List of exercises #4 (moving coordinate systems) - 7600018

1. A mass m is fastened by a spring (spring constant k) to a point of support which moves back and forth along the x-axis in simple harmonic motion at frequency ω and amplitude A. Assuming the mass moves only along the x-axis, set up and solve the equation of motion in a coordinate system whose origin is at the point of support.

2. Find the velocity relative to an inertial frame (in which the center of the Earth is at rest) of a point on the Earth's equator. An aircraft is flying above the equator at 1000 km/h. Assuming that it flies straight and level (i.e., at a constant altitude above the surface) what is its velocity relative to the inertial frame

   (a) if it flies north?

   (b) if it flies west?

   (c) and if it flies east?

   (d) The apparent weight of the aircraft when on the ground at the equator is 100 t weight. What is its apparent weight in each of the three cases (a)–(c)?

3. Water in a rotating container of radius 50 mm is 30 mm lower in the center than at the edge. Find the angular velocity of the container.

4. Consider a freely falling body near the Earth's surface.

   (a) Solve the problem by introducing a translating coordinate system with an acceleration $\mathbf{g}$. Set up and solve the equations of motion in this accelerated coordinate system and transform the result back to a coordinate system fixed relative to the Earth. (Neglect the Earth's rotation.)

   (b) In the same accelerated coordinate system, set up the equations of motion for a falling body subject to an air resistance proportional to its velocity (relative to the fixed air).

5. (Optional) A projectile is launched due north from a point in colatitude θ at an angle $\pi/4$ to the horizontal, and aimed at a target whose distance is y (small compared to Earth's radius R). Show that if no allowance is made for the effects of the Coriolis force, the projectile will miss its target by a distance $x = \omega\sqrt{\frac{2y^3}{g}}\left(\cos\theta - \frac{1}{3}\sin\theta\right)$. Evaluate this distance if $\theta = 45°$ and $y = 40$ km. Why is it that the deviation is to the east near the north pole, but to the west both on the equator and near the south pole? (Neglect atmospheric resistance.)

6. Westerly winds blow from west to east in the northern hemisphere with an average speed v. If the density of the air is ρ, what pressure gradient is required to maintain a steady flow of air from west to east with this speed? (On a static fluid the pressure gradient counter balances the density of external force $\mathbf{f}$. Thus, $\boldsymbol{\nabla} P = \mathbf{f}$.) Make reasonable estimates of v and ρ, and estimate the pressure gradient.

7. It has been suggested that birds may determine their latitude by sensing the Coriolis force. Calculate the force a bird must exert in level flight at velocity v against the sidewise component of Coriolis force in order to fly in a straight line.

8. A body is dropped from rest at a height h above the surface of the earth.

   (a) Calculate the Coriolis force as a function of time assuming, as a first approximation, that it has a negligible effect on the motion, and using the velocity of a freely falling body with acceleration $\mathbf{g}_e$. Neglect air resistance, and assume h is small so that $\mathbf{g}_e$ can be taken as constant.

   (b) Now, as a second approximation, calculate the net displacement of the point of impact due to the Coriolis force calculated in part (a).

   (c) Find the answer to (b) by solving for the motion in a nonrotating coordinate system. What approximations are needed to arrive at the same result?

9. Assume the Earth is a uniform sphere of mass M and radius R. Imagine a pipe extending vertically from the North pole to the center of the Earth and out again at right angles to the equator. The pipe is filled with a fluid so that the fluid level at the North pole is at the Earth's surface. Find the fluid level relative to the surface of the sphere at the equator. Does it change the answer much if the pipe runs near the surface of the earth? Does it change the answer much if you use the actual shape of the earth?

10. A gyroscope consists of a homogeneous disk of radius R and mass M. The gyroscope is rotating with angular velocity $\dot{\theta}$ about its axis, which is horizontal and is fixed relative to the Earth's surface. We choose a coordinate system at rest relative to the Earth whose x-axis coincides with the gyroscope axis and points to South, the z-axis points vertically upwards, and whose origin lies at the center of the disk. The angular velocity $\boldsymbol{\omega}$ of the Earth lies in the xz-plane, points to North, and makes an angle α with the gyroscope axis.

    (a) Find the x-, y-, and z-components of the torque $\mathbf{N}$ about the origin, due to the Coriolis force in the xyz-coordinate system, acting on a mass $\mathrm{d}M$ on the rim of the gyroscope wheel whose polar coordinates, in the zy-plane, are (r,θ).

    (b) Use this result to show that the total Coriolis torque on the gyroscope is $\mathbf{N} = -\frac{1}{2}MR^2\omega\dot{\theta}\sin\alpha\hat{y}$.

11. A particle moves in the xy-plane under the action of a force $\mathbf{F} = -k\mathbf{r}$, with $k > 0$. Find its possible motions by introducing a coordinate system rotating about the z-axis with angular velocity ω chosen so that the centrifugal force just cancels the force $\mathbf{F}$, and solving the equations of motion in this coordinate system. Describe the resulting motions, and show that your result agrees with that of 16 of list of exercises #2.

12. An electron revolves about a fixed proton in an ellipse of semimajor axis 10^{-8} cm. If the corresponding motion occurs in a magnetic field of 10,000 gausses, show that Larmor's theorem is applicable, and calculate the angular velocity of precession of the ellipse.

13. A beam of particles of charge q, mass m, and velocity v is emitted from a point source, roughly parallel with a magnetic field $\mathbf{B}$, but with a small angular dispersion. Show that the effect of the field is to focus the beam to a point at a distance $z = 2\pi mv/(|q|B)$ from the source. Calculate the focal distance for electrons of kinetic energy 500 eV in a magnetic field of 0.01 T. ($q_e = -1.6 \times 10^{-19}$ C, $m_e = 9.1 \times 10^{-31}$ kg, 1 eV $= 1.6 \times 10^{-19}$ J.)