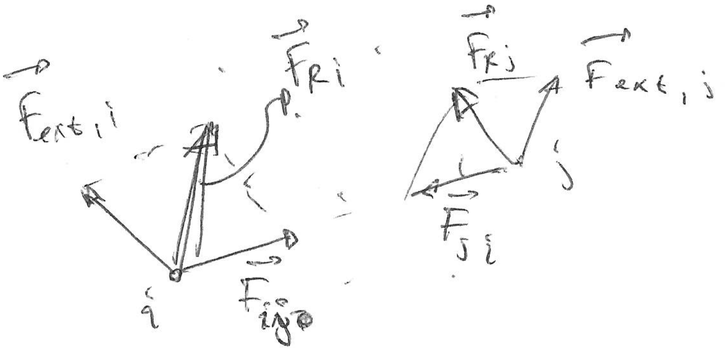
SISTEMAS DE MUITAS PARTÍCULAS

1



$$\vec{P}_i = m_i \vec{v}_i$$

$$\vec{F}_{R,i} = \frac{d\vec{P}_i}{dt}$$

$$\vec{F}_{R,j} = \frac{d\vec{P}_j}{dt}$$

$$\vec{P}_{TOT} = \vec{P}_1 + \vec{P}_2 + \dots + \vec{P}_i + \dots$$

$$\frac{d\vec{P}_{TOT}}{dt} \; ?$$ O QUE É ISSO?

$$\frac{d\vec{P}_{TOT}}{dt} = \frac{d\vec{P}_1}{dt} + \frac{d\vec{P}_2}{dt_2} + \dots$$

$$= \vec{F}_{ext\,1} + \sum_k \vec{F}_{1,k} + \vec{F}_{ext,2} + \sum_k \vec{F}_{2,k} + \dots$$

$$= \sum_i \vec{F}_{ext,i} + \sum_{ij} \vec{F}_{ij} = \vec{F}_{ext} + \sum \vec{F}_{INTERNAS}$$

mas, pela 3ª LEI DE NEWTON

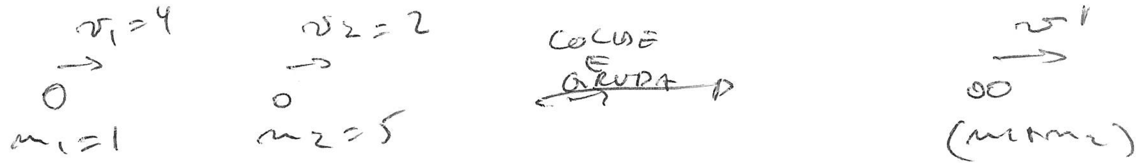
$$\vec{F}_{ij} = -\vec{F}_{ji} \Rightarrow \sum \vec{F}_{INTERNAS} = 0$$

logo $\boxed{\dfrac{d\vec{P}_{TOT}}{dt} = \vec{F}_{ext}}$

Logo, se $\vec{F}_{ext} = 0 \Rightarrow \vec{P}_{tot}$ = CONSTANTE!!!

CONSERVAÇÃO DO MOMENTO LINEAR!

VAMOS OLHAR ALGUNS EXEMPLOS:



$P_1 = m_1 v_1 = 1 \times 4 = 4 \ kg \frac{m}{s}$

$P_2 = m_2 v_2 = 5 \times 2 = 10 \ kg \frac{m}{s}$

$P' = (m_1 + m_2) v'$

Como $\vec{P}_{total}$ = constante

$$\Rightarrow 4 + 10 = 6 \times v' \Rightarrow \boxed{v' = \frac{7}{3} \frac{m}{s}}$$

$$Ec_{antes} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} \times 1 \times 16 + \frac{1}{2} \times 5 \times 4 = 8 + 10 = 18 \ J$$

$$Ec_{depois} = \frac{1}{2}(m_1 + m_2) v'^2 = \frac{1}{2} \times 6 \times \left(\frac{7}{3}\right)^2 = \frac{49}{3} = 16.3 \ J$$

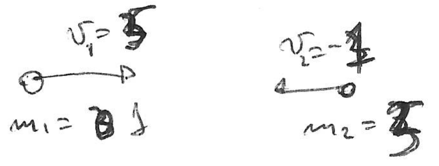
QUEM SE IMPORTA COM 1.7 J?

MAS

$$P_{total} = -6 \ kg \frac{m}{s} \Rightarrow v' = \frac{-6}{6} = -1.0 \ m/s$$

$$Ec_{depois} = \frac{1}{2}(6) \times \left(\frac{1}{6}\right)^2 = 3J =$$

QUASE TODA A ENERGIA CINÉTICA É PERDIDA!!!



$\Rightarrow$ $P_1 = 1 \times 5 = 5$ kg m/s

$P_2 = -5$ kg m/s

$\Rightarrow \vec{P}_{TOT} = 0$

OUTRA INTERPRETAÇÃO DE $\vec{P}_{TOT}$:

$$\vec{P}_{TOT} = m_1 \vec{v}_1 + m_2 \vec{v}_2 + \ldots$$

se $\vec{v}_1 = \vec{v}_2 = \vec{v}_3 = \ldots = \vec{v}$ $\Rightarrow$ $\vec{P}_{TOT} = (m_1 + m_2 + m_3 + \ldots)\,\vec{v}$

$\equiv$ $\vec{P}_{TOT}$ é como uma ÚNICA PARTÍCULA DE MASSA $M = m_1 + m_2 + \ldots$ se movendo com $\vec{v}_{CM}$

MESMO ~~p~~ ~~p~~ $\vec{v}_1 \neq \vec{v}_2 \neq \vec{v}_3 \neq \ldots$

PODEMOS DEFINIR

$$\vec{P}_{TOT} = M_{TOT} \vec{V}_{CM}$$

$\Rightarrow$ É COMO SE A $\vec{F}_{ext}$ ATUASSE NESSA PARTÍCULA FICTÍCIA DE MASSA M_{TOT} E POSIÇÃO?

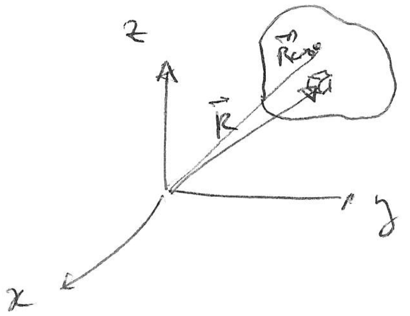
$$M_{TOT} \vec{v}_{CM} = m_1 \vec{v}_1 + m_2 \vec{v}_2 + \ldots$$

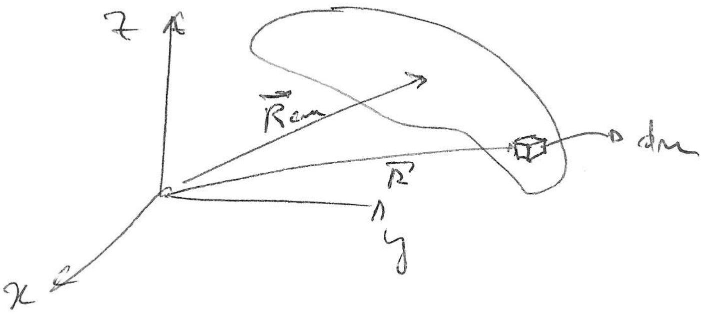
$$\frac{d}{dt} M_{TOT} \vec{R}_{CM} = \frac{d}{dt}\left(m_1 \vec{r}_1 + m_2 \vec{r}_2 + \ldots\right) \Rightarrow \boxed{M_{TOT} \vec{R}_{CM} = \sum_i m_i \vec{r}_i}$$

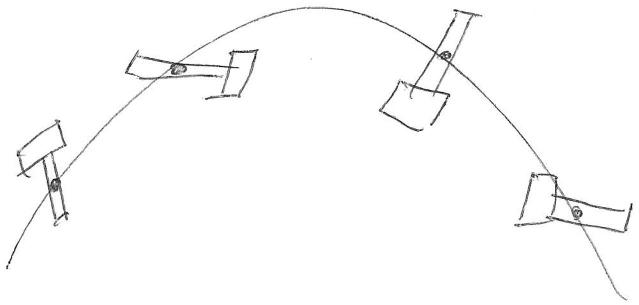
POSIÇÃO DO CENTRO DE MASSA

CENTRO DE MASSA

$$M_{tot}\vec{R}_{cm} = \sum_i m_i \vec{r}_i$$
$$= \int \vec{R}\, dm$$







→ JOGAR TRIÂNGULO

Ñ SE PODE MUDAR $\vec{V}_{cm}$ SÓ

POR FORÇAS INTERNAS !!!

— BARÃO DE MUNCHAUSSEN

EX: TIRO, $v = 300$ m/s

$m = 20$ gramas

DESAFIO

EXPLICAR O BALANÇO

ENERGIA POTENCIAL GRAVITACIONAL

$$U = \sum_i m_i g h_i = g \sum_i m_i h_i$$

$$= g\, M H_{cm}$$

ENERGIA CINÉTICA

$$E_c = \sum_i \frac{1}{2} m_i v_i^2 = \sum_i \frac{1}{2} m_i \left(\vec{v}_i \cdot \vec{v}_i\right)$$

$$\vec{v}_i = \vec{v}_{cm} + \vec{u}_i$$

$$\Rightarrow E_c = \sum_i \frac{1}{2} m_i \left(\vec{v}_{cm} + \vec{u}_i\right) \cdot \left(\vec{v}_{cm} + \vec{u}_i\right)$$

$$= \sum_i \frac{1}{2} m_i \left(v_{cm}^2 + 2\, \vec{v}_{cm} \cdot \vec{u}_i + u_i^2 \right)$$

$$= \frac{1}{2}\left(\sum_i m_i\right) v_{cm}^2 + \vec{v}_{cm} \cdot \underbrace{\left(\sum_i m_i \vec{u}_i\right)}_{\text{MOMENTO EM RELAÇÃO AO CM} = 0} + \frac{1}{2} \sum_i m_i u_i^2$$

$$\Rightarrow E_c = \frac{1}{2} M_{TOT} v_{cm}^2 + \frac{1}{2} \sum_i m_i u_i^2$$

$$= E_{c,CM} + E_{c,\text{INTERNA}}$$

AULA PASSADA

CENTRO DE MASSA:

$$M_{tot}\vec{R}_{cm} = \sum_i m_i \vec{r}_i = \int \vec{r}\, dm$$

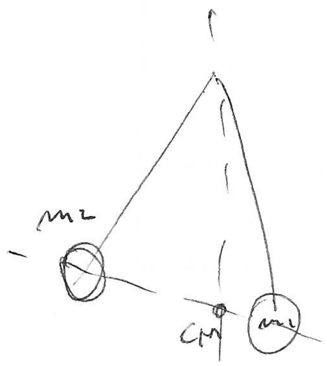
FORÇA EXTERNA:

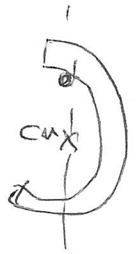
$$\vec{F}_{ext} = \frac{d\vec{P}_{tot}}{dt} = \frac{d}{dt}\left(M_{tot}\vec{V}_{cm}\right)$$

SE $\vec{F}_{ext} = 0 \Rightarrow \vec{P}_{tot} \equiv$ CONSTANTE — CONSERVAÇÃO DO MOMENTO LINEAR

ENERGIA POTENCIAL

$$U = \sum_i m_i g h_i = M_{tot} h_{cm} g$$





← MINIMIZAR U com relação ao C.M.

ENERGIA CINÉTICA

$$E_c = \sum_i \frac{1}{2} m_i v_i^2 \qquad \text{mas} \quad \vec{v}_i = \vec{V}_{cm} + \vec{u}_i$$

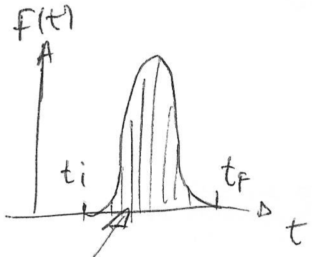
$$= \sum_i \frac{1}{2} m_i \left|\vec{V}_{cm} + \vec{u}_i\right|^2 = \sum_i \frac{1}{2} m_i \left(V_{cm}^2 + 2\vec{V}_{cm}\cdot\vec{u}_i + u_i^2\right)$$

$$= \frac{1}{2} V_{cm}^2 \sum_i m_i + \vec{V}_{cm}\cdot \underbrace{\sum_i m_i \vec{u}_i}_{\text{MOMENTO DO CM EM RELAÇÃO AO CM} = 0} + \sum_i \frac{1}{2} m_i u_i^2$$

$$\boxed{E_c = \underbrace{\frac{1}{2} M_{tot} V_{cm}^2}_{E_c \text{ DO CM}} + \underbrace{\sum_i \frac{1}{2} m_i u_i^2}_{E_c \text{ INTERNA}}}$$

IMPULSO

FORÇA SOBRE UM OBJETO :



$$\vec{I} \equiv \int_{t_i}^{t_F} \vec{F}(t)\, dt = \text{ÁREA}$$

MAS $\vec{F} = \frac{d\vec{p}}{dt} \Rightarrow \vec{I} = \int_{t_i}^{t_F} \frac{d\vec{p}}{dt}\, dt = \vec{P}_f - \vec{P}_i$

$$\boxed{\vec{I} = \Delta\vec{P}} \equiv \text{VARIAÇÃO DO MOMENTO}$$

FORÇA MÉDIA : $\vec{F}_{MED} = \frac{1}{t_f - t_i} \int_{t_i}^{t_f} \vec{F}\, dt = \frac{1}{\Delta t} \vec{I}$

EX: BATIDA DE CARRO FRONTAL
QUAL É A F_{MED}?

COLISÕES

ELÁSTICAS : CONSERVAM ENERGIA CINÉTICA

INELÁSTICAS : Ñ CONSERVAM ENERGIA "

- PERFEITAMENTE INELÁSTICA → PERDA DE ENERGIA É MÁXIMA

EM 1D

ELÁSTICA : m_1 O $\vec{v}_1$ $\quad$ m_2 $\vec{v}_2$ O $\quad$ m_1 O $\vec{v}_1'$ $\quad$ O m_2 $\vec{v}_2'$

2 SOLUÇÕES $v_1 = v_1'$ $v_2 = v_2'$

SOL. TRIVIAL

☆ $m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$

☆ $\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$

$m_1 (v_1 - v_1') = m_2 (v_2' - v_2)$

$\frac{1}{2} m_1 (v_1^2 - v_1'^2) = \frac{1}{2} m_2 (v_2'^2 - v_2^2)$

$\frac{1}{2} m_1 (v_1 + v_1')(v_1 - v_1') = \frac{1}{2} m_2 (v_2' - v_2)(v_2' + v_2)$

$$\boxed{v_1 + v_1' = v_2 + v_2'}$$

Ou $\boxed{\underbrace{v_1 - v_2}_{\text{APROXIMAÇÃO}} = \underbrace{v_2' - v_1'}_{\text{AFASTAMENTO}}}$ DESDE QUE $v_1 \neq v_1'$, $v_2 \neq v_2'$

8

↳ SUBSTITUIR NA EQ. $\vec{P_i} = \vec{P_f}$ ⟹ OBTEM-SE O RESULTADO NO FINAL DA PÁGINA

EM GERAL :

$$v_2' = v_2 + \frac{m_1}{m_2}(v_1 - v_1') = \left(v_2 + \frac{m_1}{m_1} v_1\right) - \frac{m_1}{m_2} v_1'$$

$$\Rightarrow m_1 v_1^2 + m_2 v_2^2 = m_1 v_1'^2 + m_2\left(\left(v_2 + \frac{m_1}{m_2}v_1\right)^2 - 2\frac{m_1}{m_2}\left(v_2 + \frac{m_1}{m_2}v_1\right)v_1' + \frac{m_1^2}{m_2^2}v_1'^2\right)$$

$$= m_1\left(1 + \frac{m_1}{m_2}\right)v_1'^2 - 2m_1\left(v_2 + \frac{m_1}{m_2}v_1\right)v_1' + \cancel{m_2 v_2^2} + \frac{m_1^2}{m_2}v_1^2 + 2m_1 v_1 v_2$$

$$\Rightarrow \boxed{(m_1+m_2)v_1'^2 - 2(m_1 v_1 + m_2 v_2)v_1' + (m_1 - m_2)v_1^2 + 2m_2 v_1 v_2 = 0}$$

$$v_1' = \frac{2(m_1 v_1 + m_2 v_2) \pm \sqrt{4(m_1^2 v_1^2 + 2m_1 m_2 v_1 v_2 + m_2^2 v_2^2) - 4(m_1+m_2)\left((m_1-m_2)v_1^2 + 2m_2 v_1 v_2\right)}}{2(m_1+m_2)}$$

$$= \frac{(m_1 v_1 + m_2 v_2) \pm \sqrt{(m_2^2)v_1^2 + 2m_1 m_2 v_1 v_2 + m_2^2 v_2^2 - 2(m_1+m_2)m_2 v_1 v_2}}{(m_1+m_2)}$$

$$= \frac{m_1 v_1 + m_2 v_2 \pm |m_2(v_1 - v_2)|}{m_1+m_2} \Rightarrow \begin{cases} v_1 \\ \dfrac{(m_1-m_2)v_1 + 2m_2 v_2}{m_1+m_2} \end{cases}$$

Logo $\begin{cases} 1^{\circ} \text{ SOLUÇÃO} : v_1' = v_1 \text{ e } v_2' = v_2 \\ 2^{\circ} \text{ SOLUÇÃO} : v_1' = \dfrac{(m_1-m_2)}{m_1+m_2}v_1 + \dfrac{2m_2}{m_1+m_2}v_2 \quad v_2' = \left(\dfrac{m_2-m_1}{m_2+m_1}\right)v_2 + \dfrac{2m_1 v_1}{m_1+m_2} \end{cases}$

8(b)

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$$

$$m_1 (v_1 - v_1')(v_1 + v_1') = m_2 (v_2' - v_2)(v_2' + v_2)$$

$$\Rightarrow \quad v_1 + v_1' = v_2 + v_2'$$

$$\boxed{\underbrace{v_1 - v_2}_{\text{APROX}} = \underbrace{v_2' - v_1'}_{\text{AFAST}}}$$

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 (v_1 - v_2 + v_1')$$

$$(m_1 + m_2) v_1' = (m_1 - m_2) v_1 + 2 m_2 v_2$$

$$\boxed{v_1' = \frac{(m_1 - m_2) v_1^{\circ} + 2 m_2 v_2}{m_1 + m_2}}$$

$$\boxed{v_2' = \frac{2 m_1 v_1 + (m_2 - m_1) v_2}{m_1 + m_2}}$$

EX: (a) $m_1 = m_2$ (BOLA DE BILHAR)

$\boxed{v_2 = 0}$

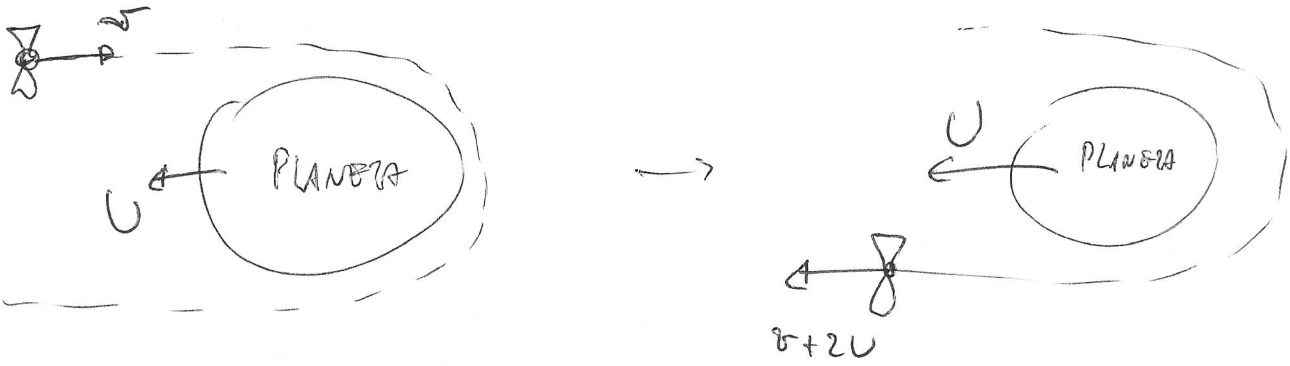
$\Rightarrow \boxed{v_1' = v_2}$

$\boxed{v_2' = v_1}$ TROCA DE VELOCIDADES

(b) $m_1 \gg m_2 \quad \Rightarrow \begin{cases} v_1' = v_1 \\ v_2' = 2 v_1 - v_2 \end{cases}$

(c) $m_1 \ll m_2 \quad \Rightarrow \begin{cases} v_1' = -v_1 + 2 v_2 \\ v_2' = v_2 \end{cases}$

SLINGSHOT



$M_P >> M_S \Rightarrow \boxed{\begin{array}{l} v_1' \approx -v_1 + 2U \\ v_2' \approx U + 0 \end{array}}$

COLISÕES PERFEITAMENTE INELÁSTICAS

m_1, v_1 $\quad m_2 v_2 \quad \longrightarrow \quad m_1+m_2, v_{cm}$

$m_1 v_1 + m_2 v_2 = (m_1+m_2) v_{cm} \rightarrow \boxed{v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1+m_2}}$

$\Delta E_c = E_{cA} - E_{cD}$ (ENERGIA CINÉTICA PERDIDA)

$= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 - \frac{1}{2}(m_1+m_2) v_{cm}^2$

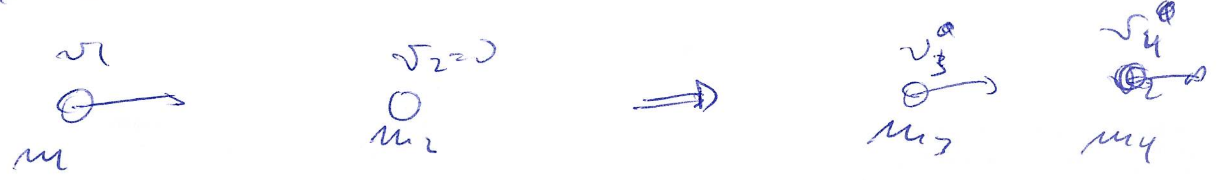
$= E_{cA} - \frac{1}{2}(m_1+m_2)\left(\frac{m_1 v_1 + m_2 v_2}{m_1+m_2}\right)^2 = E_{cA} - \frac{1}{2(m_1+m_2)}\left(m_1^2 v_1^2 + 2m_1m_2v_1v_2 + m_2^2 v_2^2\right)$

Obs: SE $m_1 v_1 + m_2 v_2 = 0 \Rightarrow \boxed{v_{cm} = 0}$ $\quad \boxed{\Delta E_c = E_{cA}}$ PERDA TOTAL DA ENERGIA CINÉTICA

$\downarrow$

ISSO ACONTECE NO REF. DO CM.

COLISÃO INELÁSTICA EM (1D)



$$Q = T_4 + T_3 - T_1$$

$$T_4 = \frac{1}{2} m_4 v_4^2 = \frac{1}{2m_4} P_4^2 = \frac{1}{2m_4}(P_1 - P_3)^2$$

$$= \frac{1}{2m_4}\left(\sqrt{2m_1 T_1} - \sqrt{2m_3 T_3}\right)^2$$

$$= \frac{1}{m_4}\left(m_1 T_1 - 2\sqrt{m_1 m_3 T_1 T_3} + m_3 T_3\right)$$

$$\Rightarrow Q = \frac{m_1}{m_4} T_1 - 2\sqrt{\frac{m_1 m_3}{m_4^2} T_1 T_3} + \frac{m_3}{m_4} T_3 + T_3 - T_1$$

$$\boxed{Q = \left(\frac{m_1}{m_4} - 1\right) T_1 - 2\sqrt{} + \left(\frac{m_3}{m_4} + 1\right) T_3}$$

NO CASO TOTALMENTE INELÁSTICO

$$v_f = \frac{m_1}{m_1 + m_2} v_1$$

$$Q = T_f - T_i = \frac{1}{2}(m_1 + m_2) v_f^2 - \frac{1}{2} m_1 v_i^2$$

$$= \frac{1}{2}(m_1 + m_2) \frac{m_1^2}{(m_1 + m_2)^2} v_1^2 - \frac{1}{2} m_1 v_1^2$$

$$= \frac{1}{2}\left(\frac{m_1^2}{m_1 + m_2} - \frac{m_1(m_1 + m_2)}{m_1 + m_2}\right) v_1^2 = -\frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} v_1^2$$

$$= -\frac{m_2}{m_1 + m_2} T_1$$

COLISÕES INELÁSTICAS

EM GERAL AS COLISÕES NÃO SÃO NEM ~~RE~~ ELÁSTICAS NEM PERFEITAMENTE INELÁSTICAS

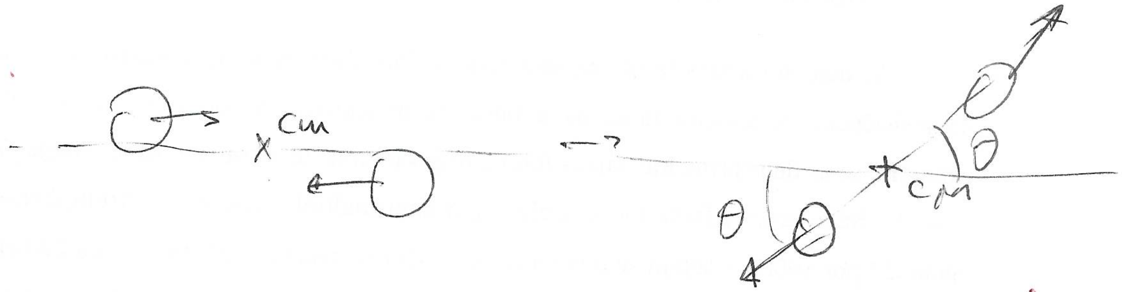
$$\downarrow \qquad\qquad\qquad\qquad \downarrow$$

$$v_2' - v_1' = v_1 - v_2 \qquad\qquad v_2' - v_1' = 0$$

COEFICIENTE DE RESTITUIÇÃO : $e \equiv \dfrac{|v_2' - v_1'|}{|v_2 - v_1|} = \dfrac{|v_{AFAST}|}{|v_{APROX}|}$

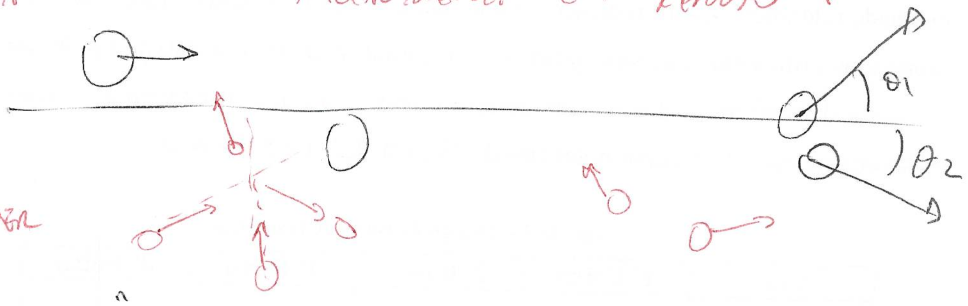
EM 2D

REF. CM.

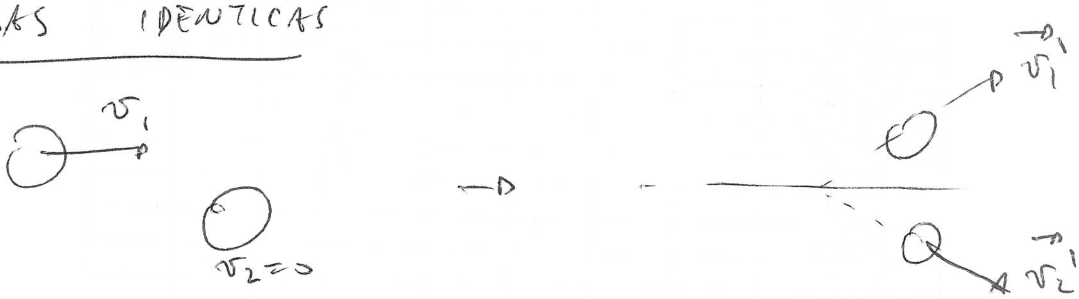


REF ONDE A PART. 2 ESTÁ INICIALMENTE EM REPOUSO (REF. LAB)

REF. QUALQUER

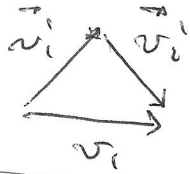


MASSAS IDÊNTICAS



$$\vec{P} = m_1\vec{v}_1 = m_1\vec{v}_1' + m_1\vec{v}_2' \Rightarrow \vec{v}_1 = \vec{v}_1' + \vec{v}_2'$$

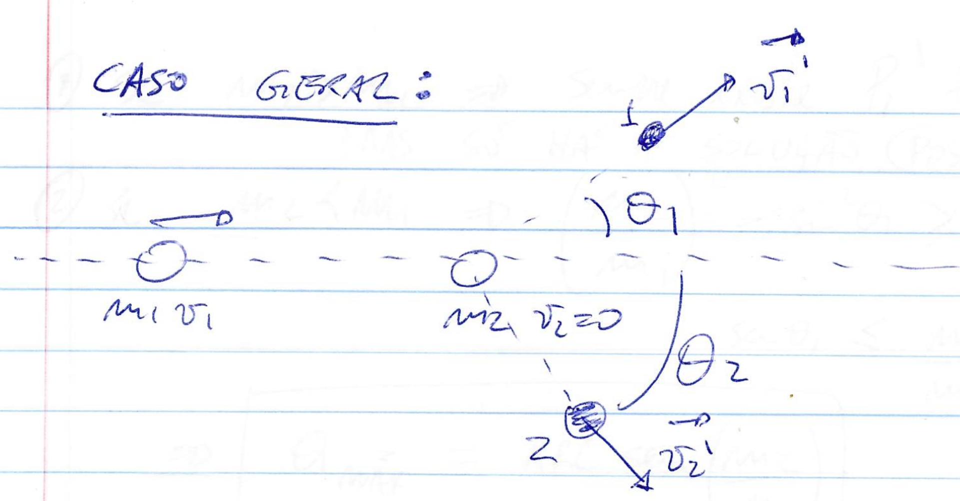
OU SEJA



SE A COLISÃO FOR ELÁSTICA

$$\Rightarrow \tfrac{1}{2}m_1 v_1^2 = \tfrac{1}{2}m_1 v_1'^2 + \tfrac{1}{2}m_2 v_2'^2 \Rightarrow \boxed{v_1^2 = v_1'^2 + v_2'^2 \Rightarrow \text{TRIÂNGULO RETÂNGULO}}$$

CASO GERAL:



MOMENTO: $\vec{P}_1 = \vec{P}_1' + \vec{P}_2' \Rightarrow \begin{cases} P_1 = P_1'\cos\theta_1 + P_2'\cos\theta_2 \\ 0 = P_1'\operatorname{sen}\theta_1 - P_2'\operatorname{sen}\theta_2 \end{cases}$

ENERGIA: $E_c = \frac{1}{2}mv^2 = \frac{1}{2m}p^2 \Rightarrow$

$$\frac{1}{2m_1}P_1^2 = \frac{1}{2m_1}P_1'^2 + \frac{1}{2m_2}P_2'^2$$

$$\boxed{P_1^2 = P_1'^2 + \frac{m_1}{m_2}P_2'^2}$$

3 EQS, 4 INCÓGNITAS $\rightarrow$ 1 GRAU DE LIBERDADE

$P_1', P_2', \theta_1 \;e\; \theta_2$

SUPONHA θ_1 é MEDIDO

$$\vec{P}_2' = \vec{P}_1 - \vec{P}_1' \Rightarrow P_2'^2 = P_1^2 - 2P_1P_1'\cos\theta_1 + P_1'^2$$

$$\Rightarrow P_1^2 = P_1'^2 + \left(\frac{m_1}{m_2}\right)\left(P_1^2 - 2P_1P_1'\cos\theta_1 + P_1'^2\right)$$

$$\left(1+\frac{m_1}{m_2}\right)P_1'^2 - 2\frac{m_1}{m_2}P_1\cos\theta_1\,P_1' + \left(\frac{m_1}{m_2}-1\right)P_1^2 = 0$$

$$P_1' = \frac{2m_1P_1\cos\theta_1 \pm \sqrt{4m_1^2P_1^2\cos^2\theta_1 - 4(m_1+m_2)(m_1-m_2)P_1^2}}{2(m_1+m_2)}$$

$$P_1' = \left(\frac{m_1\cos\theta_1 \pm \sqrt{m_2^2 - m_1^2\operatorname{sen}^2\theta_1}}{m_1+m_2}\right)P_1$$

$$P_1' = \left(\frac{\cos\theta_1 \pm \sqrt{\left(\frac{m_2}{m_1}\right)^2 - \operatorname{sen}^2\theta_1}}{1+\frac{m_2}{m_1}}\right)P_1$$

① Se $m_2 > m_1 \Rightarrow$ sempre existe $P_1' \ \forall \theta_1$ 10C

MAS SÓ HÁ A SOLUÇÃO POSITIVA!

② se $m_2 < m_1 \Rightarrow \left(\frac{m_2}{m_1}\right)^2 - \sin^2\theta_1 \geq 0$

$$\sin\theta_1 \leq \frac{m_2}{m_1}$$

$$\Rightarrow \boxed{\theta_{MÁX} = \text{ARC}\ \text{sen}\left(\frac{m_2}{m_1}\right)}$$

$$P_1 = P_1' \cos\theta_1 + P_1' \frac{\sin\theta_1}{\sin\theta_2}\cos\theta_2$$

$$\Rightarrow 1 = \left(\frac{\cos\theta_1 \pm \sqrt{\left(\frac{m_2}{m_1}\right)^2 - \sin^2\theta_1}}{1+\frac{m_2}{m_1}}\right)\left(\cos\theta_1 + \sin\theta_1 \cot g\,\theta_2\right)$$

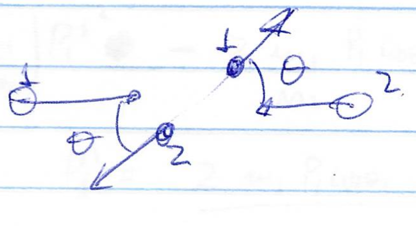
NO CASO $m_1 = m_2 \Rightarrow 2 = \left(\cos\theta_1 \pm |\cos\theta_1|\right)\left(\cos\theta_1 + \sin\theta_1 \cot g\,\theta_2\right)$

$$1 = \cos\theta_1\left(\cos\theta_1 + \sin\theta_1 \cot g\,\theta_2\right)$$

$$\sin\theta_1 \cot g\,\theta_2 = \frac{\sin^2\theta_1}{\cos\theta_1} \rightarrow \boxed{\cot g\,\theta_2 = \tan\theta_1}$$

NO REF. DO CM:

$$\vec{v}_{cm} = \frac{1}{m_1+m_2} m_1\vec{v}_1 = \frac{\vec{P}_1}{m_1+m_2}$$



$$\vec{P}_1 + \vec{P}_2 = \vec{P}_1' + \vec{P}_2' = 0$$

$$P_1 = P_2$$

$$\begin{cases} P_1'\cos\theta = P_2'\cos\theta \\ P_1'\sin\theta = P_2'\sin\theta \end{cases} \Rightarrow \text{ok}\ \forall\theta$$

θ é GRAU DE LIBERDADE

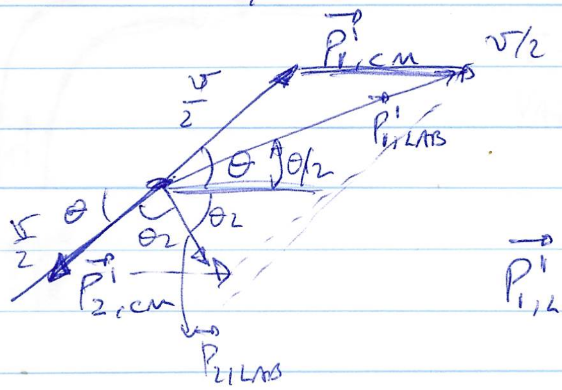
$$m_1 P_1^2 + m_2 P_2^2 = m_1 P_1'^2 + m_2 P_2'^2$$

$P_1 = P_1'$ → Ñ MUDA DE MAGNITUDE!!!

$P_2 = P_2'$

CASO $m_1 = m_2$

$\Rightarrow \theta_1 + \theta_2 = \pi/2$



$$\vec{P}'_{1,LAB} = \vec{P}'_{1,CM} + m_1 \vec{v}_{CM}$$
$$= \vec{P}'_{1,CM} + \underbrace{m_1 \frac{\vec{v}}{2}}_{\frac{P_{1,LAB}}{2}}$$