EXPERIMENT

V

The Millikan Oil Drop Experiment

I. References

R. A. Millikan: *Electrons (+ and -)*
R. M. Eisberg: *Fundamentals of Modern Physics*, pp. 73-75

II. Purpose

To demonstrate experimentally that electric charge is a quantized parameter, i.e., there is a basic unit of charge, e, of which all charges are multiples, thus

$$q_n = ne.$$

III. Introduction

One of the most important concepts in physics is that of the electron as a small indivisible particle of diameter approximately 10^{-12} cm, mass 9.1×10^{-28} g carrying a negative charge of 4.8×10^{-10} esu. Ordinary neutral matter consists of an equal number of electrons and protons (the basic unit of positive charge). Positively charged matter has lost electrons, while negatively charged matter has an excess of electrons, i.e., the positive charges are normally fixed in matter while the electrons can be caused to move. This postulate of the electron as the basic unit of transferable charge requires that the charge on any particle or macroscopic body be a whole multiple of the charge e, and that if the charge changes by a non-zero amount, the minimum change is just one electron charge, e.

Since the charge on the electron is extremely small, ordinary electrical methods cannot be used to measure it, e.g., a current of 1 ampère represents a flow of 10^{19} electrons per sec. Millikan (1917), following the work of Townsend, Thomson and Wilson, measured the smallest charge observed on single small oil drops to determine e as 4.774×10^{-10} esu. A good account of this work is given by Millikan in "Electrons (+ and –)." The oil drop method is not the most accurate way of measuring the charge e but does show quite clearly that smaller charges than this do not exist.

IV. Principles

When a fine spray of oil drops is produced by an atomizer, the drops are often charged by the frictional processes occurring in the atomizer. If these oil drops are allowed to drift into a region of space containing a uniform vertical electric field such that an individual oil drop is accelerated in the upward direction, the dynamic equilibrium of the oil drop can be analyzed Fig. (V•1a).

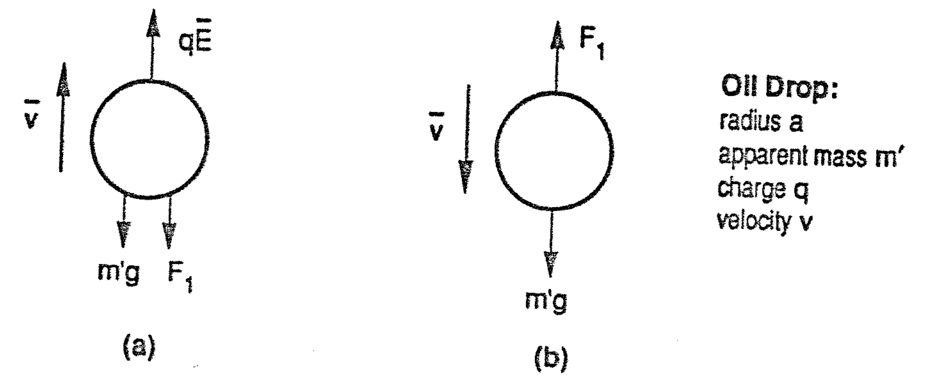

Figure V•1

The upward force caused by the electric field E and the charge q on the drop is

$$qE.$$

There is the downward gravitational force

$$m'g$$

where m' is the apparent mass of the drop and g the gravitational acceleration and the downward viscous drag force F_1 given by Stokes' Law

$$F_1 = 6\pi\eta a v$$

where η is the viscosity of air, a is the radius of the drop and v is its upward velocity. Since the object of the experiment is to measure the smallest possible value of q or the smallest change in the value of q, the charge q should be only a few electron charges, i.e., it is very small. Therefore to make the upward force qE comparable to the gravitational force for reasonable values of E, we must use very small oil drops. In fact the oil drops become so small that when travelling through air they collide with individual molecules and, during the time between collisions, travel as though in a vacuum. This being the case, Stokes law must be corrected as

$$F_1 = \frac{6\pi\eta a v}{(1 + b/pa)}$$

i.e., the viscous drag is reduced. This correction to Stokes law was deduced by Millikan during his oil drop experiments. b is a constant and p is the atmospheric pressure in centimeters of mercury.

The oil drop of Fig. (V•1a) will continue to accelerate in the upward direction until it reaches its terminal velocity v_u, then

$$qE = \frac{6\pi\eta a v_u}{(1+b/pa)} + \frac{4}{3}\pi a^3(\rho - \rho_A)g \qquad \text{V•1}$$

where ρ is the density of the oil used and ρ_A is the density of air. The radius a of the oil drop can readily be found by observing the terminal velocity v_d with which the drop falls under gravity when there is no electric field, (Fig. (V•1b)). Now the viscous drag force acts in the upward direction so

$$\frac{6\pi\eta a v_d}{(1+b/pa)} = \frac{4\pi}{3}a^3(\rho - \rho_A)g \qquad \text{V•2}$$

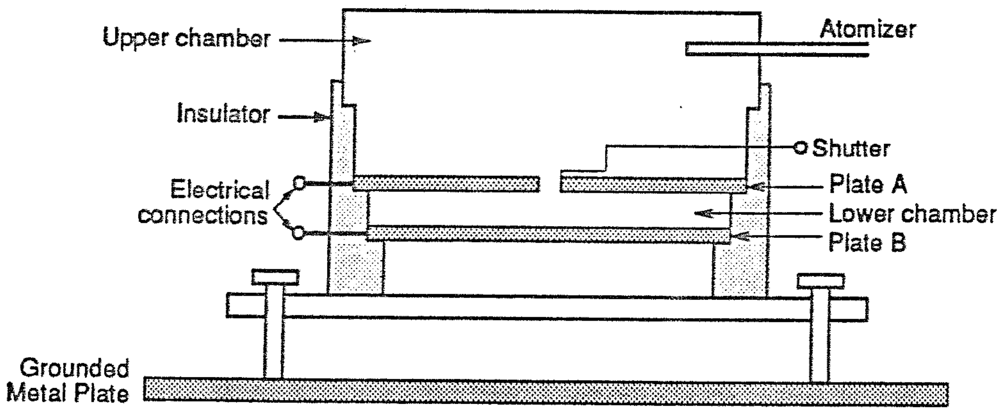

Figure V•2

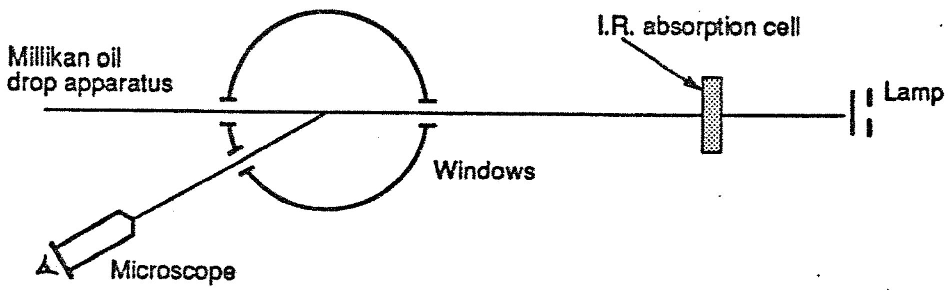

Figure V•3

The apparatus used for the oil drop experiment is shown schematically in Fig. (V•2) and Fig. (V•3). If a fine cloud of oil drops is sprayed into the upper chamber from the atomizer and the shutter is opened, some of the drops will descend into the lower chamber through the hole in the metal plate *A*. These drops can be seen through the microscope (Fig. (V•3)) as silver spheres against the black walls of the lower chamber. A potential *V* (volts) applied between the circular metal plates *A* and *B* produces a near uniform electric field in the region of their centers and the motion of the oil drops in this field can be observed through the microscope.

To measure v_d accurately requires that the oil drop falls several times through a known fixed distance *L*. If the distance *L* is about 2.5 mm, which is reasonable in a chamber where *D* is approximately 5 mm, the time taken for each fall should be between 10 and 30 seconds. Therefore to obtain several readings of this time for the same drop requires that the drop remain of constant size for several minutes, i.e. it must neither evaporate nor coalesce with another drop. The oil used has a very low vapor pressure (evaporates very slowly) and, providing only a few drops are allowed to enter the low chamber, the chance of two drops colliding is very small.

Measurement of v_u is more difficult, for the upward transit should also take at least 10 secs and if the average velocity v_u during this time is to have any meaning, the charge q on the drop must remain constant over the period. The charge q on an oil drop is (usually) initially caused by frictional charge separation in the atomizer. However, during the life of the oil drop, the charge will change each time the drop sweeps up a free ion from the atmosphere. Clearly, then, the time interval over which the charge q on the oil drop remains constant, depends on the ion density in the atmosphere which, in turn, depends very much on the proximity of radioactive material, etc.

Method I. One way to ensure that q has remained constant, is to observe several upward transits for the same drop over the distance *L* under the influence of a constant applied voltage *V*, and to accept as usable measurements only those values of v_u which occur for at least two consecutive transits. The argument is that if q is to remain constant over one transit time, there should be a reasonable chance of q remaining constant long enough for two upward transits to be observed. This procedure requires that q remain constant for several minutes. This may not be realizable in any particular laboratory. If q does remain constant, the above procedure leading to a mean value of v_u over several observations is the most accurate way of determining q.

Method II. If q changes too rapidly for a satisfactory determination of v_u to be made, it may still be possible to determine q by adjusting the potential *V* until v_u becomes zero, i.e., the drop is stationary. This measurement requires only 10 to 20 seconds and gives the values of v_u (zero) while ensuring that q is constant at the time of measurement.

It turns out that under normal laboratory conditions the chance of the charge on the oil drop remaining constant for any length of time is very small unless you deliberately stop free atmospheric ions from getting to the drop.

This can be done very easily since the geometry of the apparatus ensures that any ion reaching the oil drop has to travel between the plates *A* and *B*. However, any ion finding itself in this region is attracted to one of the plates and neutralized, providing there is always an electric field between the plates. Therefore, when determining the upward terminal velocity v_u under the influence of an electric field, instead of allowing the drop to fall freely under gravity between measurements, simply reverse the electric field and accelerate the drop downwards. This procedure will both speed up the process of collecting data and prevent changes in the charge on the oil drop. When several measurements have been made of v_u at one charge q, the charge can be changed by simply allowing the drop to fall freely.

V. Experimental Procedure

Dismantle the Millikan oil drop apparatus and clean (with Kimwipes) the inside of the upper and lower chambers. Make sure the three windows in the lower chamber are clean. While reassembling the apparatus, measure the separation *D* (cm) of the surfaces of *A* and *B* with the Vernier depth gauge. This gauge can also be used to calibrate the eyepiece scale in the microscope.

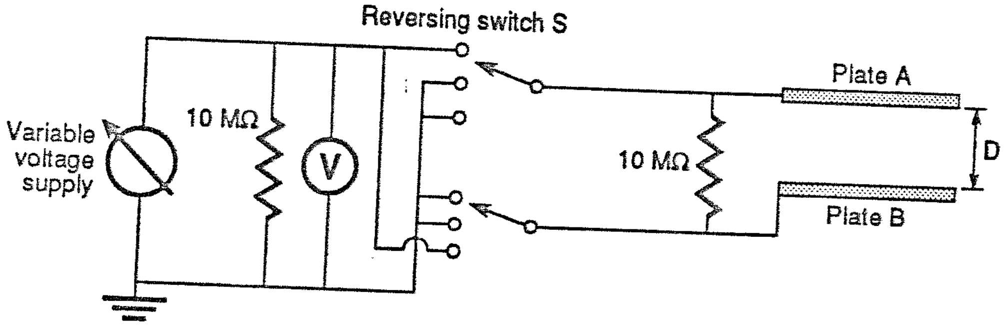

Figure V•4

Set up the experimental system as in Fig. (V•3), filling the infrared absorption cell with tap water. To get the line of sight at a reasonable height above the bench, stand the Millikan apparatus on the aluminum plate provided which also has provision for holding the microscope. Place the aluminum plate on the wooden box containing the high voltage switch, and place this on the wooden blocks. Adjust the legs of the Millikan apparatus until the surface of the plate *B* lies in the horizontal plane. A level is provided. Connect the electrical circuit shown in Fig. (V•4), putting the bleed resistance *R* (10 Meg ohms) across the terminals of the Millikan apparatus and making the voltmeter connections at the input terminals of the high voltage switch assembly. The voltages you will be using will vary up to 500 volts so be sure that the circuit connections are securely made and that no power is on while you are assembling the system! A separate standard voltmeter is used because the meter on the high voltage power supply is not sufficiently accurate. The high voltage switch *S* has three positions: i) Plate *A* positive and *B* grounded, ii) both *A* and *B* grounded, iii) *A* grounded and *B* positive. Set *S* in position (ii) except when you are making observations.

Remove the cover to the upper chamber of the Millikan apparatus and insert a pin through the hole in plate *A*; focus the microscope on the pin. Remove the pin and replace the cover. Spray oil drops into the upper chamber from the atomizer, then observing through the microscope, allow a few drops to fall into the lower chamber. Close the shutter over the hole in *A*. Switch on the high voltage power supply and move switch *S* to position i) or iii). With no electric field, all the oil drops were falling (upwards in the microscope) under gravity. Some will now move upwards. Choosing the smallest drop you can find, use a single measurement of the potential *V* at which v_u is zero, and the downward drift velocity v_d to determine the charge *q* and ascertain that *q* is of the order 10^{-9} esu. If *q* is much larger than 10^{-9} esu, perhaps the atomizer requires cleaning.

Again, choosing the smallest oil drop you can find, determine v_d from the average of many downward transits with no electric field and use method (I) for determining v_u. When timing the oil drop over the distance *L*, start the drop well outside the limit of *L* so that it has attained its terminal velocity before you start timing.

To determine the radius of the oil drop from v_d one can simply solve Eq. (V•2) which is a quadratic equation in *a*. The negative root should be ignored since the positive root is the radius.

The value of *q* can be determined from the combination of Eq. (V•1) and Eq. (V•2), as

$$q = \frac{1}{E}\frac{4\pi}{3}(\rho - \rho_A) g a^3 \frac{v_u + v_d}{v_d}$$

and if *V* (volts) is the potential difference between the plates *A* and *B* which caused the upward terminal velocity v_u

$$q = \frac{300D}{V}\frac{4\pi}{3}(\rho - \rho_A) g a^3 \frac{v_u + v_d}{v_d} \qquad \text{V•3}$$

Determine either the different values of *q* appearing on several different oil drops, or different values of *q* appearing on the same oil drop at different times. Examine your values of *q* on the assumption that each charge q_n is a multiple of some basic charge, *e*, such that

$$q_n = ne$$

where *n* is integral. Determine the largest value of *e*, consistent with your results. According to the initial postulate this must be the charge on the electron.

Units and Numerical Constants		
charge	q	esu
electric field	E	e.s.u.
potential	V	volts
density	ρ	gm cm^{-3}
gravitational acceleration	g	cm sec^{-2}
radius	a	cm
plate separation	D	cm
velocity	v	cm sec^{-1}
pressure	p	(cm of mercury)
constant	b	cm (cm of mercury)
viscosity	η	gm cm^{-1} sec^{-1} (or poise)

X Density of Apiezon type *B* oil: 0.8568 ± .0003 gm cm^{-3}
Density of air at N.T.P: 0.0013 gm cm^{-3}
Viscosity of air at 23°C (1830 ± 2.5) x 10^{-7} gm cm^{-1}sec^{-1}
Temperature coefficient of viscosity of air 4.8×10^{-7} gm cm^{-1}sec^{-1} °C^{-1}
The constant *b*: 6.17×10^{-4} cm (cm of mercury)
NB. The viscosity of air is independent of the pressure.

VI. Possible Discussion Questions (need not be answered in lab report)

A. Explain the purpose of the infrared absorption cell.

B. What error will your reaction time cause in your measurements of the oil drop velocity?

C. If graphical methods were not sufficiently accurate to determine the radius a from v_d, how would you calculate a?

D. Calculate the voltage V required to hold an oil drop of radius 10^{-4} cm. stationary if D is 5 mm and the charge on the drop is 4.8×10^{-10} esu.

E. How and to what extent do fluctuations in room temperature and atmospheric pressure affect the value of e determined from this experiment?

F. Why should the time of transit over the distance L be more than 10 sec?

APPENDIX - 1
STOKES' LAW

A particle falling due to gravity in a viscous liquid is acted upon by three forces: a gravitational force acting downward, a buoyant force acting upward, and a drag force acting upward. The descriptive equation of this motion is

$$mg - m_0 g - F_D = m_0 \frac{dv}{dt} \qquad (1)$$

where m is the mass of the particle, m_0 the mass of a volume of liquid equal to the volume of the particle, g the acceleration of gravity, F_D the drag force, v the particle velocity, and t the time.

Small particles reach a stable, or terminal, velocity very rapidly; hence, dv/dt quickly becomes zero. The equation of motion for a sphere of diameter D and density ρ falling in a liquid of density ρ_0 then becomes

$$F_D = \frac{\pi}{6} (\rho - \rho_0) g D^3 \qquad (2)$$

Dimensional analysis reveals that stable particle motion through a liquid is governed by two dimensionless groups: the particle Reynolds number Re

$$Re = \frac{D v \rho_0}{\eta} \qquad (3)$$

where η is the liquid viscosity, and a drag coefficient C_D expressed by

$$C_D = \frac{\text{(drag force)}}{\begin{pmatrix}\text{cross-sectional}\\ \text{area of particle}\end{pmatrix} \begin{pmatrix}\text{dynamic pressure}\\ \text{on particle}\end{pmatrix}} = \frac{F_D}{\frac{\rho D^2}{4} \times \frac{\rho_0 v^2}{2}} \qquad (4)$$

The relationship between Re and C_D for laminar (not turbulent) flow conditions is well established experimentally for low values of the Reynolds number to be

$$C_D = \frac{24}{Re} \qquad (5)$$

Combining equations (2) through (5) yields

$$D^2 = \frac{18 v \eta}{(\rho - \rho_0) g} \qquad (6)$$

which has come to be identified as the Stokes law equation*.

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*Stokes, G.G., Mathematical and Physical Paper III, Cambridge University Press, 1891.