Aulas passadas

Eq. de onda na corda: $$\frac{\partial^2 y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} = 0$$ $$c = \sqrt{\frac{T_0}{\mu}}$$ (velocidade da onda)

Soluções (ondas propagantes): $$y(x,t) = f(x-ct) + g(x+ct)$$

Condições iniciais:

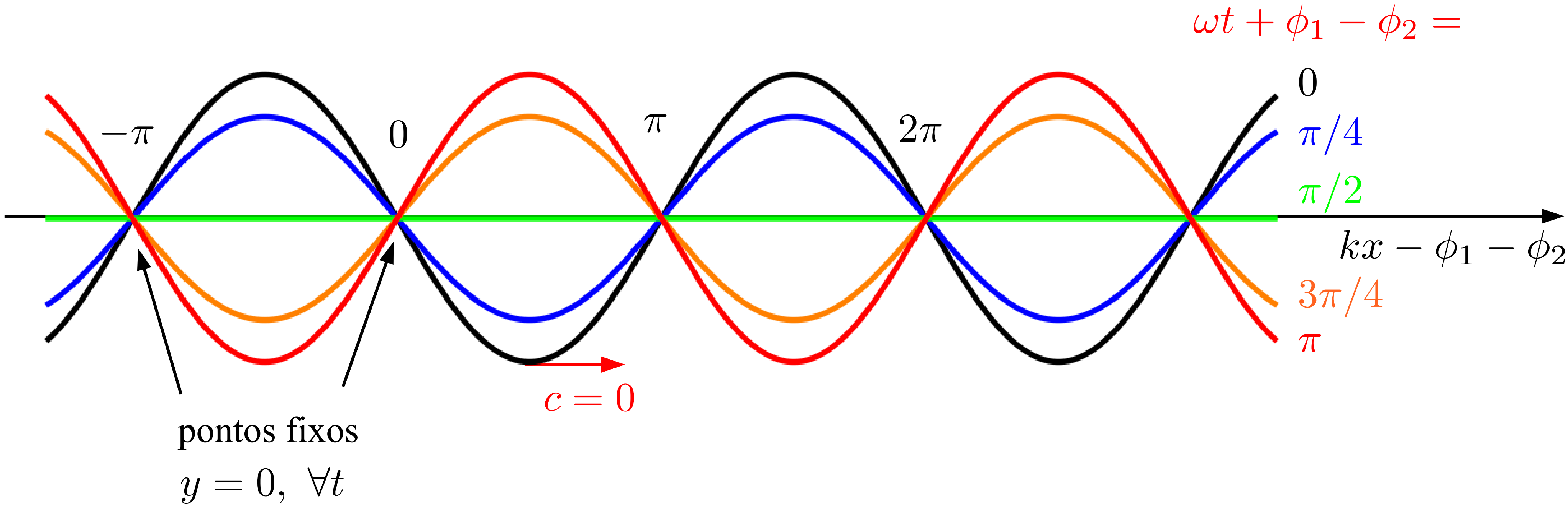
$$\begin{aligned} f(x) &= \frac{1}{2}y_0(x) - \frac{1}{2c}\int_{-\infty}^{x} v_0(z)\mathrm{d}z \\ g(x) &= \frac{1}{2}y_0(x) + \frac{1}{2c}\int_{-\infty}^{x} v_0(z)\mathrm{d}z \end{aligned}$$

Ondas harmônicas: $$y(x,t) = A\cos(kx - \omega t - \phi),\ \omega = ck$$

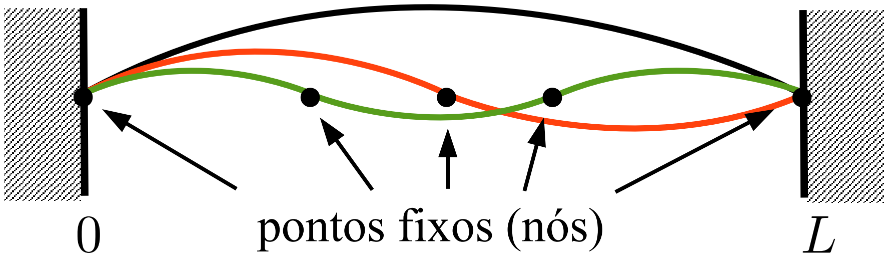
Aulas passadas

Ondas estacionárias:

$$
\begin{aligned}
y(x,t) &= A\sin(kx-\omega t-2\phi_1)+A\sin(kx+\omega t-2\phi_2)\\
&= 2A\sin(kx-\phi_1-\phi_2)\cos(\omega t+\phi_1-\phi_2)
\end{aligned}
$$

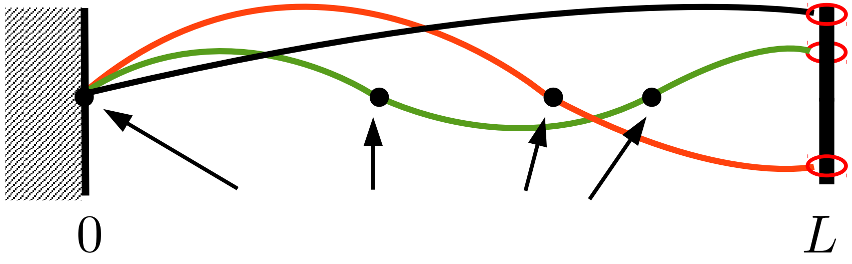


Aulas passadas



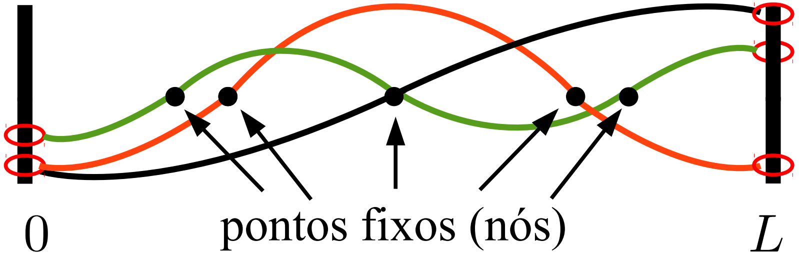
$$y(x,t) = \sum_{n=1}^{\infty} C_n \sin(k_n x)\cos(\omega_n t - \varphi_n)$$

$$k_n = \frac{\pi}{L} n, \quad n \in \mathbb{N}^*$$



$$y(x,t) = \sum_{n=1}^{\infty} C_n \sin(k_n x)\cos(\omega_n t - \varphi_n)$$

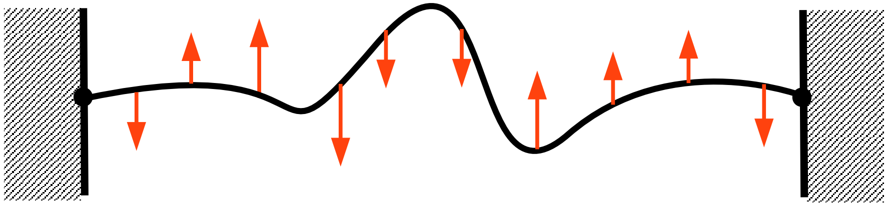
$$k_n = \frac{(2n-1)\pi}{2L}, \quad n \in \mathbb{N}^*$$



$$y(x,t) = \sum_{n=1}^{\infty} C_n \cos(k_n x)\cos(\omega_n t - \varphi_n)$$

$$k_n = \frac{\pi}{L} n, \quad n \in \mathbb{N}^*$$

Aulas passadas



$$y(x,t) = \sum_{n=1}^{\infty} C_n \sin(k_n x)\cos(\omega_n t - \varphi_n)$$

$$= \sum_{n=1}^{\infty} \sin(k_n x)\left(A_n \cos(\omega_n t) + B_n \sin(\omega_n t)\right)$$

$$k_n = \frac{\pi}{L} n, \quad \omega_n = c k_n, \quad n \in \mathbb{N}^*$$

$\{A_n, B_n\}$ determinados pelas condições iniciais:

$$y_0(x) = \sum_{n=1}^{\infty} A_n \sin(k_n x) \longrightarrow A_n = \frac{2}{L}\int_0^L y_0(x)\sin(k_n x)\mathrm{d}x$$

$$v_0(x) = \sum_{n=1}^{\infty} B_n \omega_n \sin(k_n x) \longrightarrow B_n = \frac{2}{\omega_n L}\int_0^L v_0(x)\sin(k_n x)\mathrm{d}x$$

$$C_n^2 = A_n^2 + B_n^2, \quad \tan\varphi_n = B_n / A_n$$

Aulas passadas

Série de Fourier: Representação de uma função integrável de período L em termos de funções harmônicas:

$$f(x+L) = f(x)$$

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos(k_n x) + \sum_{n=1}^{\infty} b_n \sin(k_n x)$$

$$= \frac{1}{2}c_0 + \sum_{n=1}^{\infty} c_n \cos(k_n x - \phi_n)$$

$$k_n = \frac{2\pi}{L}n, \quad n \in \mathbb{N}$$

Conjunto discreto de funções harmônicas.

Coeficientes de “Fourier”:

$$a_n = \frac{2}{L}\int_0^L f(x)\cos(k_n x)\mathrm{d}x$$

$$b_n = \frac{2}{L}\int_0^L f(x)\sin(k_n x)\mathrm{d}x$$

$$c_n^2 = a_n^2 + b_n^2, \quad \tan\phi_n = b_n/a_n$$

Note que $b_0 = 0$

Aulas passadas

Transformada de Fourier: Representação de uma função integrável (não necessariamente periódica) em termos de ondas planas

$$\tilde{f}(k) \equiv \mathcal{F}\left[f(x)\right] = \int_{-\infty}^{\infty} \mathrm{d}x f(x) e^{-ikx}$$

Transformada inversa:

$$f(x) = \mathcal{F}^{-1}\left[\tilde{f}(k)\right] = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{f}(k) e^{ikx}\mathrm{d}k$$

Vetores de onda:

$$k_n \;\rightarrow\; k$$

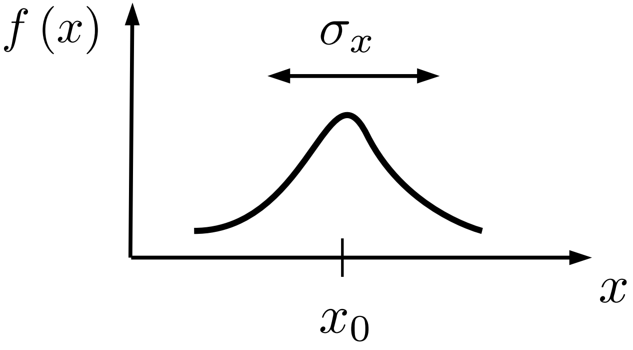
Amplitudes e fases:

$$c_n,\, \phi_n \;\rightarrow\; \tilde{f}(k)$$

Aulas passadas

Gaussiana:

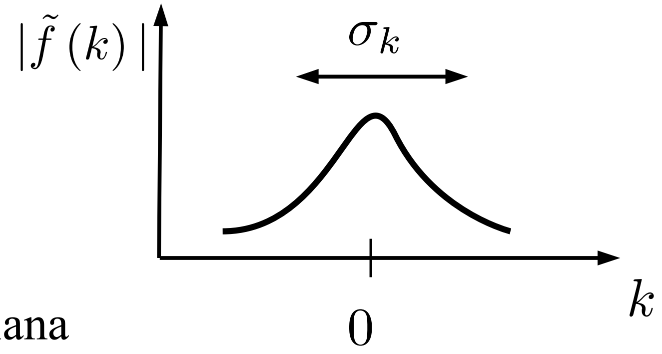
$$f(x) = \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{(x-x_0)^2}{2\sigma_x^2}}$$



Transformada:

$$\tilde{f}(k) = \frac{1}{\sqrt{2\pi}\sigma_x} \int_{-\infty}^{\infty} e^{-\frac{(x-x_0)^2}{2\sigma_x^2} - ikx} \mathrm{d}x = e^{-ikx_0} \times e^{-\frac{k^2}{2\sigma_k^2}}$$

fase   Gaussiana



$$\sigma_k = 1/\sigma_x$$

Relação de incerteza: $\Delta x \Delta k = \sigma_x \sigma_k = 1$

3 – Transformada de Fourier e a equação de onda

$$\frac{\partial^2 y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} = 0$$

Usando que $y\left(x,t\right) = \int_{-\infty}^{\infty} \frac{\tilde{y}\left(k,t\right)}{2\pi} e^{ikx} \mathrm{d}k = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\tilde{\tilde{y}}\left(k,\omega\right)}{(2\pi)^2} e^{i(kx-\omega t)} \mathrm{d}k\mathrm{d}\omega$

Então $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left(-k^2+\omega^2/c^2\right)\tilde{\tilde{y}}\left(k,\omega\right) e^{i(kx-\omega t)} \mathrm{d}k\mathrm{d}\omega = 0,\ \forall x,t$

Logo, qualquer onda plana é permitida desde que $\omega = \pm ck$

Relação de dispersão

$\Rightarrow \tilde{\tilde{y}} \equiv \tilde{\tilde{y}}(k)$ porque $\omega \equiv \omega_k$

3 – Transformada de Fourier e a equação de onda

$$\frac{\partial^2 y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} = 0$$

Logo,

$$y(x,t) = \int_{-\infty}^{\infty} \frac{\tilde{y}_+(k)}{2\pi} e^{i(kx-\omega_k t)} \mathrm{d}k + \int_{-\infty}^{\infty} \frac{\tilde{y}_-(k)}{2\pi} e^{i(kx+\omega_k t)} \mathrm{d}k$$

onde

$$\omega_k = ck$$

As condições iniciais determinam as amplitudes das ondas planas

$$\tilde{y}_+(k) = \frac{1}{2}\left[\int_{-\infty}^{\infty} y(x,0)\, e^{-ikx} \mathrm{d}k - \frac{1}{i\omega_k}\int_{-\infty}^{\infty} \dot{y}(x,0)\, e^{-ikx} \mathrm{d}k\right]$$

$$\tilde{y}_-(k) = \frac{1}{2}\left[\int_{-\infty}^{\infty} y(x,0)\, e^{-ikx} \mathrm{d}k + \frac{1}{i\omega_k}\int_{-\infty}^{\infty} \dot{y}(x,0)\, e^{-ikx} \mathrm{d}k\right]$$

3 – Transformada de Fourier e a equação de onda

Prova: $$y(x,t) = \int_{-\infty}^{\infty} \frac{\tilde{y}_+(k)}{2\pi} e^{i(kx-\omega_k t)} \mathrm{d}k + \int_{-\infty}^{\infty} \frac{\tilde{y}_-(k)}{2\pi} e^{i(kx+\omega_k t)} \mathrm{d}k$$

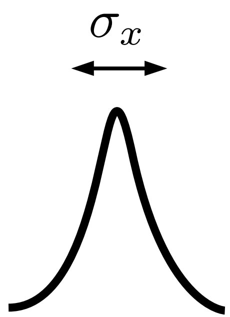
$$y(x,0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left(\tilde{y}_+(k) + \tilde{y}_-(k)\right) e^{ikx} \mathrm{d}k$$

$$\Rightarrow \tilde{y}_+(k) + \tilde{y}_-(k) = \int_{-\infty}^{\infty} y(x,0)\, e^{-ikx} \mathrm{d}k$$

$$\dot{y}(x,0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} i\omega_k \left(-\tilde{y}_+(k) + \tilde{y}_-(k)\right) e^{ikx} \mathrm{d}k$$

$$\Rightarrow i\omega_k \left(-\tilde{y}_+(k) + \tilde{y}_-(k)\right) = \int_{-\infty}^{\infty} \dot{y}(x,0)\, e^{-ikx} \mathrm{d}k$$

3 – Transformada de Fourier e a equação de onda



Exemplo: $y(x,0) = 2 \times \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{x^2}{2\sigma_x^2}}$ $\qquad \dot{y}(x,0) = 0$

$$\tilde{y}_+(k) = \tilde{y}_-(k) = \frac{1}{2}\int_{-\infty}^{\infty} y(x,0)\, e^{-ikx} \mathrm{d}k = e^{-\frac{1}{2}k^2\sigma_x^2}$$

$$y(x,t) = \frac{1}{2\pi}\left[\int_{-\infty}^{\infty} \tilde{y}_+(k)\, e^{ik(x-ct)} \mathrm{d}k + \int_{-\infty}^{\infty} \tilde{y}_-(k)\, e^{ik(x+ct)} \mathrm{d}k\right]$$

$$= \frac{1}{\sqrt{2\pi}\sigma_x}\left[e^{-\frac{(x-ct)^2}{2\sigma_x^2}} + e^{-\frac{(x+ct)^2}{2\sigma_x^2}}\right]$$

Resultado idêntico àquele predito por ondas propagantes

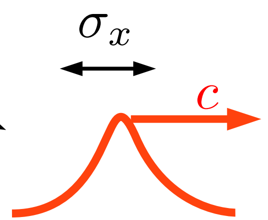
3 – Transformada de Fourier e a equação de onda

Detalhes matemáticos:

$$\int_{-\infty}^{\infty} \tilde{y}_+(k)\, e^{ik(x-ct)}\mathrm{d}k = \int_{-\infty}^{\infty} e^{-\frac{1}{2}k^2\sigma_x^2+ik(x-ct)}\mathrm{d}k = \int_{-\infty}^{\infty} e^{-\frac{1}{2}\sigma_x^2\left(k^2+2\overbrace{\frac{i(x-ct)}{\sigma_x^2}}^{k_0}k\right)}\mathrm{d}k$$

$$= \int_{-\infty}^{\infty} e^{-\frac{1}{2}\sigma_x^2\left(k^2+2k_0k+k_0^2-k_0^2\right)}\mathrm{d}k = e^{\frac{1}{2}\sigma_x^2k_0^2}\int_{-\infty}^{\infty} e^{-\frac{1}{2}\sigma_x^2\left(k^2+k_0\right)^2}\mathrm{d}k$$

$$= \frac{\sqrt{2\pi}}{\sigma_x}e^{\frac{1}{2}\sigma_x^2k_0^2} = \frac{\sqrt{2\pi}}{\sigma_x}e^{-\frac{(x-ct)^2}{2\sigma_x^2}}$$

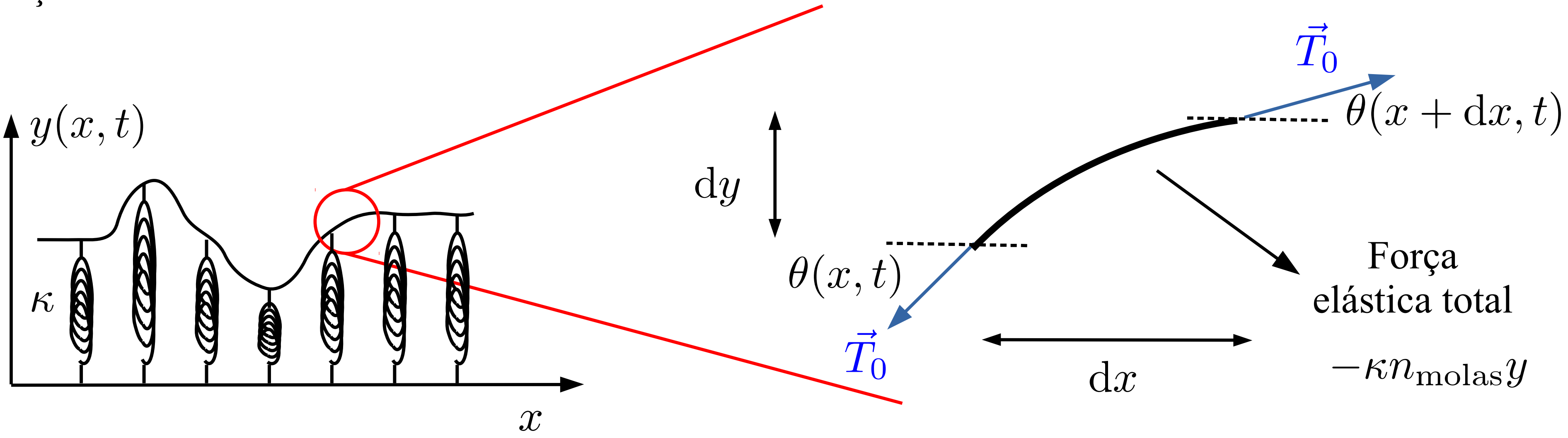


Integral Gaussiana: $$\int_{-\infty}^{\infty} e^{-\frac{1}{2\sigma^2}(x-x_0)^2}\mathrm{d}x = \sqrt{2\pi}\sigma$$

4 – Transformada de Fourier e a equação de onda dispersiva I

$$\frac{\partial^2 y}{\partial t^2} - c^2 \frac{\partial^2 y}{\partial x^2} + \omega_0^2 y = 0$$

Motivação: corda num meio elástico



2ª lei de Newton: $\rho \frac{\partial^2 y}{\partial t^2} = T \frac{\partial^2 y}{\partial x^2} - \kappa \frac{N_{\mathrm{molas}}}{L} y$

4 – Transformada de Fourier e a equação de onda dispersiva I

$$\frac{\partial^2 y}{\partial t^2} - c^2 \frac{\partial^2 y}{\partial x^2} + \omega_0^2 y = 0$$

OBS: Ondas propagantes não são mais soluções: $y(x,t) \neq f(x-ct) + g(x+ct)$

Usando que $$y(x,t) = \int_{-\infty}^{\infty} \frac{\tilde{y}(k,t)}{2\pi} e^{ikx} \mathrm{d}k = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\tilde{\tilde{y}}(k,\omega)}{(2\pi)^2} e^{i(kx-\omega t)} \mathrm{d}k\mathrm{d}\omega$$

Então $$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left(-\omega^2 + c^2k^2 + \omega_0^2\right) \tilde{\tilde{y}}(k,\omega)\, e^{i(kx-\omega t)} \mathrm{d}k\mathrm{d}\omega = 0, \ \forall x, t$$

Logo, qualquer onda plana é solução desde que $\boxed{\omega = \pm\sqrt{\omega_0^2 + c^2k^2}}$

Relação de dispersão

4 – Transformada de Fourier e a equação de onda dispersiva I

$$\frac{\partial^2 y}{\partial t^2} - c^2 \frac{\partial^2 y}{\partial x^2} + \omega_0^2 y = 0$$

Logo,
$$y\left(x,t\right) = \int_{-\infty}^{\infty} \frac{\tilde{y}_+\left(k\right)}{2\pi} e^{i(kx-\omega_k t)} \mathrm{d}k + \int_{-\infty}^{\infty} \frac{\tilde{y}_-\left(k\right)}{2\pi} e^{i(kx+\omega_k t)} \mathrm{d}k$$

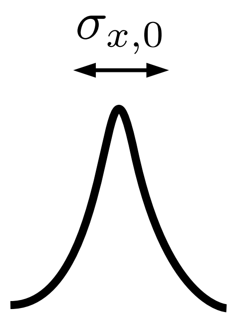
onde
$$\omega_k = \sqrt{\omega_0^2 + c^2 k^2}$$

As condições iniciais determinam as amplitudes das ondas planas

$$\tilde{y}_+\left(k\right) = \frac{1}{2}\left[\int_{-\infty}^{\infty} y\left(x,0\right) e^{-ikx} \mathrm{d}k - \frac{1}{i\omega_k} \int_{-\infty}^{\infty} \dot{y}\left(x,0\right) e^{-ikx} \mathrm{d}k\right]$$

$$\tilde{y}_-\left(k\right) = \frac{1}{2}\left[\int_{-\infty}^{\infty} y\left(x,0\right) e^{-ikx} \mathrm{d}k + \frac{1}{i\omega_k} \int_{-\infty}^{\infty} \dot{y}\left(x,0\right) e^{-ikx} \mathrm{d}k\right]$$

4 – Transformada de Fourier e a equação de onda dispersiva I



Exemplo: $$y(x,0) = 2 \times \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{x^2}{2\sigma_x^2}} \qquad \dot{y}(x,0) = 0$$

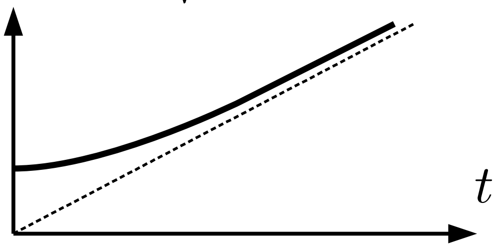
$$\tilde{y}_+(k) = \tilde{y}_-(k) = \frac{1}{2}\int_{-\infty}^{\infty} y(x,0)\, e^{-ikx} \mathrm{d}k = e^{-\frac{1}{2}k^2\sigma_x^2}$$

$$y(x,t) = \frac{1}{2\pi}\left[\int_{-\infty}^{\infty} \tilde{y}_+(k)\, e^{ik(x-\tilde{c}_k t)} \mathrm{d}k + \int_{-\infty}^{\infty} \tilde{y}_-(k)\, e^{ik(x+\tilde{c}_k t)} \mathrm{d}k\right]$$

$=$

ondas dispersivas

$$\sigma_{x,t} = \sigma_{x,0}\sqrt{1 + \Gamma^2 t^2/\sigma_{x,0}^4}$$



4 – Transformada de Fourier e a equação de onda dispersiva I

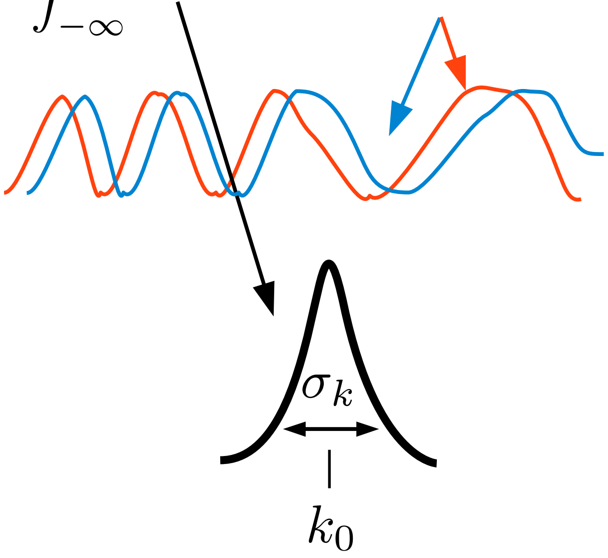
Detalhes matemáticos:

$$\int_{-\infty}^{\infty} e^{-\frac{\sigma^2}{2}(k-k_0)^2} e^{i(kx-\omega_k t)} \mathrm{d}k \approx \int_{-\infty}^{\infty} e^{-\frac{\sigma^2}{2}(k-k_0)^2} e^{i\left[kx-\left(\omega_{k_0}+\left.\frac{\mathrm{d}\omega_k}{\mathrm{d}k}\right|_{k_0}(k-k_0)+\frac{1}{2}\left.\frac{\mathrm{d}^2\omega_k}{\mathrm{d}k^2}\right|_{k_0}(k-k_0)^2\right)t\right]} \mathrm{d}k$$

$$= \int_{-\infty}^{\infty} e^{-\frac{\sigma^2}{2}q^2} e^{i\left((q+k_0)x-\left(\omega_{k_0}+c_g q+\frac{1}{2}\Gamma q^2\right)t\right)} \mathrm{d}q$$

$$= e^{i\left(k_0 x-\omega_{k_0} t\right)} \int_{-\infty}^{\infty} e^{-\frac{\sigma^2}{2}q^2} e^{i\left(qx-c_g qt-\frac{1}{2}\Gamma q^2 t\right)} \mathrm{d}q$$

$$= e^{ik_0(x-c_f t)} \int_{-\infty}^{\infty} e^{-\left(\frac{\sigma^2+i\Gamma t}{2}\right)q^2} e^{iq(x-c_g t)} \mathrm{d}q$$



Contribuição principal vem dos k's tal que

$$|k-k_0| \lessapprox \sigma_k$$

Onda plana com velocidade (de fase)

$$c_f = \omega_{k_0}/k_0$$

Envelope (grupo) com velocidade

$$c_g = \left.\frac{\mathrm{d}\omega_k}{\mathrm{d}k}\right|_{k=k_0}$$

4 – Transformada de Fourier e a equação de onda dispersiva I

Detalhes matemáticos:

$$\int_{-\infty}^{\infty} e^{-\frac{\sigma^2}{2}(k-k_0)^2} e^{i(kx-\omega_k t)}\mathrm{d}k \approx \int_{-\infty}^{\infty} e^{-\frac{\sigma^2}{2}(k-k_0)^2} e^{i\left[kx-\left(\omega_{k_0}+\left.\frac{\mathrm{d}\omega_k}{\mathrm{d}k}\right|_{k_0}(k-k_0)+\frac{1}{2}\left.\frac{\mathrm{d}^2\omega_k}{\mathrm{d}k^2}\right|_{k_0}(k-k_0)^2\right)t\right]}\mathrm{d}k$$

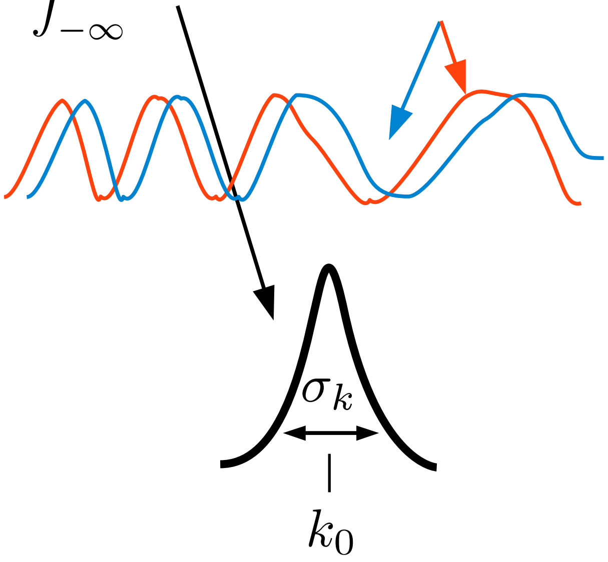
$$= \int_{-\infty}^{\infty} e^{-\frac{\sigma^2}{2}q^2} e^{i\left((q+k_0)x-\left(\omega_{k_0}+c_g q+\frac{1}{2}\Gamma q^2\right)t\right)}\mathrm{d}q$$

$$= e^{i\left(k_0 x-\omega_{k_0} t\right)} \int_{-\infty}^{\infty} e^{-\frac{\sigma^2}{2}q^2} e^{i\left(qx-c_g q t-\frac{1}{2}\Gamma q^2 t\right)}\mathrm{d}q$$

$$= e^{ik_0(x-c_f t)} \int_{-\infty}^{\infty} e^{-\left(\frac{\sigma^2+i\Gamma t}{2}\right)q^2} e^{iq(x-c_g t)}\mathrm{d}q$$

$$= e^{ik_0(x-c_f t)} \sqrt{\frac{2\pi}{\sigma^2+i\Gamma t}}\, e^{-\frac{(x-c_g t)^2}{2(\sigma^2+i\Gamma t)}}$$

$$= \frac{\sqrt{2\pi}}{\sigma\left(1+\Gamma^2 t^2/\sigma^4\right)^{1/4}} e^{-\frac{1}{2}\left(\frac{x-c_g t}{\sigma\sqrt{1+\Gamma^2 t^2/\sigma^4}}\right)^2} e^{i\phi(x,t)}$$



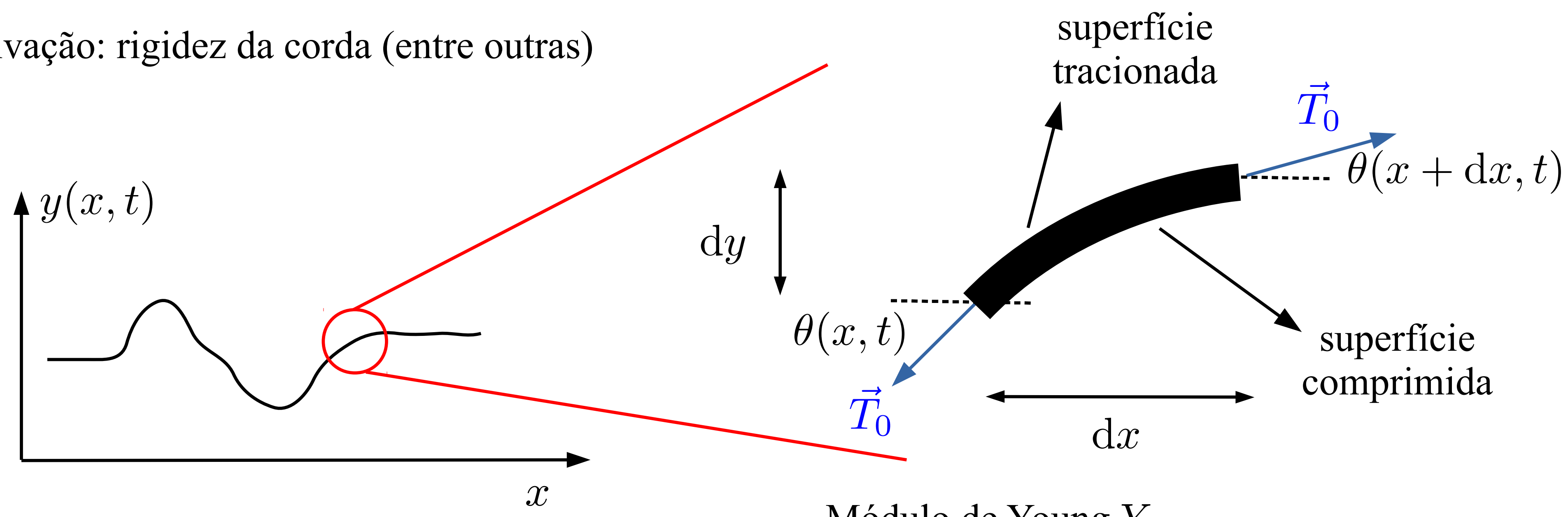
Contribuição principal vem dos k's tal que

$$|k - k_0| \lesssim \sigma_k$$

4 – Transformada de Fourier e a equação de onda dispersiva II

$$\frac{\partial^2 y}{\partial t^2} - c^2 \left(\frac{\partial^2 y}{\partial x^2} - \epsilon \frac{\partial^4 y}{\partial x^4} \right) = 0$$

Motivação: rigidez da corda (entre outras)



Teoria elástica: $\rho \frac{\partial^2 y}{\partial t^2} = T \frac{\partial^2 y}{\partial x^2} - YI \frac{\partial^4 y}{\partial x^4} = 0$

Módulo de Young Y

Momento de Inércia superficial $I = \pi R^4/4$

(para uma corda cilíndrica)

4 – Transformada de Fourier e a equação de onda dispersiva II

$$\frac{\partial^2 y}{\partial t^2} - c^2 \left(\frac{\partial^2 y}{\partial x^2} - \epsilon \frac{\partial^4 y}{\partial x^4} \right) = 0$$

OBS: Ondas propagantes não são mais soluções: $y(x,t) \neq f(x-ct) + g(x+ct)$

Usando que $$y\,(x,t) = \int_{-\infty}^{\infty} \frac{\tilde{y}\,(k,t)}{2\pi} e^{ikx} \mathrm{d}k = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\tilde{\tilde{y}}\,(k,\omega)}{(2\pi)^2} e^{i(kx-\omega t)} \mathrm{d}k \mathrm{d}\omega$$

Então $$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(-\omega^2 + c^2(k^2 + \epsilon k^4)\right) \tilde{\tilde{y}}\,(k,\omega)\, e^{i(kx-\omega t)} \mathrm{d}k \mathrm{d}\omega = 0, \ \forall x, t$$

Logo, qualquer onda plana é permitida desde que $\boxed{\omega = \pm ck\sqrt{1+\epsilon k^2}}$

Relação de dispersão

4 – Transformada de Fourier e a equação de onda dispersiva II

$$\frac{\partial^2 y}{\partial t^2} - c^2\left(\frac{\partial^2 y}{\partial x^2} - \epsilon\frac{\partial^4 y}{\partial x^4}\right) = 0$$

Logo,
$$y\left(x,t\right) = \int_{-\infty}^{\infty}\frac{\tilde{y}_+\left(k\right)}{2\pi}e^{i(kx-\omega_k t)}\mathrm{d}k + \int_{-\infty}^{\infty}\frac{\tilde{y}_-\left(k\right)}{2\pi}e^{i(kx+\omega_k t)}\mathrm{d}k$$

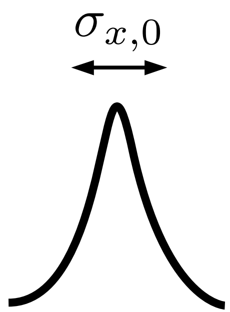
onde
$$\omega_k = \tilde{c}_k k = c\sqrt{1+\epsilon k^2}k$$

As condições iniciais determinam as amplitudes das ondas planas

$$\tilde{y}_+\left(k\right) = \frac{1}{2}\left[\int_{-\infty}^{\infty} y\left(x,0\right)e^{-ikx}\mathrm{d}k - \frac{1}{i\omega_k}\int_{-\infty}^{\infty}\dot{y}\left(x,0\right)e^{-ikx}\mathrm{d}k\right]$$

$$\tilde{y}_-\left(k\right) = \frac{1}{2}\left[\int_{-\infty}^{\infty} y\left(x,0\right)e^{-ikx}\mathrm{d}k + \frac{1}{i\omega_k}\int_{-\infty}^{\infty}\dot{y}\left(x,0\right)e^{-ikx}\mathrm{d}k\right]$$

4 – Transformada de Fourier e a equação de onda dispersiva II

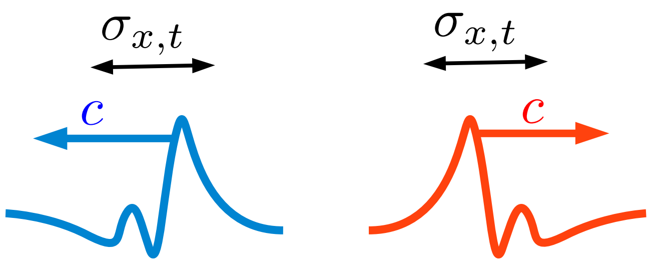


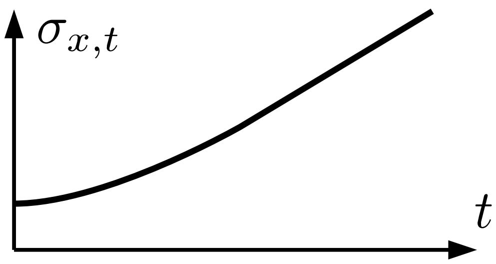
Exemplo: $$y(x,0) = 2 \times \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{x^2}{2\sigma_x^2}} \qquad \dot{y}(x,0) = 0$$

$$\tilde{y}_+(k) = \tilde{y}_-(k) = \frac{1}{2}\int_{-\infty}^{\infty} y(x,0)\, e^{-ikx} \mathrm{d}k = e^{-\frac{1}{2}k^2\sigma_x^2}$$

$$y(x,t) = \frac{1}{2\pi}\left[\int_{-\infty}^{\infty} \tilde{y}_+(k)\, e^{ik(x-\tilde{c}_k t)} \mathrm{d}k + \int_{-\infty}^{\infty} \tilde{y}_-(k)\, e^{ik(x+\tilde{c}_k t)} \mathrm{d}k\right]$$

$=$





5 – Relações de dispersão

$$y(x,t) = \int_{-\infty}^{\infty} \frac{\tilde{y}_+(k)}{2\pi} e^{i(kx-\omega_k t)} \mathrm{d}k + \int_{-\infty}^{\infty} \frac{\tilde{y}_-(k)}{2\pi} e^{i(kx+\omega_k t)} \mathrm{d}k$$

onde $\omega_k = ck$ (meio não-dispersivo)

$\omega_k = \sqrt{c^2k^2 + \epsilon k^4}$ (corda rígida; meios elásticos realistas)

$\omega_k = \sqrt{\omega_0^2 + c^2k^2}$ (luz num plasma; Klein-Gordon; corda num substrato elástico)

$\omega_k = \sqrt{gk \tanh(hk)}$ (ondas de gravidade num fluido)

$\omega_k = \frac{c}{n}k, \quad n^2 = 1 + \frac{\alpha}{1-(k/k_0)^2}$ (luz num meio transparente)

$\omega_k = 2\omega_0 \sin(ka/2)$ (conjunto de osciladores acoplados)

5 – Relações de dispersão

$$y\left(x,t\right)=\int_{-\infty}^{\infty}\frac{\tilde{y}_{+}\left(k\right)}{2\pi}e^{i\left(kx-\omega_k t\right)}\mathrm{d}k+\int_{-\infty}^{\infty}\frac{\tilde{y}_{-}\left(k\right)}{2\pi}e^{i\left(kx+\omega_k t\right)}\mathrm{d}k$$

onde

$$\omega_k = ck$$

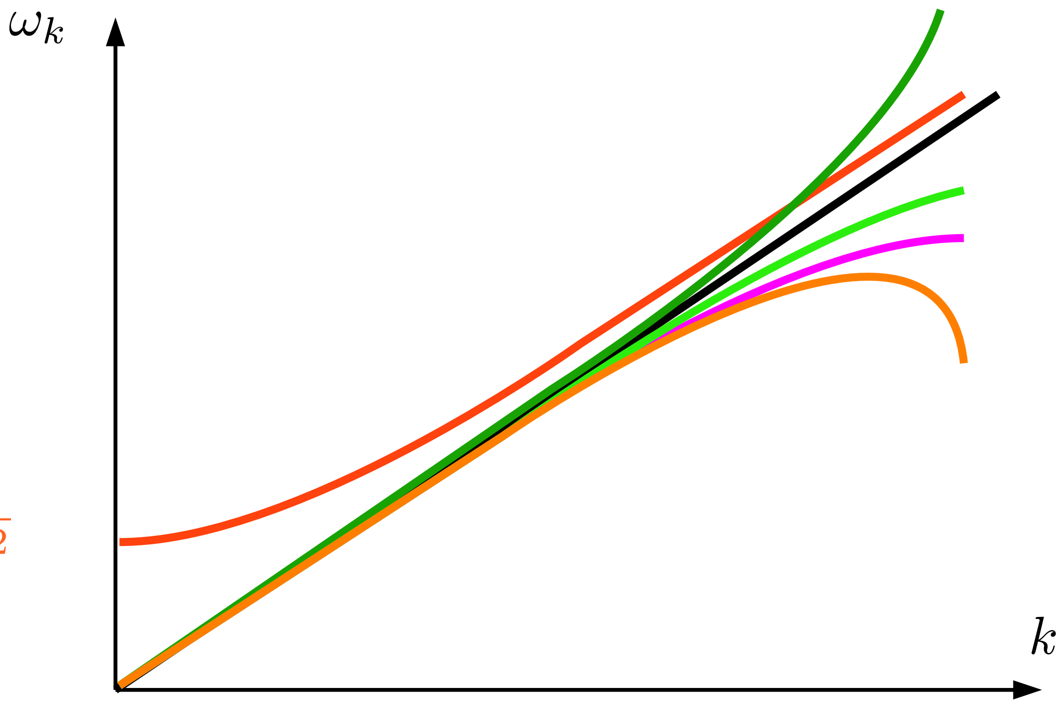
$$\omega_k = \sqrt{c^2k^2+\epsilon k^4}$$

$$\omega_k = \sqrt{\omega_0^2+c^2k^2}$$

$$\omega_k = \sqrt{gk\tanh(hk)}$$

$$\omega_k = \frac{c}{n}k,\quad n^2 = 1+\frac{\alpha}{1-(k/k_0)^2}$$

$$\omega_k = 2\omega_0\sin(ka/2)$$



5 – Relações de dispersão

$$y\left(x,t\right)=\int_{-\infty}^{\infty}\frac{\tilde{y}_{+}\left(k\right)}{2\pi}e^{i(kx-\omega_k t)}\mathrm{d}k+\int_{-\infty}^{\infty}\frac{\tilde{y}_{-}\left(k\right)}{2\pi}e^{i(kx+\omega_k t)}\mathrm{d}k$$

onde

$$\omega_k = ck$$

$$\omega_k = \sqrt{c^2k^2+\epsilon k^4}$$

$$\omega_k = \sqrt{\omega_0^2+c^2k^2}$$

$$\omega_k = \sqrt{gk\tanh(hk)}$$

$$\omega_k = \frac{c}{n}k, \quad n^2 = 1+\frac{\alpha}{1-(k/k_0)^2}$$

$$\omega_k = 2\omega_0\sin(ka/2)$$

Meio não-dispersivo: velocidade de fase não depende do comprimento de onda λ (ou de k)

$$c_f \equiv \frac{\omega_k}{k} = c, \quad \Rightarrow \quad c_g \equiv \frac{\mathrm{d}\omega_k}{\mathrm{d}k} = c = c_f$$

Meio dispersivo “normal”: $c_f > c_g$

Meio dispersivo “anômalo”: $c_f < c_g$

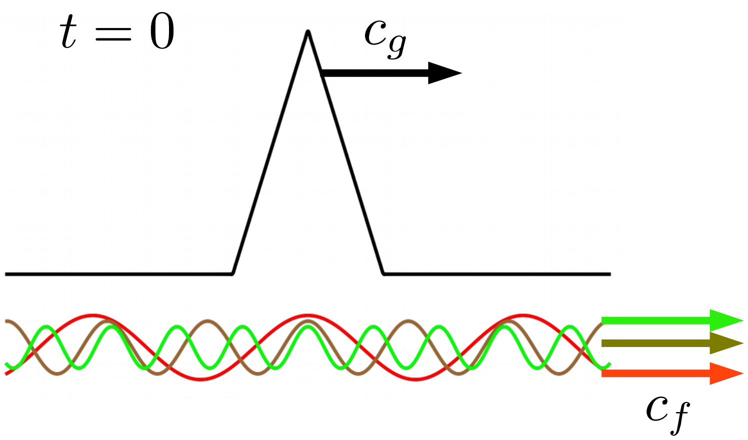
5 – Relações de dispersão

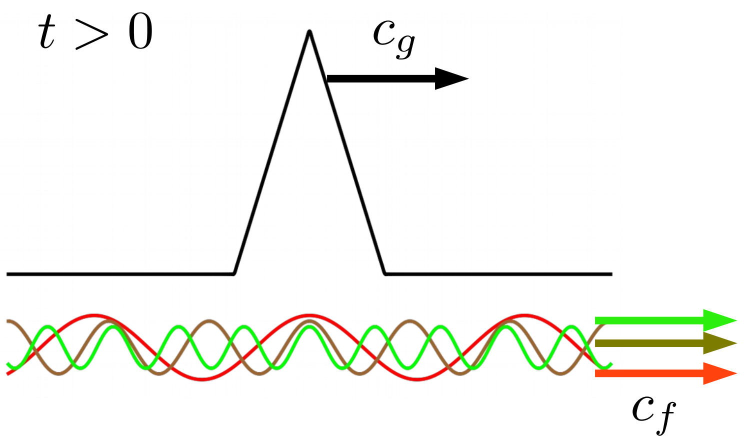
$$y\left(x,t\right)=\int_{-\infty}^{\infty}\frac{\tilde{y}_{+}\left(k\right)}{2\pi}e^{i\left(kx-\omega_{k}t\right)}\mathrm{d}k+\int_{-\infty}^{\infty}\frac{\tilde{y}_{-}\left(k\right)}{2\pi}e^{i\left(kx+\omega_{k}t\right)}\mathrm{d}k$$

onde $\omega_k = ck$

Meio não-dispersivo: velocidade de fase não depende do comprimento de onda λ (ou de k)

$$\mathrm{c}_f \equiv \frac{\omega_k}{k} = c_g \equiv \frac{\mathrm{d}\omega_k}{\mathrm{d}k}$$



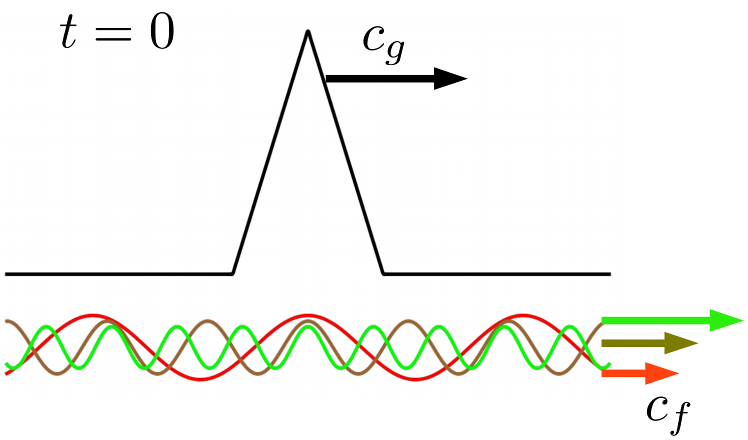


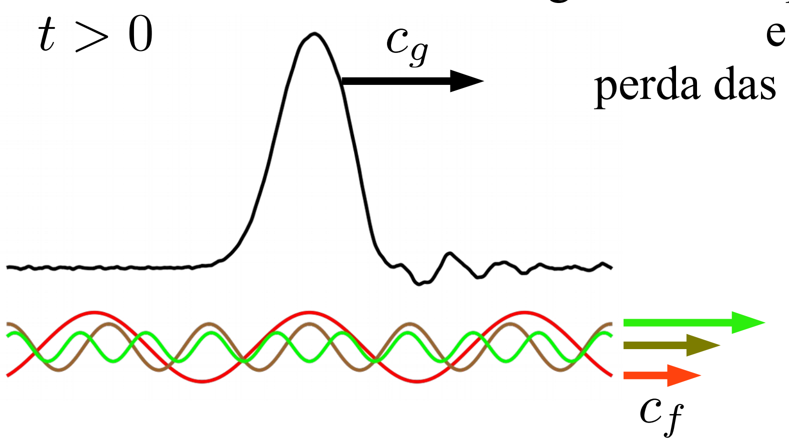
5 – Relações de dispersão

$$y\left(x,t\right)=\int_{-\infty}^{\infty}\frac{\tilde{y}_{+}\left(k\right)}{2\pi}e^{i(kx-\omega_k t)}\mathrm{d}k+\int_{-\infty}^{\infty}\frac{\tilde{y}_{-}\left(k\right)}{2\pi}e^{i(kx+\omega_k t)}\mathrm{d}k$$

onde $\omega_k=\sqrt{c^2k^2+\epsilon k^4}$

Meio dispersivo "anômalo": $c_f < c_g$





alargamento do pacote de onda
e
perda das "pontas"