Strong coupling and strange quark mass from lattice QCD

Rainer Sommer

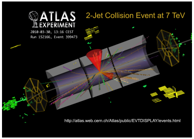
IFSC – USP, Sao Carlos, April 3, 2013

The talk is based on

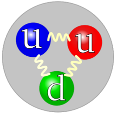
- old work [ALPHA Collaboration]
- $N_\mathrm{f} = 2$ running
  [Nucl.Phys. B713 (2005) 378, Michele Della Morte, Roberto Frezzotti, Jochen Heitger, Juri Rolf, RS, Ulli Wolff]
  [Nucl.Phys. B729 (2005) 117, Michele Della Morte, Roland Hoffmann, Francesco Knechtli, Juri Rolf, RS, Ines Wetzorke, Ulli Wolff]
- Scale setting
  [*Strange quark mass and the Lambda parameter in two flavor QCD,*
  Fritzsch, Leder, Knechtli, Marinkovic, Schaefer, S, Virotta, 2012]
- Recent development
  [Fritzsch & Ramos, `arXiv:1301.4388`]

Fascinating strong interactions

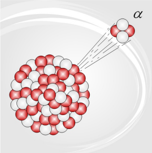
- jets at large energies



- hadrons at small energies



- nuclei at even smaller energies



Fascinating strong interactions THEORY

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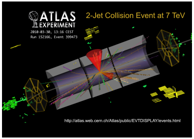
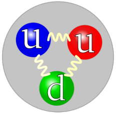


Q C D

- hadrons at small energies



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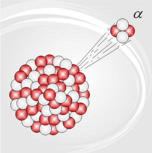


Q C D

Believed to be described by a most beautiful theory: Q C D

QCD

$$\mathcal{L}_{\rm QCD} = -\frac{1}{2{g_0}^2}\mathrm{tr}\{F_{\mu\nu}F_{\mu\nu}\} + \sum_{f=1}^{N_{\rm f}} \overline{\psi}_f\{D + m_{0f}\}\psi_f$$

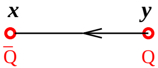
- $N_{\rm f} + 1 = 7$ (bare) parameters
- at low energy essentially 4 parameters
- in the following: u, d quarks mass-degenerate
  s-quark quenched

  3 parameters

Strong force

A way to define the strong force is

$$F(r) = \frac{\mathrm{d}}{\mathrm{d}r} V(r)\,,\ \ r = |\mathbf{x} - \mathbf{y}|$$



Perturbation theory (Feynman graphs)

$$F(r) \;=\; \frac{4}{3}\frac{1}{4\pi r^2} g^2 + \mathrm{O}(g^4)$$

coulombic

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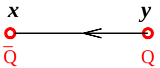

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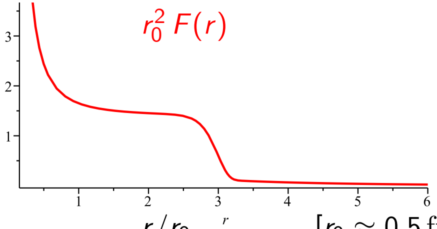
$$F(r) \;=\; \frac{4}{3}\frac{1}{4\pi r^2} g^2 + \mathrm{O}(g^4)$$ coulombic

$$\downarrow$$



r/r_0 $[r_0 \approx 0.5\,\mathrm{fm}]$

Strong coupling

Perturbation theory (Feynman graphs)

$$F(r) = \frac{4}{3}\frac{1}{4\pi r^2} g^2 + \mathrm{O}(g^4)$$

coulombic

Strong coupling

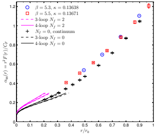
Perturbation theory (Feynman graphs)

$$F(r) = \frac{4}{3}\frac{1}{4\pi r^2} g^2 + \mathrm{O}(g^4)$$ coulombic

A way to define the strong coupling is

$$\alpha_{\rm qq}(\mu) = \frac{\bar{g}^2_{\rm qq}(r)}{4\pi} \equiv \frac{3}{4} r^2 F(r)\,, \quad \mu \equiv 1/r$$

It is r-dependent: "it runs"



Running and Renormalization Group Invariants

$$
\begin{aligned}
\text{RGE:}\quad \mu\frac{\partial\bar g}{\partial\mu} &= \beta(\bar g) \qquad\qquad \bar g(\mu)^2 = 4\pi\alpha(\mu) \qquad\qquad .\\
\beta(\bar g) &\overset{\bar g\to 0}{\sim} -\bar g^3\left\{b_0 + b_1\bar g^2 + b_2\bar g^4 + \ldots\right\}\\
b_0 &= \frac{1}{(4\pi)^2}\left(11 - \frac{2}{3}N_\mathrm{f}\right)
\end{aligned}
$$

Λ-parameter ($\bar g \equiv \bar g(\mu)$) $=$ Renormalization Group Invariant
$=$ intrinsic scale of QCD

$$
\Lambda \;=\; \mu\,(b_0\bar g^2)^{-b_1/2b_0^2}\,\mathrm{e}^{-1/2b_0\bar g^2}\ \exp\left\{-\int_0^{\bar g}\mathrm{d}g\left[\frac{1}{\beta(g)} + \frac{1}{b_0 g^3} - \frac{b_1}{b_0^2 g}\right]\right\}
$$

- there is a similar formula relating $\overline{m}_i(\mu)$ and M_i: RGI quark masses
- Λ, M_i have a trivial dependence on the "scheme"

  scheme $\leftrightarrow$ definition of $\bar g$, $\overline{m}$

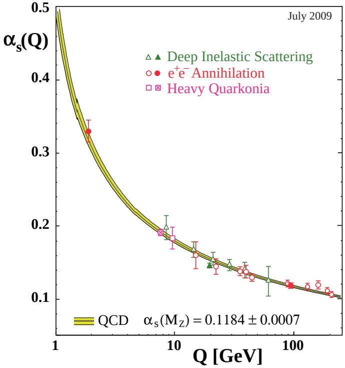
- they are the fundamental parameters of QCD

Uses of α (or Λ)

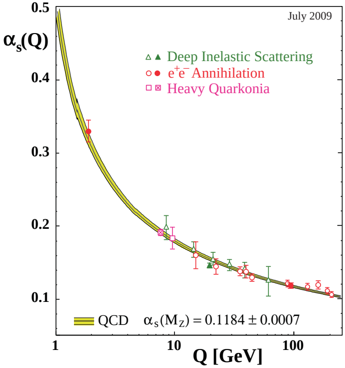
- Λ is a fundamental constant of Nature, just like $\alpha_{\rm em} = 1/137.0359997$ for atomic physics
- $\alpha(\mu)$ at high scale is needed for the search of new particles at the LHC.

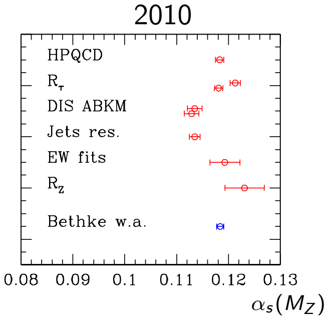
  E.g. the Higgs.
- It is an important constraint for possible theories at higher energy scales.

“Experimental” determinations of $\alpha_{\overline{\mathrm{MS}}}$



"Experimental" determinations of $\alpha_{\overline{\rm MS}}$

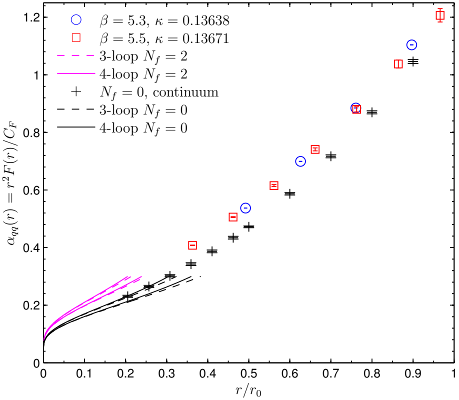




There is a considerable spread. Errors seem often too agressive.
Theory uncertainties are usually dominating.

Determination of Λ from lattice QCD

Lattice determination of α_{qq}

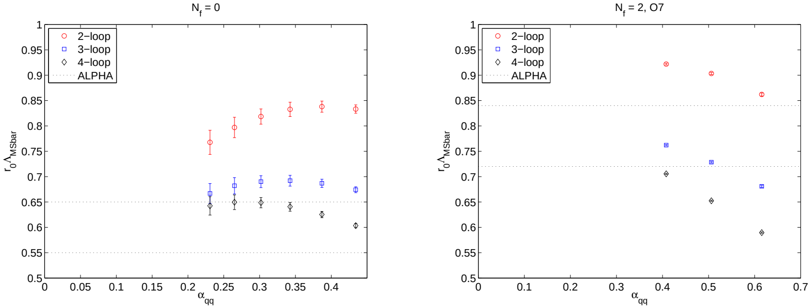


graph from [Leder & Knechtli, 2011]

$N_\mathrm{f} = 0$, continuum limit [Necco & S., 2001]

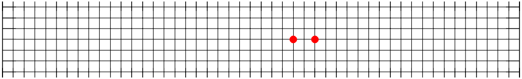
$N_\mathrm{f} = 2$, small lattice spacing [Leder & Knechtli, 2011]

Lambda parameter from α_{qq}



A realistic estimate of the uncertainty is impossible.
(Other members of the group come to a different conclusion
[Jansen, Karbstein, Nagy, Wagner, 2011])

The basic problem



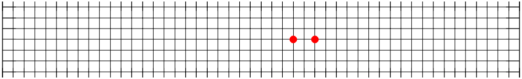
$$L \gg \frac{1}{0.2\mathrm{GeV}} \gg \frac{1}{\mu} \sim \frac{1}{10\mathrm{GeV}} \gg a$$

$\uparrow$ box size $\quad$ $\uparrow$ confinement scale, m_π $\quad$ $\uparrow$ spacing

$$\Downarrow$$

$$L/a \gg 50$$

The basic problem and its solution



$$L \gg \frac{1}{0.2\mathrm{GeV}} \gg \frac{1}{\mu} \sim \frac{1}{10\mathrm{GeV}} \gg a$$

$\uparrow$ box size; $\uparrow$ confinement scale, m_π; $\uparrow$ spacing

$$\Downarrow$$

$$L/a \gg 50$$

Solution: $L = 1/\mu$ $\longrightarrow$ left with $L/a \gg 1$ [Lüscher, Weisz, Wolff]

Finite size effect as a physical observable; finite size scaling!

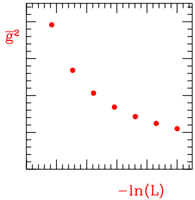
The step scaling function

It is a discrete β function:

$$\sigma(\bar{g}^2(L)) = \bar{g}^2(2L)$$

determines the
non-perturbative running:

$$\begin{aligned} u_0 &= \bar{g}^2(L_{\max}) \\ &\downarrow \\ \sigma(u_{k+1}) &= u_k \\ &\downarrow \\ u_k &= \bar{g}^2(2^{-k}L_{\max}) \end{aligned}$$



The step scaling function: $\sigma(u) = \bar{g}^2(2L)$ with $u = \bar{g}^2(L)$

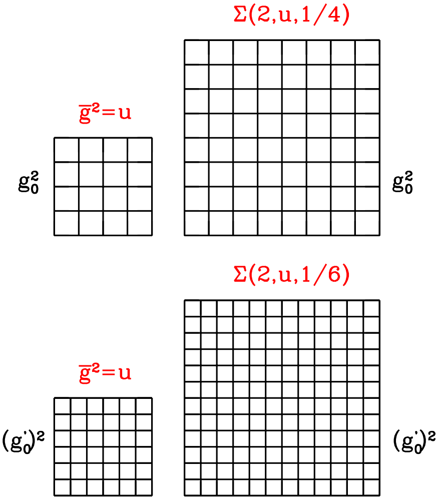
On the lattice:
additional dependence on the resolution a/L

g_0 fixed, L/a fixed:

$$\bar{g}^2(L) = u, \qquad \bar{g}^2(2L) = u'\,,$$
$$\Sigma(u, a/L) \quad = \quad u'$$

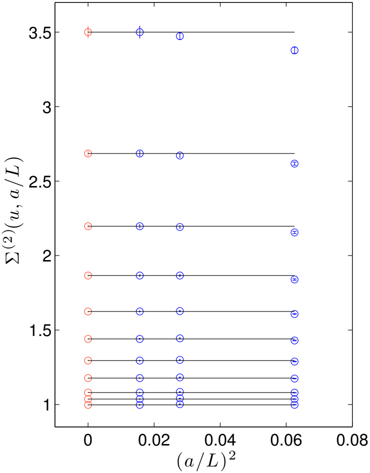
continuum limit:

$$\Sigma(u, a/L) = \sigma(u) + \mathrm{O}(a/L)$$



everywhere: $m = 0$ (PCAC mass defined in $(L/a)^4$ lattice)

Continuum limit ($N_f = 4$)



- *Constant fit:*

$$\Sigma^{(2)}(u, a/L) = \sigma(u)$$

for $L/a = 6, 8$

- *Global fit:*

$$\Sigma^{(2)}(u, a/L) = \sigma(u) + \rho\, u^4\, (a/L)^2$$

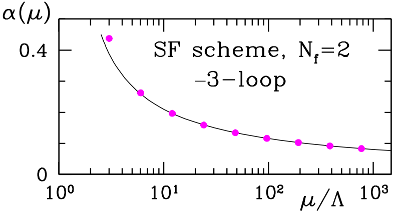
for $L/a = 6, 8$
$\rightarrow \rho = 0.007(85)$

- *$L/a = 8$ data:*

$$\sigma(u) = \Sigma^{(2)}(u, 1/8)$$

[ALPHA Collaboration (S., Tekin & Wolff, 2010); update: M. Marinkovic, 2013]

Non-perturbative running of α



[ALPHA Collaboration, 2005]

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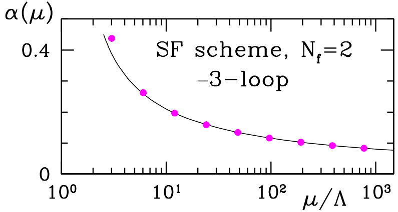

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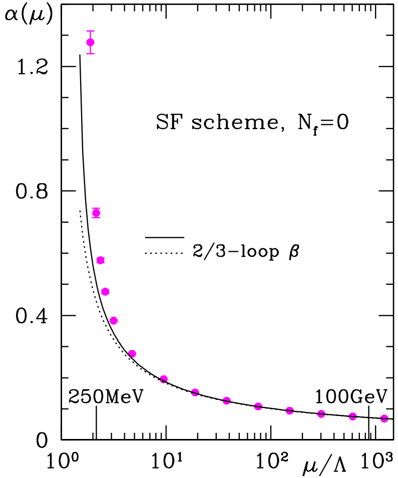

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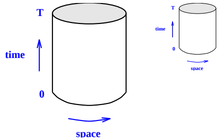
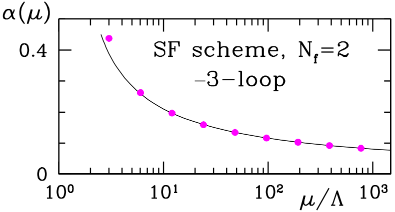
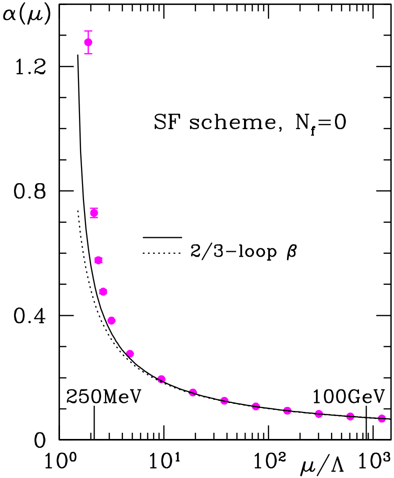




[ALPHA Collaboration , 2005]



[ALPHA Collaboration , 2001]

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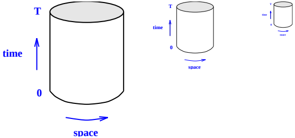

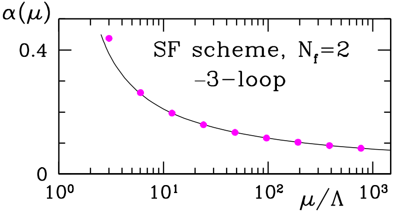

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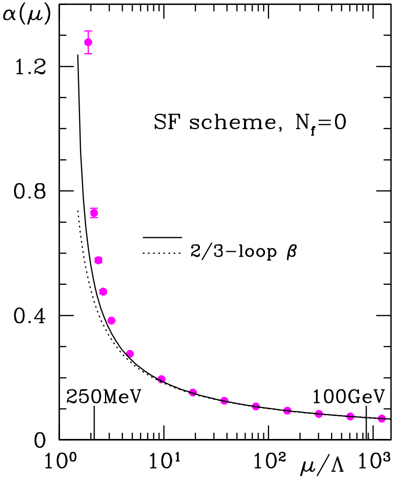

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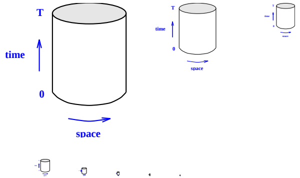

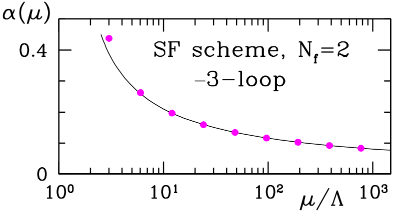

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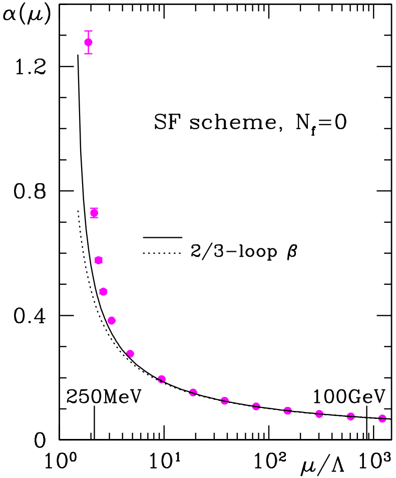


[ALPHA Collaboration, 2001]

The master formula of the ALPHA Collaboration strategy

$$\frac{\Lambda^{(2)}_{\overline{\rm MS}}}{F_{\rm K}} = \frac{1}{F_{\rm K}\,L_{\rm max}} \times \frac{L_{\rm max}}{L_k} \times L_k \Lambda^{(2)}_{\rm SF} \times \frac{\Lambda^{(2)}_{\overline{\rm MS}}}{\Lambda^{(2)}_{\rm SF}}$$

$\Gamma(K \to \mu\nu_\mu)$

$aF_{\rm K}$ $\quad L_{\rm max}/a$

$\bar{g}^2(L_{\rm max})$ → non-perturbative SSF's, massless $N_{\rm f} = 2$ theory → $\bar{g}^2(L_k)$ → 3-lp → $\Lambda^{(2)}_{\rm SF}$ → 1-lp (exact) → $\Lambda^{(2)}_{\overline{\rm MS}}$

μ

$\mu = 0$ $\quad m_b$ $\quad M_Z$ $\quad \infty$

Setting the scale

$$\frac{\Lambda_{\overline{\mathrm{MS}}}^{(2)}}{F_\mathrm{K}} = \frac{1}{F_\mathrm{K} L_\mathrm{max}} \times \frac{L_\mathrm{max}}{L_k} \times L_k \Lambda_{\overline{\mathrm{MS}}}^{(2)}$$

- F_K has been missing
- $F_\mathrm{K}\, m_\mathrm{K} = \langle K(\mathbf{p}=0)|\bar{s}\gamma_0\gamma_5 u|0\rangle$
- needs K-meson $\rightarrow$ "large" volume: $L > 2\,\mathrm{fm}\,,\ 4/m_\pi$

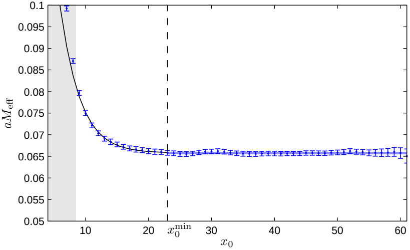
Setting the scale

$$\frac{\Lambda^{(2)}_{\overline{\mathrm{MS}}}}{F_{\mathrm{K}}} = \frac{1}{F_{\mathrm{K}} L_{\mathrm{max}}} \times \frac{L_{\mathrm{max}}}{L_k} \times L_k \Lambda^{(2)}_{\overline{\mathrm{MS}}}$$

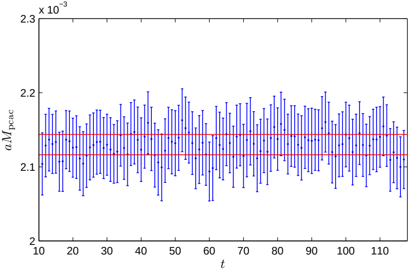
- F_{K} has been missing
- $F_{\mathrm{K}}\, m_{\mathrm{K}} = \langle K(\mathbf{p}=0)|\bar{s}\gamma_0\gamma_5 u|0\rangle$
- needs K-meson $\to$ "large" volume: $L > 2\,\mathrm{fm}\,,\ 4/m_\pi$
- issues
  - extracting ground state "plateau"
  - autocorrelations (sufficient statistics) and error analysis
  - extrapolation to physical quark masses
  - renormalization ...

Plateaux $64^3 128$ lattice $m_\pi = 270\,\mathrm{MeV}$

- m_π

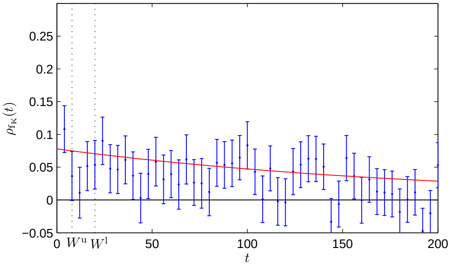


- PCAC quark mass



Error analysis, F_{K}, $64^3 128$ lattice $m_\pi = 270\,\mathrm{MeV}$

- Done as proposed in
  Critical slowing down and error analysis in lattice QCD simulations
  [Schaefer, S, Virotta, 2011]
- Autocorrelation function



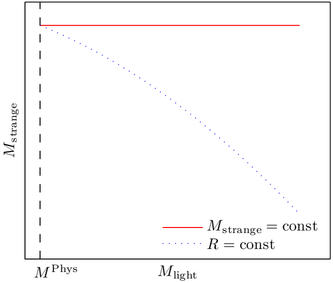
- The tail contributes about 60% of the error.

Extrapolation to physical light quark masses

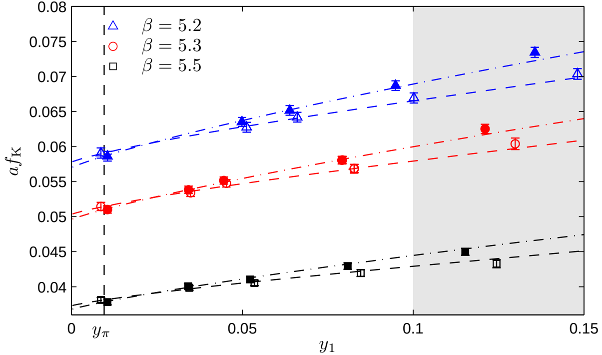
Strategies

1) $R = \frac{m_{\rm K}^2}{F_{\rm K}^2} = \text{constant}$

2) $M_s = \text{constant}$



Extrapolation to physical light quark masses

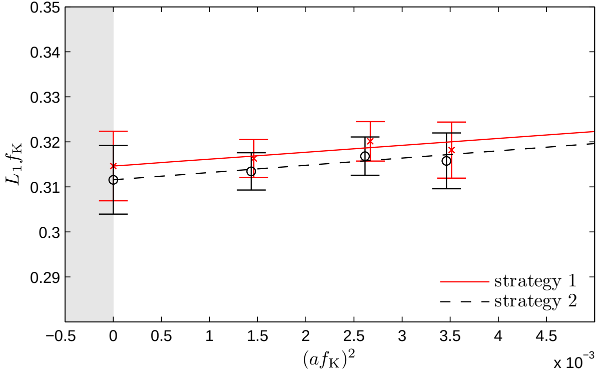


$$y_1 = \frac{m_\pi^2}{8\pi^2 F_{\rm K}^2}$$

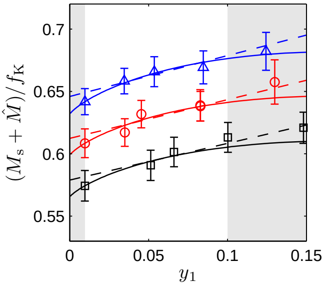
- systematic expansion including
  M, $M\log(M)$, a^2, not $M \times a^2$ [there are some checks]
- agreement also with simple linear extrpolation (no $M\log(M)$)

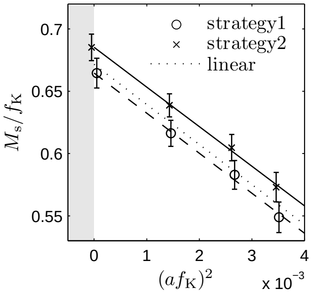
$F_K L_{max}$

combine $L_{max}/a = L_1/a$ with aF_K



Similar for the RGI mass of the strange quark





Results for the strange quark mass (no complete review of results)

N_f	$\overline{m}_s(2\mathrm{GeV})$	Experiment	Theory
0	97(4)	m_K + scale	LGT, ALPHA Collaboration
2	103(4)	m_K + scale	LGT, ALPHA Collaboration
3	96(3)	m_K + scale	LGT, BMW

- Firm results
- Before lattice gauge theory, (still in the 90's) values between $100\mathrm{MeV}$ and $200\mathrm{MeV}$ were given

N_f dependence of $\Lambda_{\overline{MS}}$ and comparison to phenomenology

$\Lambda_{\overline{\mathrm{MS}}}[\mathrm{MeV}]$

Experiment	Theory	N_f: 0	2	3	4	5
M_K, $K \to l2, l3$	SF [ALPHA Collaboration]	238(19)	310(20)			
M_K, M_ρ	SF [PACS-CS]			362(23)(25)		239(10) (6)(-22)
DIS, HERA	PT, PDF-fits [ABM11]				234(14)	160(11)
DIS, HERA	PT, PDF-fits [MSTW09]				285(23)	198(16)
"world av. "[2011]	PT					212(12)
$e^+e^- \to$ had (LEP)	4-loop PT					275(57)

- Non-trivial, non-perturbative N_f-dependence.
- Small errors are cited, but overall consistency is not that great.
- More precision and rigor (PT only at high energy) will be very useful.

Towards even better precision: Gradient Flow and SF

- Gradient flow [Lüscher, 2010; Lüscher & Weisz, 2011]

  new observables

  - UV finite (proven to all orders of PT)
  - excellent numerical precision
  - renormalized coupling in finite volume with pbc [BMW, 2012]
- Flow in finite volume, SF [Fritzsch & Ramos, arXiv:1301.4388]
  - lowest order PT to define a new coupling
  - numerical investigation shows excellent precision
- General idea

$$x = (x_0, \mathbf{x}) \,, \quad t = \text{ flow time}$$

$$A_\mu(x) = \text{ quantum gauge fields } : \quad \mathcal{Z} = \int \mathrm{D}[A_\mu(x)] \ldots$$

$$B_\mu(x,t) = \text{ smoothed gauge fields } , \quad B_\mu(x,0) = A_\mu(x)$$

$$\frac{\mathrm{d}B_\mu(x,t)}{\mathrm{d}t} = D_\nu G_{\nu\mu}(x,t) + \text{gauge fixing}$$

$$\sim -\frac{\delta S_{YM}[B]}{\delta B_\mu}$$

correlation functions of B-fields at arbitrary points are finite

Gradient Flow

$$\frac{\mathrm{d}B_\mu(x,t)}{\mathrm{d}t} \equiv \dot{B}_\mu(x,t) = \underbrace{D_\nu G_{\nu\mu}(x,t)}_{\uparrow} + \underbrace{D_\mu \partial_\nu B_\nu(x,t)}_{\uparrow} \qquad (*)$$

$\sim -\frac{\delta S_{YM}[B]}{\delta B_\mu}$ eliminate by a t-dependent gauge traf

in PT: $A_\mu(x) = g_0 \bar{A}_\mu(x)$

$$\begin{aligned} B_\mu(x,t) &= B_{\mu,1}(x,t)g_0 + B_{\mu,2}(x,t)g_0^2 + \ldots \\ & \quad G_{\nu\mu} = [\partial_\nu B_{\mu,1} - \partial_\mu B_{\nu,1}]\, g_0 + \mathrm{O}(g_0^2)\,, \quad D_\nu = \partial_\nu + \mathrm{O}(g_0) \\ \rightarrow \quad \dot{B}_{\mu,1}(x,t) &= \partial_\nu \partial_\nu B_{\mu,1}(x,t) \end{aligned}$$

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- heat equation

$$
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B_{\mu,1}(x,t) &= \int \mathrm{d}^D p\, \mathrm{e}^{ipx}\, b_\mu(p,t) \\
& \qquad \dot{b}_\mu = -p^2 b_\mu \quad \rightarrow \quad b_\mu(p,t) = b_\mu(p,0) \mathrm{e}^{-p^2 t} \\
B_{\mu,1}(x,t) &= \int \mathrm{d}^D y\, K_t(x-y)\, \bar{A}_\mu(y)\,, \quad K_t(z) = (4\pi t)^{-D/2} \mathrm{e}^{-z^2/(4t)}
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$$

- smoothing over a radius of $\sqrt{8t}$
- gaussian damping of large momenta

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- smoothing over a radius of $\sqrt{8t}$
- gaussian damping of large momenta
- all correlation functions of B_μ are finite ($t > 0$) [Lüscher & Weisz, 2011]
  in particular $\langle E(t) \rangle\,, \quad E(t) = -\frac{1}{2} \mathrm{tr}\, G_{\mu\nu} G_{\mu\nu}$

Gradient Flow

For $\langle E \rangle$, $E = -\frac{1}{2}\mathrm{tr}\, G_{\mu\nu} G_{\mu\nu}$

$$\begin{aligned} \langle E \rangle &= E_0\, g_0^2 + E_0\, g_0^4 + \dots \\ E_0 &= \langle \mathrm{tr}[\partial_\mu B_{\nu,1} \partial_\mu B_{\nu,1} - \partial_\mu B_{\nu,1} \partial_\nu B_{\mu,1}] \rangle \\ &\sim \int_p \mathrm{e}^{-p^2 2t} [p^2 \delta_{\mu\nu} - p_\mu p_\nu]\, D_{\mu\nu}(p) \end{aligned}$$

finite (also with cutoff reg'n)!

Gradient Flow and SF-coupling

use the flow in finite volume (SF): $T \times L^3$ world with Dirichlet BC in time, $T = L$
define

$$\langle E(t)\rangle \equiv -\tfrac{1}{2}\langle \mathrm{tr}\, G_{\mu\nu}G_{\mu\nu}(x,t)\rangle_{x_0=T/2} = \tfrac{\mathcal{N}}{t^2}\,\overline{g}^2_{\rm MS}(\mu)\,(1 + c_1\overline{g}^2_{\rm MS} + \ldots)$$

$$\overline{g}^2_{\rm GF}(L) \equiv \mathcal{N}^{-1}\, t^2\langle E(t)\rangle\Big|_{t=c^2L^2/8}$$

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$$\mathcal{N}(c) = \frac{c^4(N^2-1)}{128}\sum_{\mathbf{n},n_0} e^{-c^2\pi^2(\mathbf{n}^2+\frac{1}{4}n_0^2)} \times \frac{2\mathbf{n}^2 s^2_{n_0}(T/2) + (\mathbf{n}^2+\frac{3}{4}n_0^2)c^2_{n_0}(T/2)}{\mathbf{n}^2+\frac{1}{4}n_0^2}$$

- the lattice version is known (and needed)

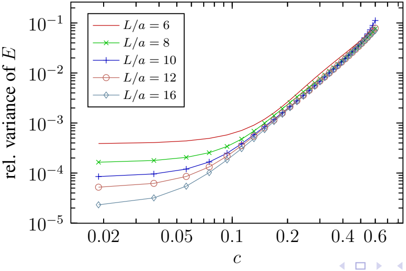
Gradient Flow and SF-coupling

statistical precision: variance

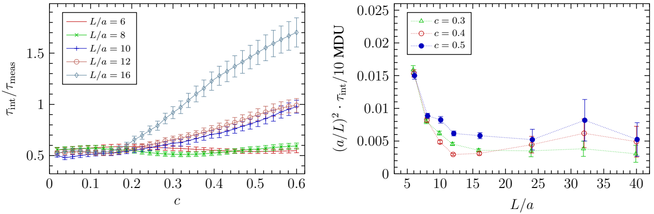
$$\text{relative variance} = \frac{\langle E^2\rangle - \langle E\rangle^2}{\langle E\rangle^2}$$

should be finite as $a \to 0$, $L/a \to \infty$

Numerically, Fritzsch & Ramos:



autocorrelations scale as expected: $\tau_{\rm int} \propto a^{-2}$



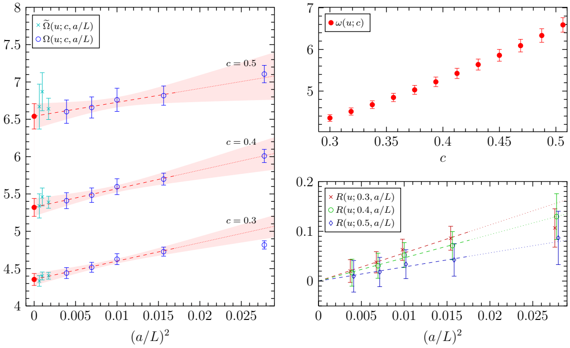
Statistical precision is good and theoretically understood.
There will be no surprises on the way to the continuum limit.

Gradient Flow and SF-coupling

systematic precision

keeping old SF-coupling $\bar{g}_{\rm SF}(L)$ fixed (defines L), compute

$$\Omega(u; c, a/L) \quad = \left[\hat{\mathcal{N}}^{-1}(c, a/L) \cdot t^2 \langle E(t, T/2) \rangle \right]^{\bar{g}^2_{\rm SF}=u,\, m=0}_{t=c^2L^2/8}$$



- small cutoff effects $\longrightarrow$ ready for applications $\longrightarrow$ . . .
  $\rightarrow$ **precise Λ-parameter**

Summary

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The fundamental parameters of QCD

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- refer to the asymptotic high energy behavior of QCD and therefore Nature
- nevertheless they can be connected non-perturbatively through lattice simulations to hadron masses (and properties)
- firm and precise results can be obtained
- the precision will be improved even further in the near future