List of exercises #1 (Newton's laws and 1D dynamics) - 7600018

For Test1, consider only the problems marked with ♣.

1. Compute the gravitational force of attraction between an electron and a proton at a separation of 0.5 Å. Compare with the electrostatic force of attraction at the same distance.

2. A fluid flows through a cylindrical pipe of length L and radius R. A pressure difference ΔP (force per unit area) causes a flux I (volume per second) to flow through the pipe. Assume that ΔP is proportional to L and depends otherwise only on I, on the radius R of the pipe, and on the viscosity η. Show from dimensional considerations that ΔP must also be proportional to η and to I and inversely proportional to R^4.

3. An object A moving with velocity $\mathbf{v}$ collides with a stationary object B. After the collision, A is moving with velocity $\frac{1}{2}\mathbf{v}$ and B with velocity $\frac{3}{2}\mathbf{v}$. Find the ratio of their masses. If, instead of bouncing apart, the two bodies stuck together after the collision, with what velocity would they then move?

4. The two components of a double star are observed to move in circles of radii R_1 and R_2. What is the ratio of their masses? (Hint: Write down their accelerations in terms of the angular velocity of rotation.)

5. Consider a transformation to a relatively uniformly moving frame of reference, where each position vector $\mathbf{r}_i$ is replaced by $\mathbf{r}_i = \mathbf{r}_i - \mathbf{v}t$. (Here $\mathbf{v}$ is a constant, the relative velocity of the two frames.) How does a relative position vector $\mathbf{r}_{ij} = \mathbf{r}_j - \mathbf{r}_i$ transform? How do momenta and forces transform?

6. The sun is about 25,000 light years from the center of the galaxy, and travels approximately in a circle at a speed of 230 Km/s. Find the approximate mass of the galaxy by assuming that the gravitational force on the sun can be calculated as if all the mass of the galaxy were at its center. Express the result as a ratio of the galactic mass to the sun's mass. You do not need to look up either G or the sun's mass to do this problem if you compare the revolution of the sun around the galactic center with the revolution of the earth about the sun.

7. ♣ A high-speed proton of electric charge e moves with constant speed v_0 in a straight line past an electron of mass m and charge $-e$, initially at rest. The electron is at a distance a from the path of the proton.

   (a) Assume that the proton passes so quickly that the electron does not have time to move appreciably from its initial position until the proton is far away. Show that the component of force in a direction perpendicular to the line along which the proton moves is
   $$F = \frac{e^2 a}{4\pi\epsilon_0 \left(a^2 + v_0^2 t^2\right)^{3/2}},$$
   where $t = 0$ when the proton passes closest to the electron.

   (b) Calculate the impulse delivered by this force.

   (c) Write the component of the force in a direction parallel to the proton velocity and show that the net impulse in that direction is zero.

   (d) Using these results, calculate the (approximate) final momentum and final kinetic energy of the electron.

   (e) Show that the condition for the original assumption in part (a) to be valid is $\frac{e^2}{4\pi\epsilon_0 a} \ll \frac{1}{2}mv_0^2$.

8. ♣ A particle of mass m at rest at $t = 0$ is subject to a force $F(t) = F_0 \sin^2 \omega t$.

   (a) Sketch the form you expect for $v(t)$ and $x(t)$, for several periods of oscillation of the force.

   (b) Find $v(t)$ and $x(t)$ and compare with your sketch.

9. ♣ A particle which had originally a velocity v_0 is subject to a force $F(t) = \frac{p_0}{\delta t}\Theta(t - t_0)\Theta(t_0 + \delta t - t)$, where $\Theta(x)$ is the Heaviside step function.

   (a) Find $v(t)$ and $x(t)$.

   (b) Show that as $\delta t \to 0$, the motion approaches that of a constant velocity with an abrupt change in velocity at $t = t_0$ of amount p_0/m.

10. A weight of mass m is hung from the end of a spring which provides a restoring force equal to k times its extension. The weight is released from rest with the spring unextended. Find its position as a function of time, assuming negligible damping.

11. ♣ A particle of mass m moves (in the region $x > 0$) under a force $F = -kx + c/x$, where k and c are positive constants. Find the corresponding potential energy function. Determine the position of equilibrium, and the frequency of small oscillations about it.

12. A particle of mass m has the potential energy function $V(x) = mk|x|$, where k is a positive constant. What is the force when $x > 0$, and when $x < 0$? Describe the motion of this particle. If it starts from rest at $x = -a$, find the time it takes to reach $x = a$.

13. A particle of mass m moves under a conservative force with potential energy function given by $V(x) = \frac{1}{2}k\left(x^2 - a^2\right)\Theta\left(a - |x|\right)$, where a and k are constants, and $a > 0$.

    (a) What is the force on the particle?

    (b) Sketch the function V, for both cases $k > 0$ and $k < 0$, and describe the possible types of motion.

    (c) Let $k = m\omega^2 > 0$. The particle is initially in the region $x < -a$, moving to the right with velocity v. When it emerges into the region $x > a$, will it do so earlier or later than if it were moving freely under no force? Find an expression for the time difference. (To do the required integral, use a substitution of the form $x = \text{const} \times \sin\theta$.)

14. The potential energy function of a particle of mass m is $V = -V_0\left((x/a)^2 - 1\right)^2$, where V_0 and a are positive constants.

    (a) Sketch this function, and describe the possible types of motion in the three cases $E \geq 0$, $E < -V_0$, and $-V_0 \leq E < 0$.

    (b) Calculate the associate force.

    (c) Find $x(t)$ for the case $E = 0$, choosing x_0 and t_0 in any convenient way. Show that your result agrees with the qualitative discussion in part (a) for this particular energy.

15. Find the solution for the motion of a body subject to a linear repelling force $F = kx$. Show that this is the type of motion to be expected in the neighborhood of a point of unstable equilibrium.

16. The equation of motion of a particle falling under constant gravity in a straight line is $m\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -mg$. Solve equation by the methods of

    (a) directly integrating a time-dependent force $\int_{t_0}^{t}\int_{t_0}^{t'} F\left(t''\right)\mathrm{d}t''$,

    (b) directly integrating a velocity-dependent force $\int_{v_0}^{v}\frac{\mathrm{d}v'}{F(v')}$,

    (c) and by energy conservation $\int_{x_0}^{x}\frac{\mathrm{d}x'}{\sqrt{E-V(x')}}$.

17. ♣ A particle falling under gravity in a straight line is subject to a retarding force proportional to its velocity. Find its position as a function of time, if it starts from rest, and show that it will eventually reach a terminal velocity.

18. A particle moves vertically under gravity with a retarding force proportional to the square of its velocity. The resulting acceleration is $\dot{v} = \mp g - kv^2$, where k is a constant.

    (a) If the particle is moving upwards $(-g)$, show that its position at time t is given by $z = z_0 + (1/k)\ln\cos\left[\sqrt{gk}(t_0 - t)\right]$, where z_0 and t_0 are integration constants. If its initial velocity at $t = 0$ is v_0, find the time at which it comes instantaneously to rest, and its corresponding height. [You may find the identity $\ln\cos x = -\frac{1}{2}\ln(1 + \tan^2 x)$ useful.]

    (b) Show that if the particle falls $(+g)$ from rest its speed after a time t is given by $v = \sqrt{g/k}\tanh\left(\sqrt{gk}t\right)$. What is its limiting speed? How long does it take to hit the ground if dropped from height h?

19. Find the motion of a body projected upward from the Earth with a velocity equal to the escape velocity. Neglect air resistance.

20. A mass m subject to a linear restoring force $-kx$ and damping $-b\dot{x}$ is displaced a distance x_0 from equilibrium and released with zero initial velocity. Find the motion in the underdamped, critically damped, and overdamped cases.

21. A force $F_0 e^{-t/\tau}$ (F_0 and $\tau > 0$ being constants) acts on a harmonic oscillator of mass m, spring constant k, and damping constant b. Find a particular solution of the equation of motion by starting from the guess that there should be a solution with the same time dependence as the applied force.

22. An undamped harmonic oscillator ($b = 0$), initially at rest, is subject beginning at $t = 0$ to an applied force $F_0 \sin \omega t$. Find the motion $x(t)$.

23. ♣ Find $x\,(t)$ for a critically damped harmonic oscillator subject to an impulse p_0 delivered at $t = 0$.

24. Find, by the Fourier-series method, the steady-state solution for an undamped harmonic oscillator subject to a force having the form of a rectified sine-wave $F(t) = F_0 \left|\sin \omega_0 t\right|$, where ω_0 is the natural frequency of the oscillator.

25. Using the result of Problem 23, find by Green's method the motion of a critically damped oscillator initially at rest and subject to a force $F(t)$.