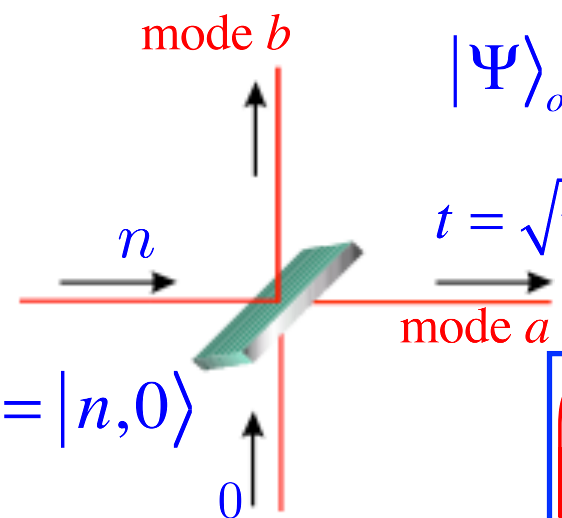

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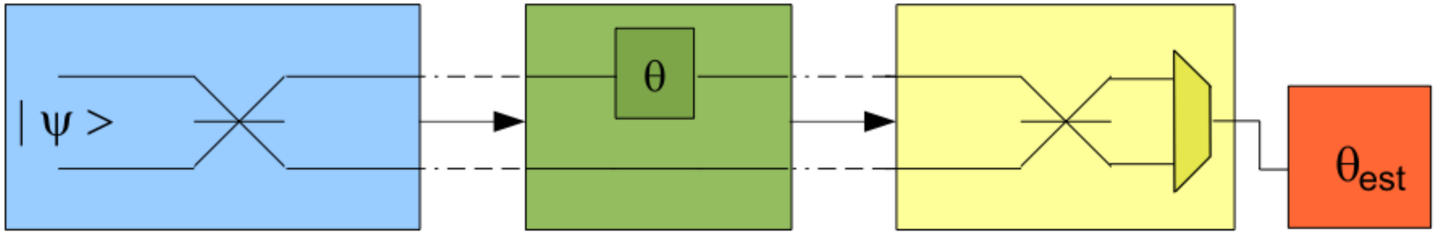
Set $\eta = \exp(-\gamma t)$, derive (A) with respect to t, find previous master equation - beam splitter is one of the possible realizations of the reservoir.

Lossy optical interferometry and Kraus operators



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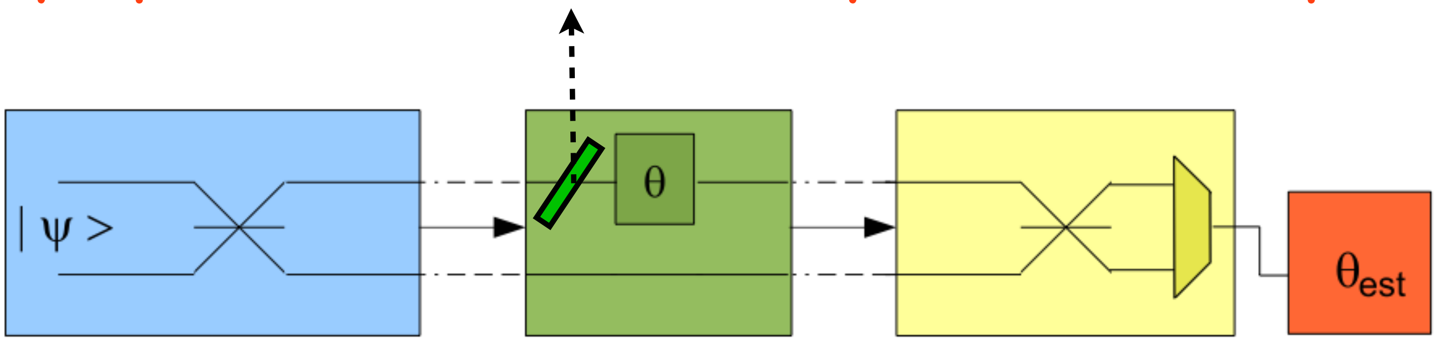


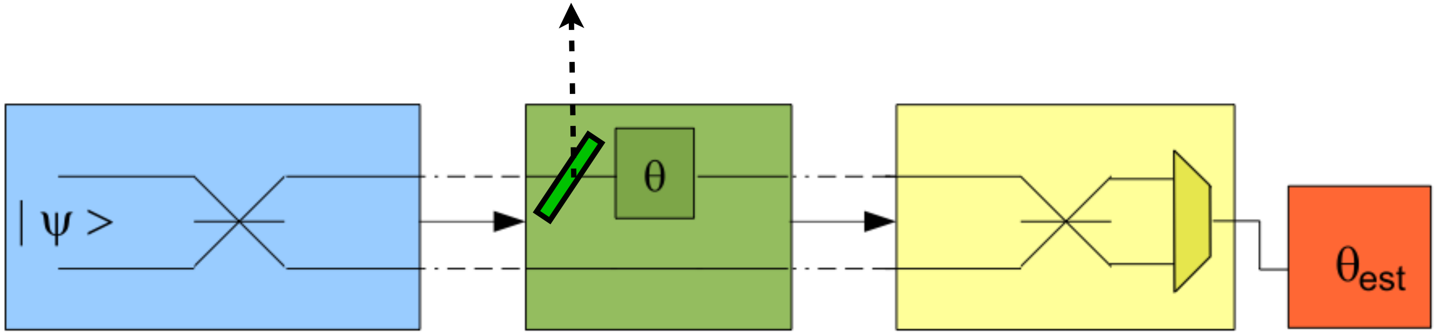
$$\hat{\Pi}_{\ell}(\theta)=\sqrt{\frac{(1-\eta)^{\ell}}{\ell!}}e^{i\theta\hat{n}}\eta^{\hat{n}/2}\hat{a}^{\ell}$$

$\rightarrow$ Beam splitter placed before dispersive element

$$\eta=e^{-\gamma t}$$

$$\theta = \omega t$$

Lossy optical interferometry and Kraus operators



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Partial recovery of information upon monitoring the environment: scattered photons do not carry phase information

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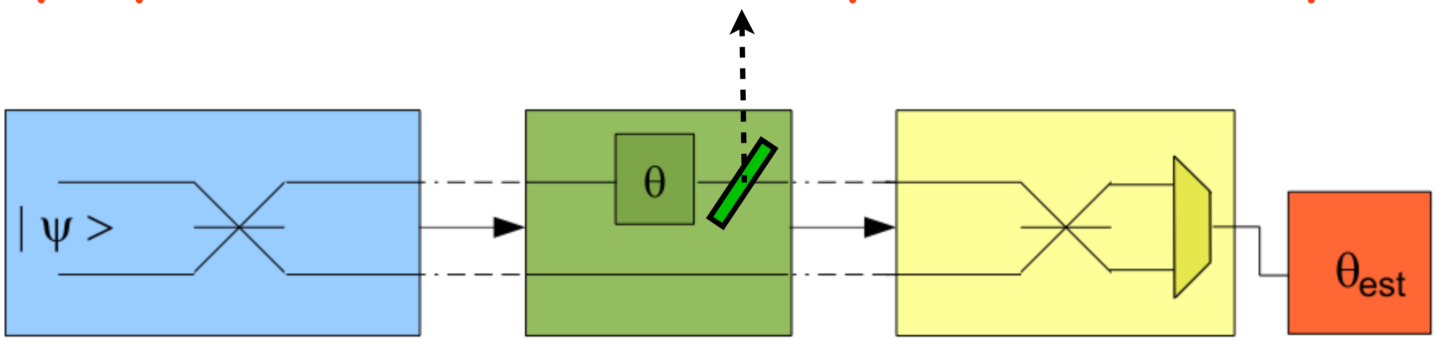

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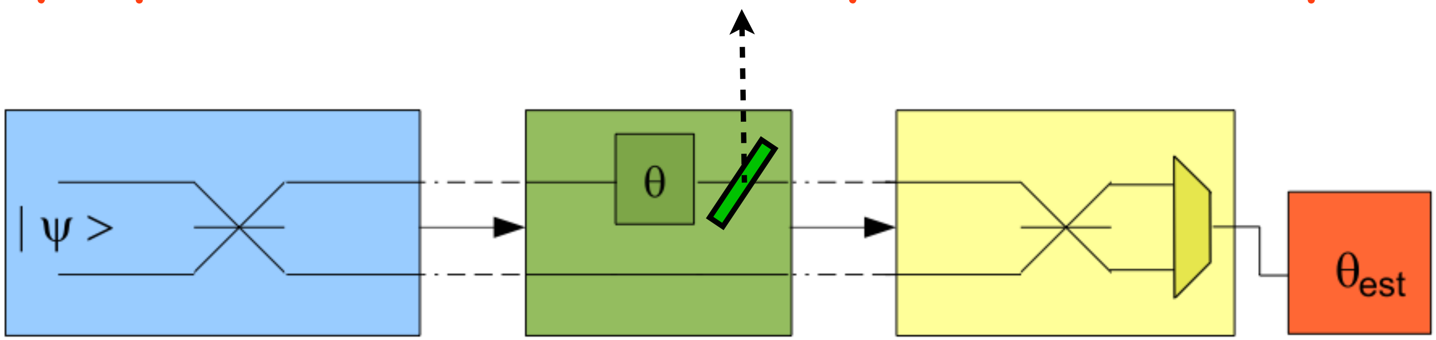

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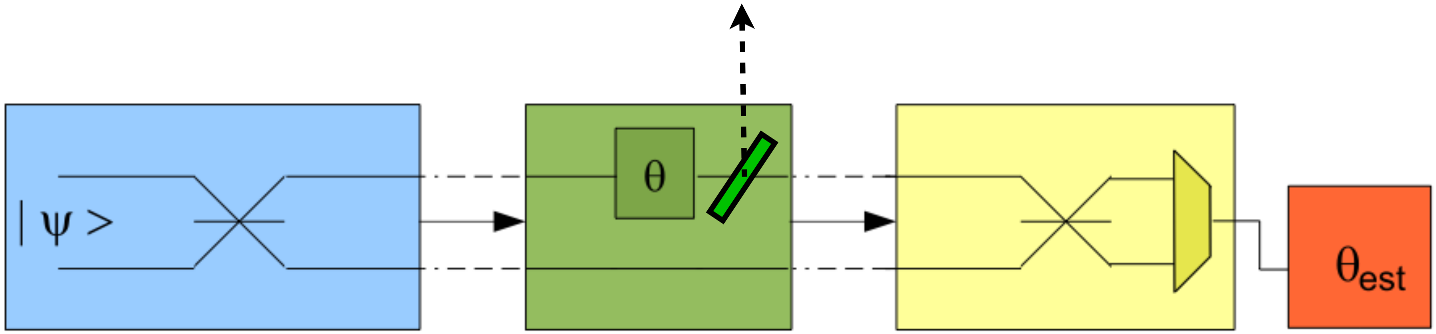

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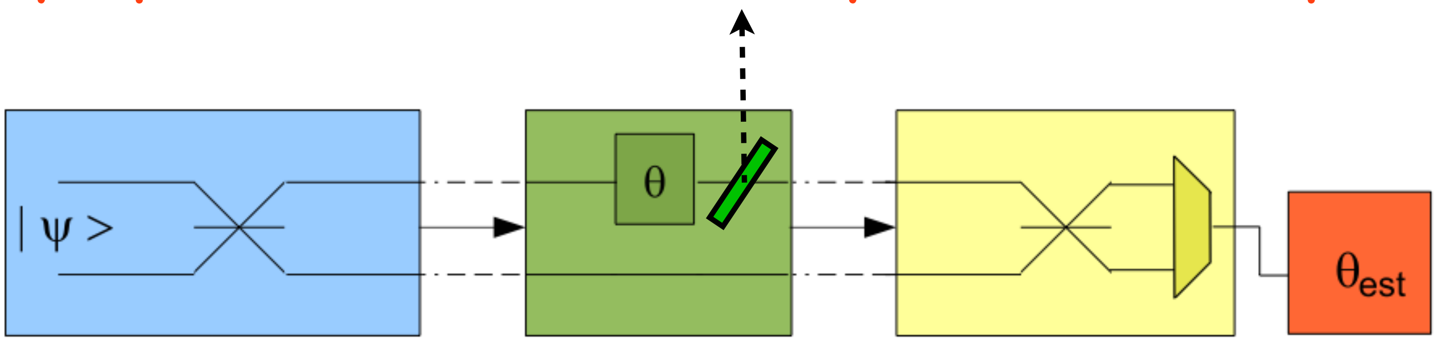
Lossy optical interferometry and Kraus operators



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Choose α that miminizes C_Q !