List of exercises #3 (system of particles) - 7600018

For Test2, consider only the problems marked with ♠.

1. Formulate and prove a conservation law for the angular momentum about the origin of a system of particles confined to a plane.

2. Water is poured into a barrel at a constant rate λ (mass per unit of time) from a height of H. The barrel mass is M, and rests on a scale. Find the scale reading as a function of time after the water has been pouring into the barrel.

3. A box of mass m falls (without bouncing) on a conveyor belt which moves with constant speed v. The coefficient of sliding friction between the box and the belt is μ.

   (a) How far does the box slide along the belt before it is moving with the same speed as the belt?

   (b) What force F must be applied to the belt to keep it moving at constant speed after the box falls on it, and for how long?

   (c) Calculate the impulse delivered by this force and check that momentum is conserved between the time before the box falls on the belt and the time when the box is moving with the belt.

   (d) Calculate the work done by the force F in pulling the belt.

   (e) Calculate the work dissipated in friction between the box and the belt.

   (f) Check that the energy delivered to the belt by the force F is just equal to the kinetic energy increase of the box plus the energy dissipated in friction. Notice that this result does not depend on μ, i.e., even if μ were infinity and the box does not slide, the result would be the same.

4. ♠ A spherical satellite of mass m and radius R moves with speed v through a tenuous atmosphere of density ρ.

   (a) Find the frictional force on it assuming that the speed of the air molecules can be neglected in comparison with v, and that each molecule which is struck becomes embedded in the skin of the satellite. Do you think these assumptions are valid? (Optional) Alternatively, you can consider that each molecule colides elastically with the satellite and neglect the interactions among the atmosphere molecules.

   (b) If the orbit is a circle 400 km above the earth (radius 6360 km), where $\rho = 10^{-11}$ kg/m^3, $R = 1$, and $m = 100$ kg, find the change in altitude and the change in period of revolution in one week.

5. ♠ A rocket is to be fired vertically upward. The initial mass is M_0, the exhaust velocity $-u$ is constant, and the rate of exhaust $-(\mathrm{d}M/\mathrm{d}t) = \gamma$ is constant. After a total mass ΔM is exhausted, the rocket engine runs out of fuel.

   (a) Neglecting air resistance and assuming that the acceleration g of gravity is constant, set up and solve the equation of motion.

   (b) Show that if M_0, u, and ΔM are fixed, then the larger the rate of exhaust γ, that is, the faster it uses up its fuel, the greater the maximum altitude reached by the rocket.

6. ♠ A particle of mass m_1 and energy T_{1I} collides elastically with a particle of mass m_2 at rest. If the mass m_2 leaves the collision at an angle θ_2 with the original direction of motion of m_1, what is the energy T_{2F} delivered to particle m_2? Show that T_{2F} is a maximum for a head-on collision, and that in this case the energy lost by the incident particle in the collision is

$$T_{1I} - T_{1F} = \frac{4m_1m_2}{(m_1+m_2)^2}T_{1I}.$$

7. A proton of mass m_1 collides elastically with an unknown nucleus in a bubble chamber and is scattered through an angle θ_1. The ratio P_{1F}/P_{1I} is determined from the curvature of its initial and final tracks. Find the mass m_2 of the target nucleus. How might it be possible to determine whether the collision was indeed elastic?

8. ♠ Calculate the energy loss $-Q$ for a head-on collision between a particle of mass m_1 and velocity v_1 with a particle of mass m_2 at rest, if the coefficient of restitution is e. What is Q when $e = 1$? What kind of collision is that?

9. A billiard ball sliding on a frictionless table ,strikes an identical stationary ball. The balls leave the collision at angles $\pm\theta$ with the original direction of motion. Show that after the collision the balls must have a rotational energy equal to $1 - \frac{1}{2\cos^2\theta}$ of the initial kinetic energy, assuming that no energy is dissipated in friction.

10. A neutral particle of unknown momentum and direction produces a reaction in a bubble chamber in which two charged particles of masses m_3 and m_4 emerge with momenta $\mathbf{P}_3$ and $\mathbf{P}_4$. The angle between their tracks is α. Find the direction and momentum of the incident particle. If the mass m_1 of the incident particle is known or suspected, find the energy Q released in the reaction. (Assume nonrelativistic velocities.)

11. ♠ Work out a generalization of Kepler's Third Law $\tau^2 = \frac{4\pi^2}{GM}a^3$ which takes into account the motion of the central mass M under the influence of the revolving mass m. A pair of stars revolve about each other, so close together that they appear in the telescope as a single star. It is determined from spectroscopic observations that the two stars are of equal mass and that each revolves in a circle with speed v and period τ under the gravitational attraction of the other. Find the mass of each star by using your formula.

12. ♠ A star of mass m and initial speed v_0, approaches a second star of mass $2m$ at rest. The first star travels initially along a line which if continued would pass the second star at a distance b. Find the final speed and direction of motion of each star.

13. Find an expression analogous to Eq. (4.116) (of Symon's book) for the angle of recoil of the target particle (θ_2 in Fig. 4.7) in terms of the scattering angle Θ in the equivalent one-body problem. Show that, for an elastic collision, $\theta_2 = \frac{1}{2}(\pi - \Theta)$.

14. ♠ An elastic sphere of radius R collides with an identical elastic sphere at rest. Assume that in the center of mass coordinate system, each sphere rebounds from the other so that the relative velocities before and after impact make equal angles with the normal to the spheres at the point of contact. Find the cross section for scattering the incident sphere through an angle θ_1.

15. ♠ A pair of masses m_1 and m_2 connected by a spring of force constant k slide without friction along the x-axis. Show that the center of mass moves with uniform velocity and that the masses oscillate with frequency $\sqrt{\frac{k(m_1+m_2)}{m_1 m_2}}$.