# Holographic superconductors and superfluids: effect of backreaction

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Distance São Carlos – Bremen:
≈ 9980 km (direct line)

Population of Bremen:
≈ 550.000
($10^{th}$ biggest city in Germany)

# Collaborations and References

Work done in collaboration with:

**Yves Brihaye** - *Université de Mons, Belgium*

References:

*Y. Brihaye and B. Hartmann, Phys. Rev. D81 (2010) 126008*
*Y. Brihaye and B. Hartmann, JHEP 1009 (2010) 002*
*Y. Brihaye and B. Hartmann, Phys. Rev. D83 (2011) 126008*

# Outline

# Outline

# The holographic principle

## The AdS/CFT correspondence (Maldacena, 1997)

A theory of **gravity** in $d$-dimensional asymptotically Anti-de Sitter ($AdS_d$) space-time is dual to a **conformal field theory** (CFT) living on the $(d-1)$-dimensional boundary of $AdS_d$.

- important result of String Theory
- **weak coupling** $\Leftrightarrow$ **strong coupling duality**
- CFT is **scale-invariant** Quantum field theory (QFT)
- $AdS_d$: Vacuum solution of $d$-dimensional Einstein's equations with **negative cosmological constant**

# Applying the AdS/CFT correspondence

use classical gravity theory (weakly-coupled) to study strongly coupled Quantum Field theories

- gravity $\Leftrightarrow$ condensed matter (Quantum phase transitions):

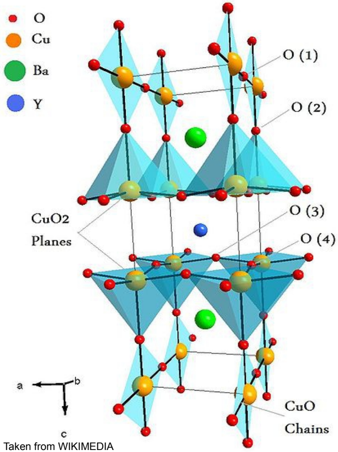
  **holographic superconductors/superfluids**

  - Superconductor/insulator phase transitions in thin metallic films
  - high temperature superconductors
  - ...
- gravity $\Leftrightarrow$ Quantumchromodynamics (QCD) (e.g. quark-gluon plasma ...)

# Quantum Phase Transitions

- phase transition (PT) at $T = 0$
- not **thermal**, but **quantum** fluctuations
  (uncertainty principle)
- PT at critical parameter, e.g. at chemical potential $\mu = \mu_c$
- Quantum critical region at $\mu \approx \mu_c$ and $T > 0$
- **believed to appear in high-$T$ superconductors**
- (believed to be) described by **strongly coupled** theories
  $\rightarrow$ "standard" (mainly weakly coupled) theories
  (BCS, Ginzburg-Landau) do **not** work well

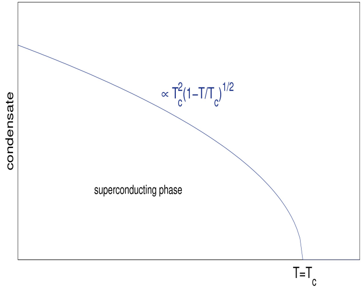
# Example of a high temperature superconductor




Taken from WIKIMEDIA

- Yttrium(Y)-barium(BA)-copper(Cu)-oxide(O)
- highest possible $T_c$ =92K (boiling point of liquid nitrogen: 77K)
- **superconductivity associated to $CuO_2$-planes**

# Holographic phase transitions




Basic ingredients

- notion of **temperature**
  $\Rightarrow$ black hole
- notion of **chemical potential**
  $\Rightarrow$ **charged** black hole
- notion of **condensate**
  $\Rightarrow$ non-trivial field outside black hole horizon

black hole for $T > T_c$
"hairy" black hole for $T < T_c$

# Properties of black holes

- **temperature** $T = \frac{\kappa}{2\pi}$ with surface gravity

$$\kappa^2 = -\frac{1}{2}(D_\mu \chi_\nu)(D^\mu \chi^\nu)\Big|_{r_h}$$

  where $\chi$ Killing

- **free energy** $\Omega$ ($\rightarrow$ canonical ensemble)

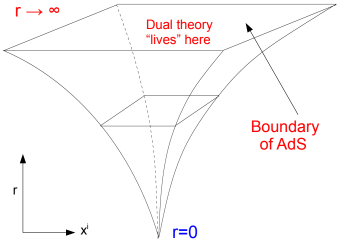
$$\Omega = TS_{\rm os}$$

  $S_{\rm os}$: action evaluated on-shell

# Planar Anti-de Sitter (AdS) space-time

- Metric in (3+1) dimensions

$$ds^2 = -\frac{r^2}{\ell^2}dt^2 + \frac{\ell^2}{r^2}dr^2 + \frac{r^2}{\ell^2}\left(dx^2 + dy^2\right)$$




Taken from arxiv: 0808.1115

- cosmological constant

$$\Lambda = -3/\ell^2$$

- Ricci scalar

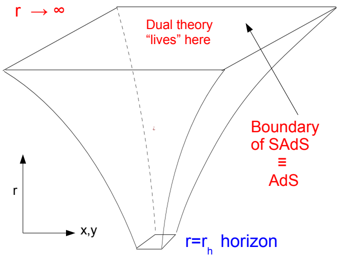
$$R = -12/\ell^2$$

$\Rightarrow \ell$ AdS radius

# Planar Schwarzschild-Anti-de Sitter (SAdS)

- Metric in (3+1) dimensions

$$ds^2 = -\left(\frac{r^2}{\ell^2} - \frac{{r_h}^3}{r\ell^2}\right) dt^2 + \left(\frac{r^2}{\ell^2} - \frac{{r_h}^3}{r\ell^2}\right)^{-1} dr^2 + \frac{r^2}{\ell^2}\left(dx^2 + dy^2\right)$$



Taken from arxiv: 0808.1115

- Black hole with planar horizon at $r = r_h$
- **temperature** $T = 3r_h/(4\pi\ell^2)$
- asymptotically $AdS_{3+1}$

# Planar Reissner-Nordström-Anti-de Sitter (RNAdS)

- Metric in (3+1) dimensions

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + \frac{r^2}{\ell^2}\left(dx^2 + dy^2\right)$$

with

$$f(r) = \frac{r^2}{\ell^2} - \frac{1}{r}\left(\frac{{r_+}^3}{\ell^2} + \frac{q^2}{4r_+}\right) + \frac{q^2}{4r^2}$$

- U(1) gauge field

$$A_M dx^M = A_t(r)dt = \frac{q}{r_+} - \frac{q}{r} = \mu\left(1 - \frac{r_+}{r}\right)$$

- planar (outer) horizon at $r = r_+$, asymptotically $AdS_{3+1}$
- $q \propto$ charge density, $\mu$ chemical potential
- **temperature** $T = 3r_+/(4\pi\ell^2) - \mu^2/(4r_+)$

# Anti-de Sitter soliton

- **double Wick rotation** ($t \to i\eta$, $y \to it$) of SAdS
- metric in (3+1) dimensions

$$ds^2 = -\frac{r^2}{\ell^2}dt^2 + \left(\frac{r^2}{\ell^2} - \frac{r_0{}^3}{r\ell^2}\right)^{-1} dr^2 + \left(\frac{r^2}{\ell^2} - \frac{r_0{}^3}{r\ell^2}\right) d\eta^2 + \frac{r^2}{\ell^2}dx^2$$

- to avoid conical singularity $\to \eta$ **periodic** with period

$$\tau_\eta = \frac{4\pi\ell^2}{3r_0} \qquad \text{where } r_0 > 0$$

- space-time unchanged for $A_M dx^M = \mu dt$
- can be considered at **any temperature** $T$
- **unstable** to decay to SAdS for large $T$
(Surya, Schleich & Witt, 2001)

# Formation of scalar hair on AdS black hole

- Action ($M, N = 0, ..., d-1$)

$$S = \int d^d x \sqrt{-g} \left[ \frac{R - 2\Lambda}{16\pi G} - (\partial_M \psi)^* \partial^M \psi - m^2 \psi^* \psi \right]$$

$\psi$: complex scalar field
$m$: mass of scalar field
$G$: Newton's constant
$\Lambda$: (negative) cosmological constant

- **Breitenlohner-Freedman (BF) bound**: $AdS_d$ stable against scalar hair formation for

$$m^2 \geq m^2_{\rm BF} = -\frac{(d-1)^2}{4\ell^2}$$

(Breitenlohner & Freedman, 1982)

# Formation of scalar hair on AdS black hole

- Action ($M, N = 0, ..., d-1$)

$$S = \int d^dx\sqrt{-g}\left[\frac{R-2\Lambda}{16\pi G} - (D_M\psi)^* D^M\psi - m^2\psi^*\psi - \frac{1}{4}F_{MN}F^{MN}\right]$$

$F_{MN} = \partial_M A_N - \partial_N A_M$: field strength of U(1) gauge field $A_M$
$D_M\psi = \partial_M\psi - ieA_M\psi$: covariant derivative
$e$: gauge coupling

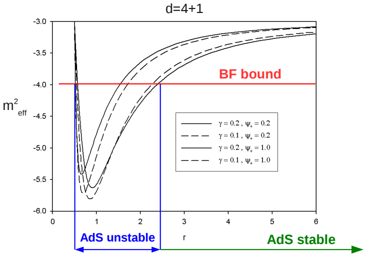
- effective mass of scalar field for $A_i = 0$, $i = 1, ..., d-1$

$$m^2_{\mathrm{eff}} = m^2 - e^2|g^{tt}|A_t^2$$

- $e^2|g^{tt}|$ sufficiently large $\Rightarrow$ **unstable**

# Formation of scalar hair on AdS black hole

- Example (Y. Brihaye & B.Hartmann, Phys.Rev. D81 (2010) 126008)




- Pioneering example: RNAdS black hole close to $T = 0$ unstable to form **scalar hair** close to horizon (Gubser, 2008)

# Formation of scalar hair on AdS soliton

- ground state energy of scalar field in AdS soliton background **finite and positive** $\Rightarrow$ **energy gap** (Witten, 1998)
- coupling to U(1) gauge field:

$$A_M dx^M = \phi(r) dt \quad \text{with} \quad \phi(r \to \infty) \to \mu$$

  **decreases** ground state energy
- for $T$ small and $\mu > \mu_c$ soliton unstable to form scalar hair (Nishioka, Ryu & Takayanagi, 2010; Horowitz & Way, 2010; Brihaye & B. Hartmann, 2011)

# AdS/CFT “dictionary” in (3+1) dimensions

(Hartnoll, Herzog & Horowitz, 2008)

- formation of scalar hair on (3+1)-dimensional AdS black hole/soliton dual to onset of superconductivity in (2+1) dimensions
- U(1) gauge field dual to global U(1) field with

$$A_t(r \gg 1) = \mu - \frac{q}{r} + ....$$

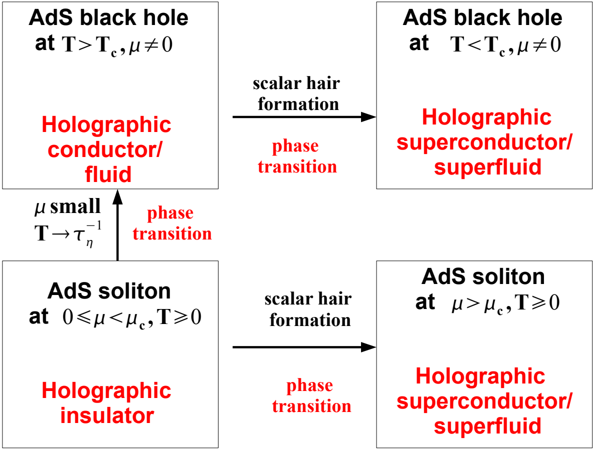
$\mu$: dual to chemical potential , $q$: dual to charge density

- scalar field with $m^2 > m^2_{\rm BF} = -9/(4\ell^2)$:

$$\psi(r \gg 1) = \frac{\psi_-}{r} + \frac{\psi_+}{r^2} + ... \quad \text{for} \quad m^2 = -2/\ell^2$$

$\psi_\pm$: dual to expectation value of operator $\mathcal{O}_\pm$
$\Rightarrow$ value of condensate

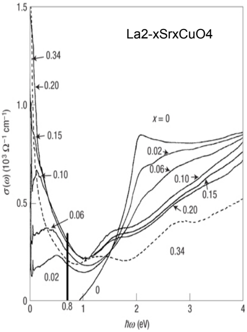
# Holographic phase transitions



# Outline

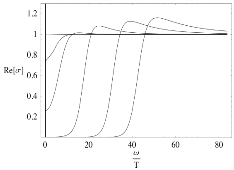
# First results on holographic superconductors

- optical conductivity: conductivity $\sigma$ as function of frequency $\omega$




Experimental

Uchida et. al (1991)




Theoretical

Hartnoll, Herzog, Horowitz (2008)

# First results on holographic superconductors

- **London equation** $\vec{E} \propto \frac{\partial \vec{J}}{\partial t}$ fulfilled
- boundary magnetic fields not dynamical:
  **Meissner effect** difficult to model
- Correlation length $\xi T_c \approx 0.1(1 - T/T_c)^{-1/2}$
  (Horowitz, and Roberts, 2008)
- Holographic superconductors are type II
  ... just like most high-T superconductors...
  $\Rightarrow$ formation of **vortices** with quantized magnetic flux
  (Montull, Pomarol, and Silva, 2009)

# First results on holographic superconductors

- **frequency gap** $\omega_g \hat{=}$ energy required to break Cooper pair
  - from BCS theory:
    $\omega_g / T_c \approx 3.5$
  - Holographic superconductors:
    $\omega_g / T_c \approx 8$
    (Hartnoll, Herzog, Horowitz, 2008)
  - compare to **experimental** $Bi_2Sr_2CaCu_2O_{8+x}$ values:
    $\omega_g / T_c = 7.9 \pm 0.5$
    (Gomes et al., 2007)

# One problem though...

- **Mermin-Wagner theorem**: spontaneous symmetry breaking forbidden in (2+1) dimensions at finite temperature, but holographic superconductors have been constructed
- BUT: Einstein gravity (..used mostly..) corresponds to large $N$ limit on QFT side

**Q: Can higher curvature corrections (e.g. Gauss-Bonnet terms) suppress condensation?**

# The model

**Gauss-Bonnet gravity** in d-dimensional Anti-de Sitter ($AdS_d$)

$$S = \frac{1}{16\pi G}\int d^d x\sqrt{-g}\left(R - 2\Lambda + \frac{\alpha}{2}\mathcal{L}_{\rm GB} + 16\pi G\mathcal{L}_{\rm m}\right) + S_{\rm ct}$$

Gauss-Bonnet Lagrangian

$$\mathcal{L}_{\rm GB} = \left(R^{MNKL}R_{MNKL} - 4R^{MN}R_{MN} + R^2\right)$$

$S_{\rm ct}$: boundary **counterterm** - necessary to make action finite
$G$: Newton's constant
$\Lambda = -(d-1)(d-2)/(2\ell^2)$: cosmological constant
$\alpha$: Gauss–Bonnet coupling

# The model

Lagrangian of **charged complex scalar field**:

$$\mathcal{L}_{\rm m} = -\frac{1}{4} F_{MN} F^{MN} - (D_M \psi)^* D^M \psi - m^2 \psi^* \psi \ , \quad M, N = 0, 1, 2, 3, d-1$$

U(1) field strength tensor

$$F_{MN} = \partial_M A_N - \partial_N A_M$$

covariant derivative

$$D_M \psi = \partial_M \psi - ie A_M \psi$$

$e$: gauge coupling
$m$: mass of scalar field

# Outline

# The Ansatz for AdS black holes

- Metric

$$ds^2 = -f(r)a^2(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2\left(dx^2 + dy^2 + dz^2\right)$$

with $f(r_h) = 0$ at **horizon** $r = r_h$

- **Electric field** only

$$A_M dx^M = \phi(r)dt$$

- Gauge freedom: scalar field chosen to be **real**

$$\psi = \psi(r)$$

# Parameters

- Equations invariant under rescalings

$$\psi \to \psi/e \quad , \quad \phi \to \phi/e \quad , \quad G \to e^2 G$$
$$r \to \ell r \quad , \quad (t,x,y,z) \to (t,x,y,z)/\ell \quad , \quad f \to \ell^2 f \quad , \quad \phi \to \ell\phi$$

$\Rightarrow e = \ell \equiv 1$ **without loss of generality**

- choose $m^2 = -\frac{(d-2)}{\ell^2} > m^2_{\rm BF} = -\frac{(d-1)^2}{4\ell^2}$ for $d = 4, 5$
- define $\gamma = 8\pi G/e^2$

# Probe limit vs. Backreaction

- $G = 0$ ($e = \infty$) **"probe limit"**: fixed space-time background
  $\Rightarrow$ coupled scalar & gauge field equations
- $G \neq 0$ ($e < \infty$): **backreaction** of matter fields on space-time
  $\Rightarrow$ coupled Einstein, scalar & gauge field equations
- results in systems of coupled, nonlinear ordinary or partial differential equations that have to be solved **numerically**

# Equations of motion

$$f' = 2r\frac{-f+2r^2}{r^2-2\alpha f} - \gamma\frac{r^3}{2fa^2}\left(\frac{2\phi^2\psi^2+f(2m^2a^2\psi^2+\phi'^2)+2f^2a^2\psi'^2}{r^2-2\alpha f}\right)$$

$$a' = \gamma\frac{r^3(\phi^2\psi^2+a^2f^2\psi'^2)}{af^2(r^2-2\alpha f)}$$

$$\phi'' = -\left(\frac{3}{r}-\frac{a'}{a}\right)\phi' + 2\frac{\psi^2}{f}\phi$$

$$\psi'' = -\left(\frac{3}{r}+\frac{f'}{f}+\frac{a'}{a}\right)\psi' - \left(\frac{\phi^2}{f^2a^2}-\frac{m^2}{f}\right)\psi$$

# Conditions at the horizon $r = r_h$

- Horizon $r = r_h$

$$f(r_h) = 0$$

- Regularity of matter fields on horizon

$$\phi(r_h) = 0 \ , \ \ \psi'(r_h) = \left. \frac{m^2 \psi r^2}{4r/\ell^2 - \gamma r^3 \left(m^2 \psi^2 + \phi'^2/(2a^2)\right)} \right|_{r=r_h}$$

# Conditions on the AdS boundary $r \gg 1$

- Electric potential

$$\phi(r \gg 1) = \mu - q/r^2$$

$\mu$: chemical potential
$q$: charge density

- Scalar field

$$\psi(r \gg 1) = \frac{\psi_-}{r^{\lambda_-}} + \frac{\psi_+}{r^{\lambda_+}}$$

with

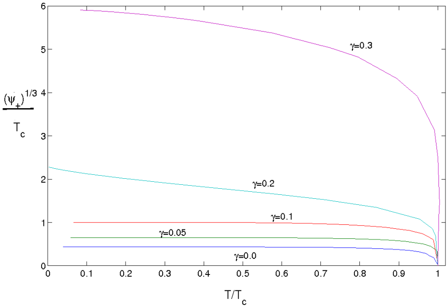
$$\lambda_\pm = 2 \pm \sqrt{4 - 3\tilde{\alpha}^2} \ , \ \tilde{\alpha}^2 \equiv \frac{2\alpha}{1 - \sqrt{1 - 4\alpha}} \ , \ \alpha \leq 1/4$$

$\psi_\pm$: value of condensate in dual theory

# Holographic superconductors: backreaction

(Y. Brihaye & B.Hartmann, Phys.Rev. D81 (2010) 126008)

- Value of condensate increases with increasing $\gamma$



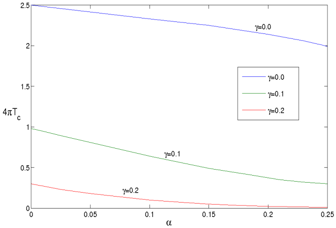
# Holographic superconductors: critical temperature $T_c$

(Y. Brihaye & B.Hartmann, Phys.Rev. D81 (2010) 126008)

- temperature $T_c$ of conductor/superconductor phase transition
  - for $\alpha = 0$:

$$T_c \approx 0.198 \cdot \exp(-10.6\gamma) q^{1/3}$$

  - for $\alpha \neq 0$:



# Outline

# The Ansatz for AdS black strings/AdS solitons

- $\alpha = 0$ in the following
- Metric of a (3+1)-dimensional **rotating AdS black string (BS)**

$$ds^2 = -b(\rho)dt^2 + \frac{1}{f(\rho)}d\rho^2 + \rho^2\left(g(\rho)dt - d\varphi\right)^2 + p(\rho)dz^2$$

with $f(\rho_h) = b(\rho_h) = 0$ at horizon $\rho = \rho_h$

- Metric of a (3+1)-dimensional **rotating AdS soliton (S)**

$$ds^2 = -p(\rho)dt^2 + \frac{1}{f(\rho)}d\rho^2 + b(\rho)\left(g(\rho)dt - d\eta\right)^2 + \rho^2 dz^2$$

with $f(\rho_0) = b(\rho_0) = 0$ and $\eta$ **periodic** with period

$$\tau_\eta = \frac{4\pi}{\sqrt{b'(\rho_0)f'(\rho_0)}}$$

# The Ansatz

- U(1) gauge field for black strings

$$A_M dx^M = \phi(\rho) dt + A(\rho) d\varphi$$

- U(1) gauge field for AdS solitons

$$A_M dx^M = \phi(\rho) dt + A(\rho) d\eta$$

- Gauge freedom: scalar field chosen to be **real**

$$\psi = \psi(\rho)$$

# Conditions on the AdS boundary $\rho \gg 1$

- U(1) potential

$$\phi(\rho \gg 1) = \mu - q/\rho \quad , \quad A(\rho \gg 1) = \sigma - \tilde{q}/\rho$$

$\mu$: chemical potential
$q$: electric charge density

$\sigma$: superfluid velocity
$\tilde{q}$: magnetic charge density

- Scalar field

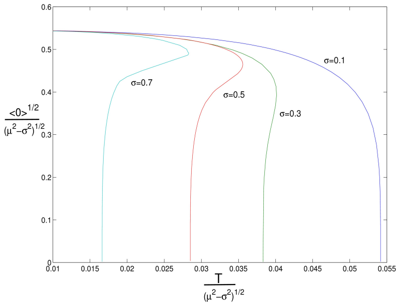
$$\psi(\rho \gg 1) = \frac{\psi_-}{\rho} + \frac{\psi_+}{\rho^2}$$

$\psi_\pm$: value of condensate in dual theory

# Holographic superfluids: order of phase transition

(Y. Brihaye & B. Hartmann, JHEP 1009 (2010) 002 )

- Probe limit $\gamma = 0$, fluid/BS superfluid phase transition

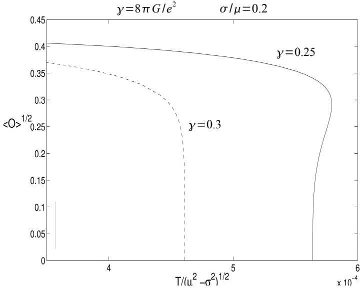



- 2nd order phase transition for $\sigma$ small
- 1st order phase transition for $\sigma$ large
- temperature $T_c$ decreases with increasing $\sigma$

# Holographic superfluids: order of phase transition

(Y. Brihaye & B. Hartmann, Phys. Rev. D83 (2011) 126008 )

- $\gamma \neq 0$, fluid/BS superfluid phase transition

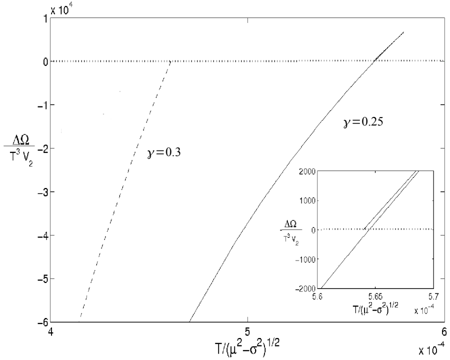



- 1st order for $\sigma$ large and $\gamma$ small
- 2nd order for $\sigma \geq 0$ and $\gamma$ large
- temperature $T_c$ decreases with increasing $\gamma$

# Holographic superfluids: free energy

(Y. Brihaye & B. Hartmann, Phys. Rev. D83 (2011) 126008)

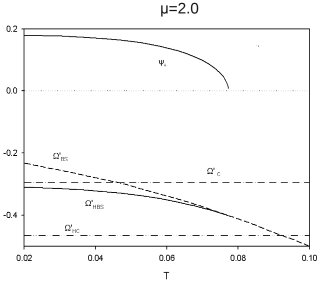
- $\gamma \neq 0$, fluid/BS superfluid phase transition

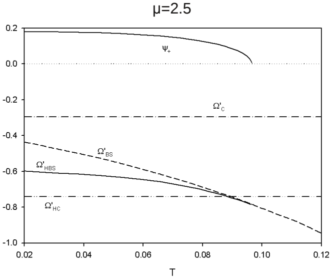


# Holographic superfluids: free energy

(Y. Brihaye & B. Hartmann, Phys. Rev. D83 (2011) 126008)

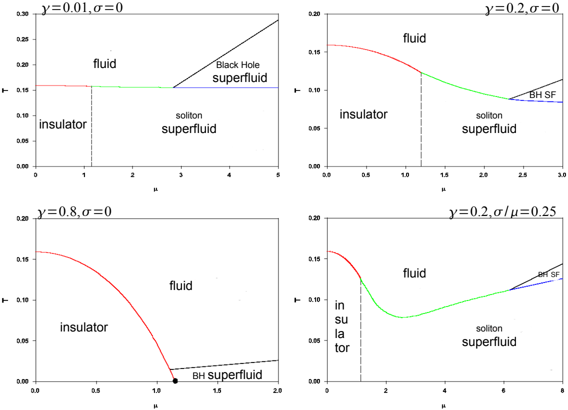
- **Free energy** of different phases with
  BS: black string $\rightarrow$ fluid   HBS: hairy black string $\rightarrow$ superfluid
  C: soliton $\rightarrow$ insulator   HC: hairy soliton $\rightarrow$ superfluid



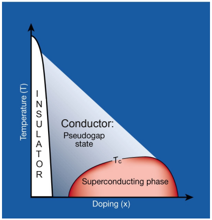
# Holographic superfluids: phase diagrams

(Y. Brihaye & B. Hartmann, Phys. Rev. D83 (2011) 126008)



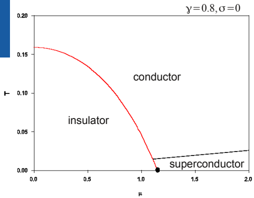
# Holographic superconductors

- $\sigma = 0 \rightarrow$ insulator/conductor/superconductor interpretation




(a) Principle phase diagram
of a cuprate superconductor

(from: http://www.pha.jhu.edu/~vstanev1/)




(b) taken from
Brihaye and B. Hartmann,
Phys. Rev. D, 2011

# Outline

# Summary

- Emerging evidence that some condensed matter phenomena are described by **strongly coupled** Quantum Field Theories (QFTs)
- **AdS/CFT** connects strongly coupled QFTs to (classical) gravity theories
- **Holographic superconductors/superfluids** ...
  - ... can be described qualitatively by "hairy" black holes/solitons
  - ... some quantitative results agree with experimental ones: $\omega_g / T_c \approx 8$
- Interesting in both directions
  - deeper insight into **"No hair" theorems** for black holes
  - "AdS/CFT in the laboratory"?


Muito obrigada
pela sua atenção.

# Holographic superfluids: Free energy

(Y. Brihaye & B. Hartmann, Phys. Rev. D83 (2011) 126008)

- Black strings (BS)

$$\left(\frac{\Omega}{V_2}\right)_{\rm BS} = c_t - 2c_z$$

- Solitonic solutions (S)

$$\left(\frac{\Omega}{V_2}\right)_{\rm S} = c_t + c_z$$

where

$$f(\rho \gg 1) = \rho^2 + \frac{(c_t + c_z)}{\rho} + O(\rho^{-2}) \ , \ \ b(\rho \gg 1) = \rho^2 + \frac{c_t}{\rho} + O(\rho^{-2}) \ ,$$

$$p(\rho \gg 1) = \rho^2 + \frac{c_z}{\rho} + O(\rho^{-2}) \ , \ \ g(\rho \gg 1) \sim O(\rho^{-3})$$