List of exercises #8 (Continuous media & field theory) - 7600040

1. Consider a perfectly elastic string of linear density ρ under tension T which one end is fixed and the other one is attached to a massless ring. A rigid rod perpendicular to the string passes through this ring. Finally, the ring can move without friction on this rod.

   (a) What are the corresponding boundary conditions?

   (b) What are the normal modes?

   (c) A small pulse propagates on this ring. Determine in details the movement of this pulse. Provide schematic drawings.

2. Lemos problem 10.4.

3. Lemos problem 10.6.

4. Study the Noether theorem in the context of field theory. See, e.g., Aguiar Sec. 11.6.

5. The Lagrangian density of a scalar field $\phi(x,t)$ is $\mathcal{L} = \frac{1}{2}\left(\frac{\partial \phi}{\partial t}\right)^2 - \frac{1}{2}c^2\left[\left(\frac{\partial \phi}{\partial x}\right)^2 - \mu\left(1-\cos\phi\right)\right]$. Compute the corresponding equation of movement (also known as the Sine-Gordon equation).

6. Consider the Lagrangian density $\mathcal{L} = \frac{\hbar^2}{2m}\boldsymbol{\nabla}\psi\cdot\boldsymbol{\nabla}\psi^* + V(\mathbf{r})\,\psi\psi^* + \frac{\hbar}{2i}\left(\psi^*\left(\frac{\partial\psi}{\partial t}\right) - \psi\left(\frac{\partial\psi^*}{\partial t}\right)\right)$ where ψ and ψ^* are two independent fields. (The field ψ is a complex field, which, for a practical purpose, can be interpreted as a real field with two components.)

   (a) Show that the equations of movement are the Schrödinger's equation $-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial\psi}{\partial t}$ and its complex conjugate.

   (b) What are the canonical momenta? Obtain the corresponding Hamiltonian density.

7. Show that $G_i = -\int \pi_k\left(\partial_i\phi_k\right)dV$ (Einstein notation being used) is a constant of the motion if the Hamiltonian density is not an explicit function of position. The quantity G_i can be identified as the total linear momentum of the field along the x_i direction. The similarity of this theorem with the usual conservation theorem for linear momentum of discrete systems should be obvious.