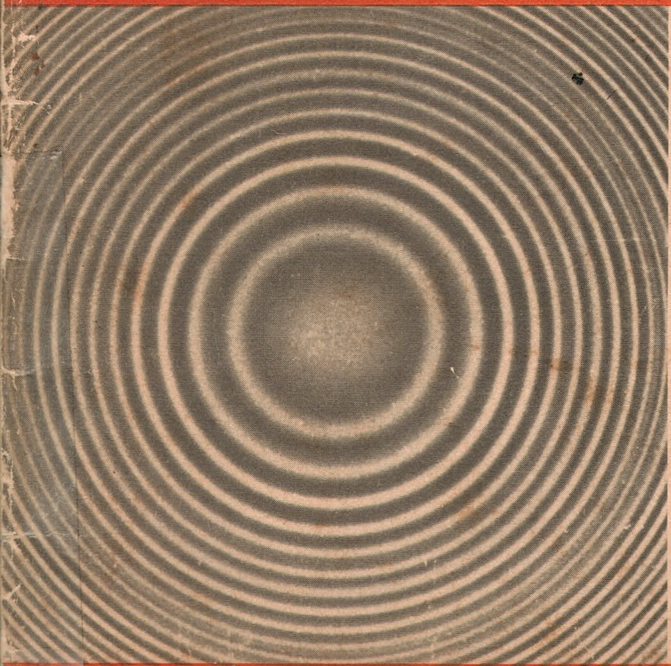
OPERATION AND EXPERIMENT MANUAL



M-4 INTERFEROMETER

ATOMIC LABORATORIES, INC., 3086 Claremont Ave., Berkeley 5, California

Subsidiary Cenco Instruments Corporation

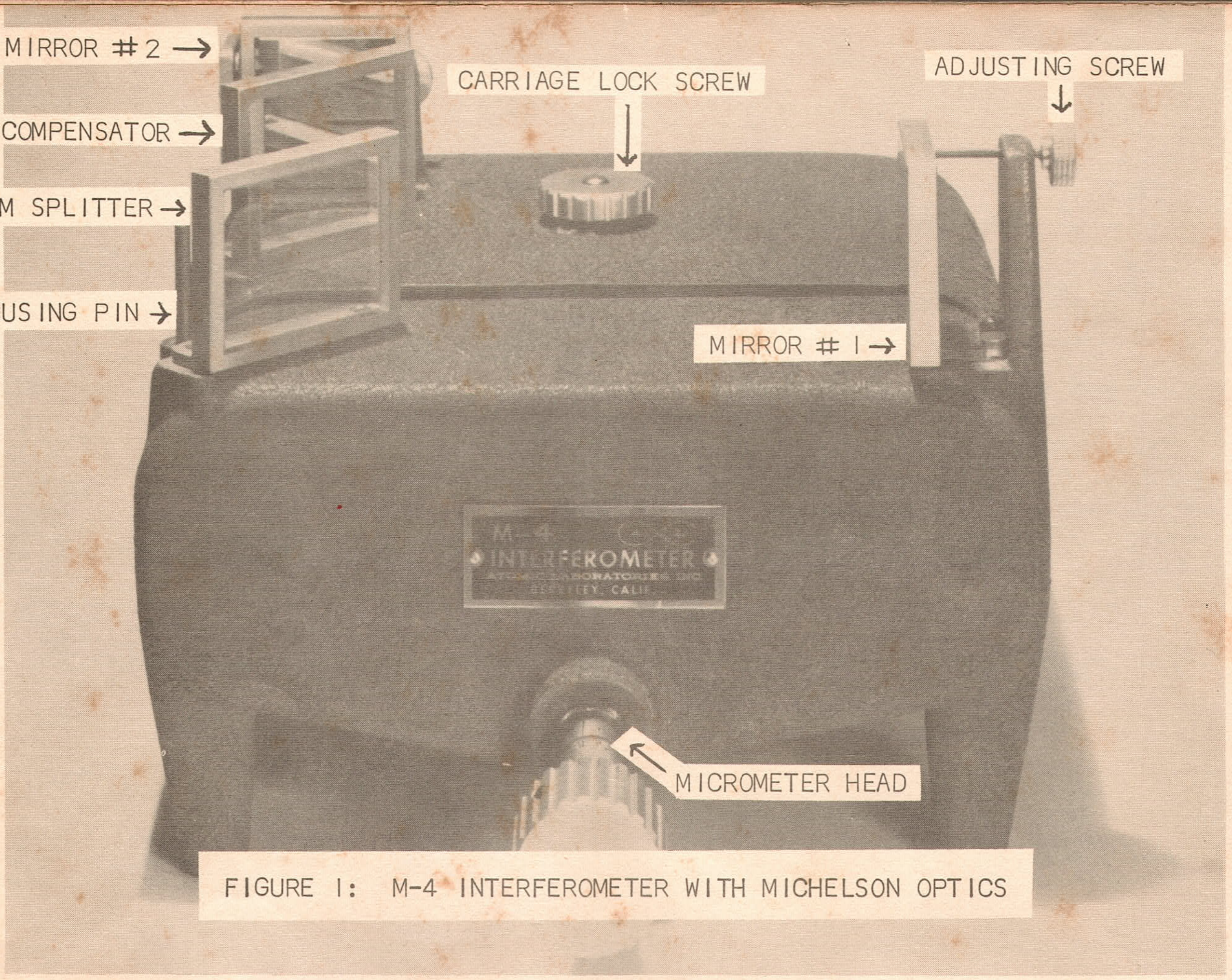


FIGURE 1: M-4 INTERFEROMETER WITH MICHELSON OPTICS

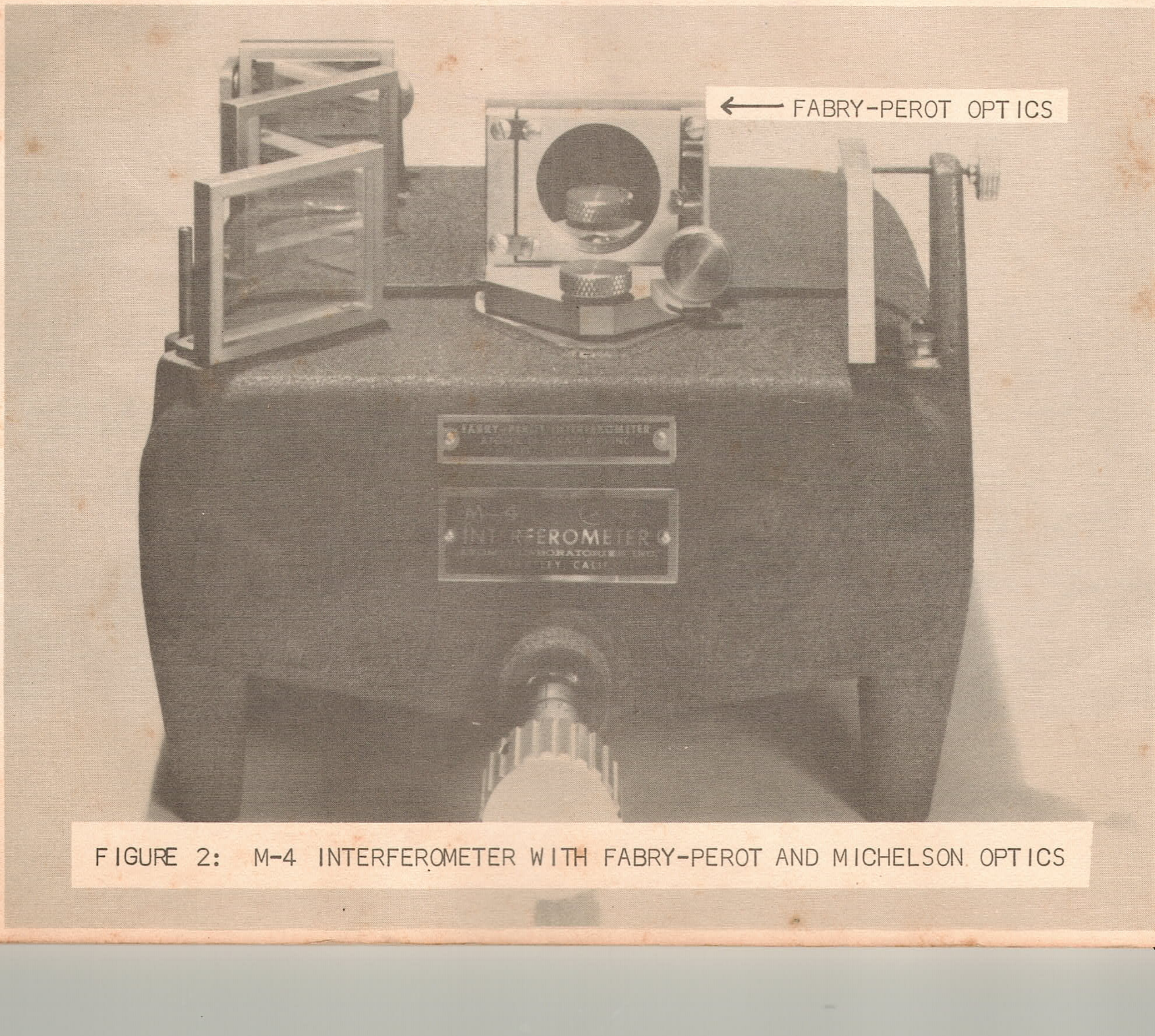


FIGURE 2: M-4 INTERFEROMETER WITH FABRY-PEROT AND MICHELSON OPTICS

ATOMIC LABORATORIES, INC., 3086 Claremont Ave., Berkeley 5, California

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M-4 INTERFEROMETER

OPERATION AND EXPERIMENT MANUAL

PART I

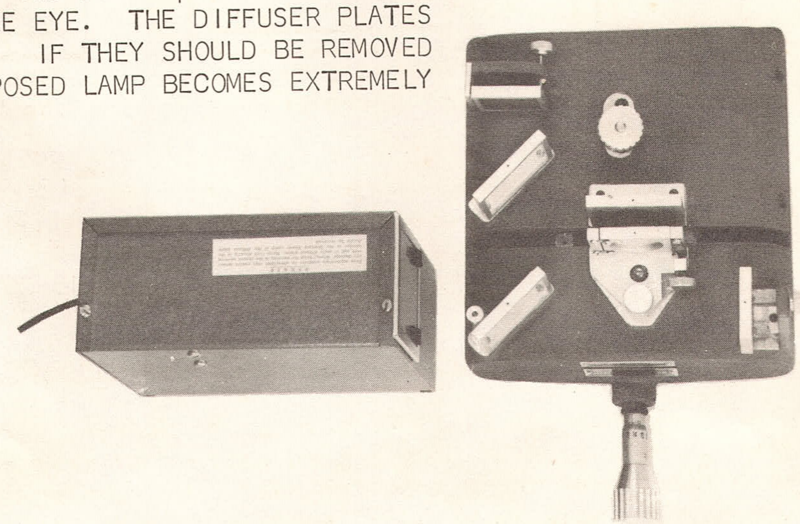
M I C H E L S O N   O P T I C S

The Atomic Laboratories M-4 Interferometer is fabricated with Michelson Optics, with Fabry-Perot Optics, or with a combination of both. The Michelson front surface mirrors and beam splitter and the Fabry-Perot mirrors must be treated with the utmost respect. They are never to be touched without wearing gloves, and then only sparingly. Dust may be cleaned from the mirrors by means of the camel's hair brush which is shipped with each instrument. Other matter may be removed, if absolutely necessary, with a soft cloth or piece of lens paper brushed ever so lightly against the surface. When the interferometer is stored, it should be covered at all times with the plastic bag. It is also a good idea to keep the lens covers over the optics when they are not in use.

When the M-4 Interferometer is ordered with only one set of optics, the other set may be ordered at any future date and easily installed by removing the masking tape on the interferometer chassis and carriage and inserting the screws in the indicated holes. The following unpacking instructions apply to the interferometer irregardless of the type of optics.

Unpacking: Gently remove the cardboard insert which holds the interferometer in place, being extremely careful not to rub or push it against the optics while lifting it out. Place the interferometer on a level surface and remove the plastic cover, [illegible] covers. Remove the tape which holds the carriage in place.

Light Source: The M-4 Interferometer will operate with any standard mercury or sodium light source, including Atomic Laboratories' own monochromatic mercury light source. To operate the source, insert lamp, either one or both of the two diffuser plates, and, if desired, the green Wratten filter. If used with the Michelson Optics, the light source should be positioned as shown in the photograph. The light should enter the beam splitter at a 45° angle. DANGER: ULTRAVIOLET RAYS CAN SEVERELY DAMAGE THE EYE. THE DIFFUSER PLATES ABSORB THESE RAYS: IF THEY SHOULD BE REMOVED OR BROKEN, THE EXPOSED LAMP BECOMES EXTREMELY DANGEROUS:



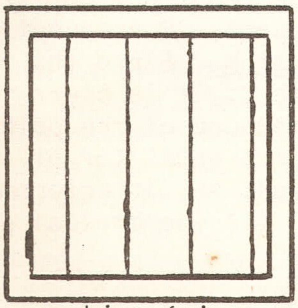
Adjustment: Move the carriage until mirror number 2 is approximately the same distance from the beam splitter as the fixed mirror number 1. This distance is generally about 12.5 cm., but it should be checked with a ruler. It should be measured from the coated side (left side) of the beam splitter in each case. Now tighten the carriage lock screw to hold the carriage firmly in place. Turn on the light source, which is in the position previously described. The next step is to bring the mirrors into exact perpendicularity. The adjustment can be accomplished as follows:

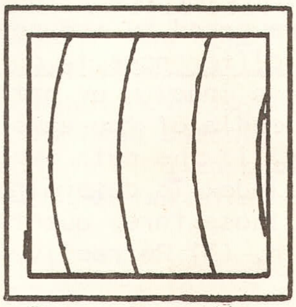
Turn on the light source and observe the focusing pin* which is situated between the light source and the beam splitter. Two images of the pin will be seen, one coming from the reflection at the front surface of the beam splitter, the other from the reflection at its back surface.

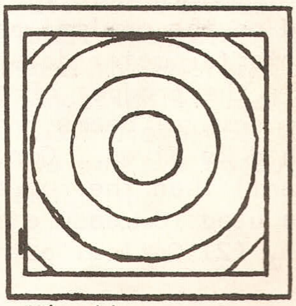
Line up the vertical positions of the focusing pin. This can be done by means of the adjusting screw of mirror number 1. The adjusting screw of mirror number 2 will line up the focusing pin horizontally. When only one image of the pin is achieved, fringes should appear. To best observe these fringes, look straight into the back mirror from the front of the interferometer.

It takes a little practice to obtain the fringes. As stated previously, before touching the adjusting screws, make certain that the two mirrors are equally distant from the beam splitter. When the fringes first appear they can be sharpened by very careful and minute adjustment of the screws. If the adjustment is accomplished in a room with a great deal of vibration or on an unsteady table, the fringes will soon disappear.

Often only very thin and blurred fringes will be seen in the beginning. A good technique is to line them up vertically or horizontally (see below) by adjusting one mirror. Then by centering in with the other mirror, the fringe curvature will be increased until finally the center appears. Be sure while making these adjustments that the mirror is moved in the direction of the fringe's decreased radius of curvature.


Line fringes up vertically


Decrease radius of curvature


Finally-- the bull's-eye

MAINTENANCE: Every six months remove carriage and wipe steel ways with clean rag containing a few drops of three-in-one oil. Ways must be spotlessly clean and free of dust, dirt, or rust for efficient operation.

EXPERIMENT I: Determination of the Wave Length of Monochromatic Light.

Procedure: Use Atomic Laboratories' Mercury Light Source, with a Wratten Number 74 filter or equivalent. Once a good pattern of fringes has been obtained, take a reading of the micrometer head. By turning the head, the carriage can be moved slowly in either direction. Count the fringes as they pass by the focusing pin or as they appear or disappear in the bull's-eye. For satisfactory precision, count at least one hundred fringes. After counting them off, take a new reading of the micrometer head. From the number of fringes passed over, Δn, and the distance traversed by the mirror, we may determine the wave length of the monochromatic light by means of

* The focusing pin is removable from the carriage.

the formula:

$$2(d_1 - d_2) = \lambda \Delta n$$

The distance travelled by the mirror in centimeters is given by

$$(d_1 - d_2) = 0.10\ (D_1 - D_2) \times K$$

where $(D_1 - D_2)$ is the change of the micrometer reading in millimeters and K is the ratio of carriage movement to micrometer screw reading. For the M-4 Interferometer $K = 0.020$

Whence:
$$\lambda = \frac{2(0.10)(D_1 - D_2)K}{\Delta n} \text{ cm.}$$

The correct value for the wavelength of green mercury light is 5460.740 Å ($1\ \text{Å} = 1 \times 10^{-8}$ cm). You may wish to use this correct value of the wavelength to obtain a more exact value for K, since there are some variations in manufacturing conditions of the instrument.

<u>Discussion</u>: An Interferometer is generally defined as an optical instrument which produces interference patterns by the division of one beam of light into one or more parts. These parts travel different paths and are then ultimately brought together to yield the interference effects. The resultant patterns depend on the optical paths traversed by the several beams. Consequently, the Interferometer determines <u>differences in optical paths</u>. Since the optical path is the product of refractive index μ by path length d, it is clear that if the several beams traverse media of the same μ, a measure of the path length d is given. Conversely, if the path lengths d are equal (or at least constant) then the refractive index is determined. Thus, the Interferometer may be used to measure any of these three quantities: (1) Geometrical path length, (2) Optical path length, (3) Refractive index.

Determination ot optical path length is of importance in technical applications. Measurement of indices of refraction will be the subject of experiments numbers 4 and 5. In this experiment, however, we have been concerned with geometrical path length.

If the difference between the separations of the two full-silvered mirrors from the half-silvered one (beam splitter) is d, then the difference of geometrical path length for the two central (normal) rays is 2d, because the distance d is traversed once in each direction. Consequently, the condition for constructive interference for the central rays is:

$$2d = n\lambda$$

Where λ is the wave length of the light and n is an integer. Actually because of the difference between internal and external reflections at silvered surfaces a phase reversal of one of the rays may result, in which case the above condition would be appropriate to destructive interference, and that for constructive interference would be:

$$2d = (n + 1/2)\lambda$$

Since we measure fringe shifts here, which condition applies is of no consequence and the formula which is applicable is:

$$2\,(d_1 - d_2) = \lambda \Delta n$$

Where $(d_1 - d_2)$ is the distance the carriage is moved to cause the appearance or disappearance of Δn fringes at the center.

Using the calibration given above and the readings of the micrometer head (before and after carriage motion) the wave length is determined by use of the above formula, by substituting the appropriate values of $(d_1 - d_2)$ and Δn into it.

EXPERIMENT 2: Measurement of Sodium Doublet Separation.

Procedure: The sodium doublet consists of two yellow spectral lines, having wavelengths of 5890 and 5896 A. The 5890 A line is twice as intense as the 5896 A line.

Use a sodium light source (e.g. Cenco Number 87300) to establish a straight line fringe pattern. There are now two sets of fringe patterns formed, one for each line of the doublet. Loosen the carriage lock screw and move the carriage by hand and observe that the yellow fringes pass alternately from a condition of high contrast to one of almost complete disappearance. This latter condition occurs when one set of fringes is half way between the other set. Fix the carriage at one of the conditions of most complete disappearance by tightening the carriage lock screw and read the micrometer head. By turning the micrometer head, move to the next condition of most complete disappearance and read the head again. Repeat. Calculate the average distance d between conditions of disappearance. We can now calculate the difference in wavelengths of these two lines as follows:

At the first micrometer reading:

$$2d_1 = m_1\lambda_1 = (m_1 + n + 1/2)\,\lambda_2$$

Where λ_1 is greater than λ_2. The term on the right hand side indicates that the order of the shorter wavelength fringe differs from that of the longer wavelength fringe system by an odd half integer. This is true since the condition of disappearance of the fringe system occurs when one system is just halfway between the other. For the second reading, we have:

$$2\,d_2 = m_2\lambda_1 = (m_2 + n + 3/2)\,\lambda_2$$

sub. (2) − (1)

By subtraction we obtain:

$$2(d_2 - d_1) = \frac{\lambda_1\lambda_2}{\lambda_1 - \lambda_2}$$

$2(d_2 - d_1) = \Delta n\,\lambda_1 = (\Delta n + 1)\lambda_2$

order $\Delta n = m_2 - m_1$

Since λ_1 and λ_2 are approximately equal, we then obtain:

$$\lambda_1 - \lambda_2 = \Delta\lambda = \lambda^2/2d$$

Where λ is the average wavelength, and $d = d_2 - d_1 = 0.10\ (D_2 - D_1)K$.

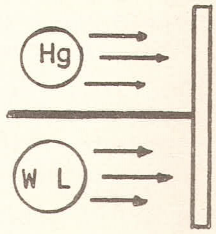
The average wavelength can be measured by repeating Experiment I using the Sodium lamp as the light source.

EXPERIMENT 3: Observation of White Light Fringes.

Procedure: In order to observe fringes with a source of white light, it is best to adjust the Interferometer to obtain the so-called localized fringes. This is done by adjusting one of the mirrors slightly so as to destroy the condition of exact perpendicularity. The fringe pattern now will consist of curved, horizontal or vertical stripes. Next, a position of nearly zero path length is searched for. This position is characterized by the fact that the striped pattern will become most nearly straight when the condition is achieved.

At this point an extended white light source may be substituted for the monochromatic source, and a very slow motion of the carriage will bring the white light fringes into view. White fringes are especially important in the Michelson Interferometer in that they give a precise indication of the position of zero optical path length difference.

An ordinary, frosted, incandescent lamp or even bright sunlight will serve very well as the light source for this experiment. The lamp can be placed behind the monochromatic light and turned on in the beginning of the experiment. When zero path length is obtained, simply turn off the monochromatic light.



Another means of obtaining zero path length is to construct a "T" by separating a diffuser plate with cardboard or black paper (see drawing). Place a monochromatic light source on one side, a white light source on the other, and turn both of them on. With this method it is not necessary to obtain localized fringes. Once any fringes have been obtained--they will appear in the left side of the mirror--loosen carriage lock screw and move carriage slowly by hand until the right side of the mirror flashes briefly with color. Tighten carriage lock screw. Search either way from this direction, and the white light fringes, which have the appearance of small spectra, will be finally located.

Discussion: Only a few white light fringes are observed. This is accounted for when we recall that white light consists of all wave lengths of visible light. Apart from the central fringe, the various interference patterns (for different wave lengths) will overlap. A colored fringe is violet on the side nearer to the central fringe and red on the other.

EXPERIMENT 4: Index of Refraction of a Transparent Solid.

Procedure: For the purpose of performing this experiment it will be necessary to construct a sample holder which is capable of positioning the sample accurately between the beam splitter and the fixed mirror (mirror number I).

The holder must be capable of giving a slow rotation of the sample, through a measureable angle.

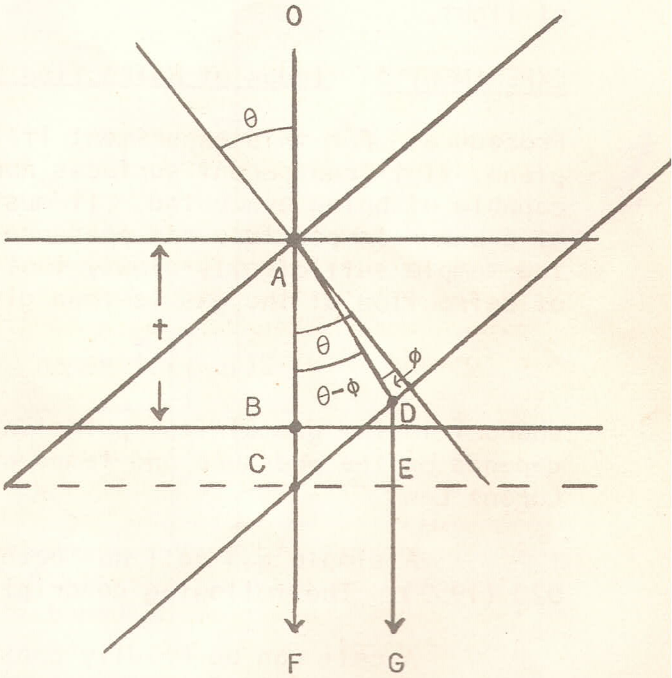
When the holder has been attached to the Interferometer and with the sample positioned normal to the beam, the instrument is aligned to produce circular monochromatic fringes. When this has been achieved, rotate the sample through an angle sufficient to produce a shift of a few hundred fringes. Count the number of fringes. The index of refraction of the sample is then given by the formula:

$$2\Delta(\mu x) = \lambda \Delta n$$

where $\Delta(\mu x)$ is the increase in optical path produced by the rotation.

For a given angle of rotation θ, fringe shift Δn, and wave length λ, μ is evaluated as follows:

Referring to the diagram:



Optical path before rotation $= \mu\,\overline{AB} + \overline{BC}$

Optical path after rotation $= \mu\,\overline{AD} + \overline{DE}$

Angle of rotation $= \theta$

$\overline{AD} = t \sec\phi$

$\overline{AB} = t$

$\overline{DE} = \overline{CE} \tan\theta$

$\overline{CE} = \overline{AD} \sin(\theta - \phi) = t \sec\phi \sin(\theta - \phi)$

$\overline{DE} = t \sec\phi \sin(\theta - \phi) \tan\theta$

$\overline{BC} = t \sec\theta - t$

$$\frac{\lambda \Delta n}{2} = \mu t \sec\phi + t \sec\phi \sin(\theta - \phi) \tan\theta - \mu t - t \sec\theta + t$$

$$\sec\phi \tan\theta \sin(\theta - \phi) = (\tan\theta - \tan\phi) \sin\theta$$

$$\frac{\lambda \Delta n}{2} = t\left(\frac{\mu - \sin\theta \sin\phi}{\cos\phi}\right) + t(1 - \cos\theta - \mu)$$

ϕ and θ are related by $\frac{\sin\theta}{\sin\phi} = \mu$ so

$$t\left(\frac{\mu - \frac{1}{\mu}\sin^2\theta}{\sqrt{1 - \frac{1}{\mu^2}\sin^2\theta}}\right) + t(1 - \cos\theta - \mu) = \frac{\lambda \Delta n}{2}$$

$$\frac{\lambda\Delta n}{2} = t\sqrt{\mu^2 - \sin^2\theta} + t(1 - \cos\theta - \mu)$$

$$t^2(\mu^2 - \sin^2\theta) = \left(\frac{\lambda\Delta n}{2}\right)^2 + t^2(\mu^2 + \cos^2\theta + 1 + 2\mu\cos\theta - 2\mu - 2\cos\theta)$$
$$+ \lambda t\Delta n(\mu + \cos\theta - 1)$$

Neglecting the term $\left(\frac{\lambda\Delta n}{2}\right)^2$ (because it is very small) and simplifying gives

$$\mu = \frac{\left(t - \frac{\lambda\Delta n}{2}\right)(1 - \cos\theta)}{t(1 - \cos\theta) - \frac{\lambda\Delta n}{2}}$$

Any transparent material available in suitable shape and size will be satisfactory. One must bear in mind that index of refraction depends on wave length and that, therefore, different results will be obtained for different colors of light.

EXPERIMENT 5: Index of Refraction of a Gas.

Procedure: For this experiment it is necessary to construct a gas cell with plane, flat transparent surfaces normal to the beam direction. The cell must be capable of being evacuated. It must also be possible to introduce the gas sample at a known temperature and pressure. It is, of course, necessary to introduce the sample sufficiently slowly that the fringe shift can be determined. The index of refraction of the gas is then given by

$$2(\mu - 1)t = \lambda\Delta n$$

where t is the geometrical path length through the cell. Here the quantity μ depends on the pressure and temperature of the gas according to the Lorentz-Lorenz Law.

A simple gas cell has been described by T. G. Bullen in Am. J. Phys. 27, 520 (1959). The following description is from his article:

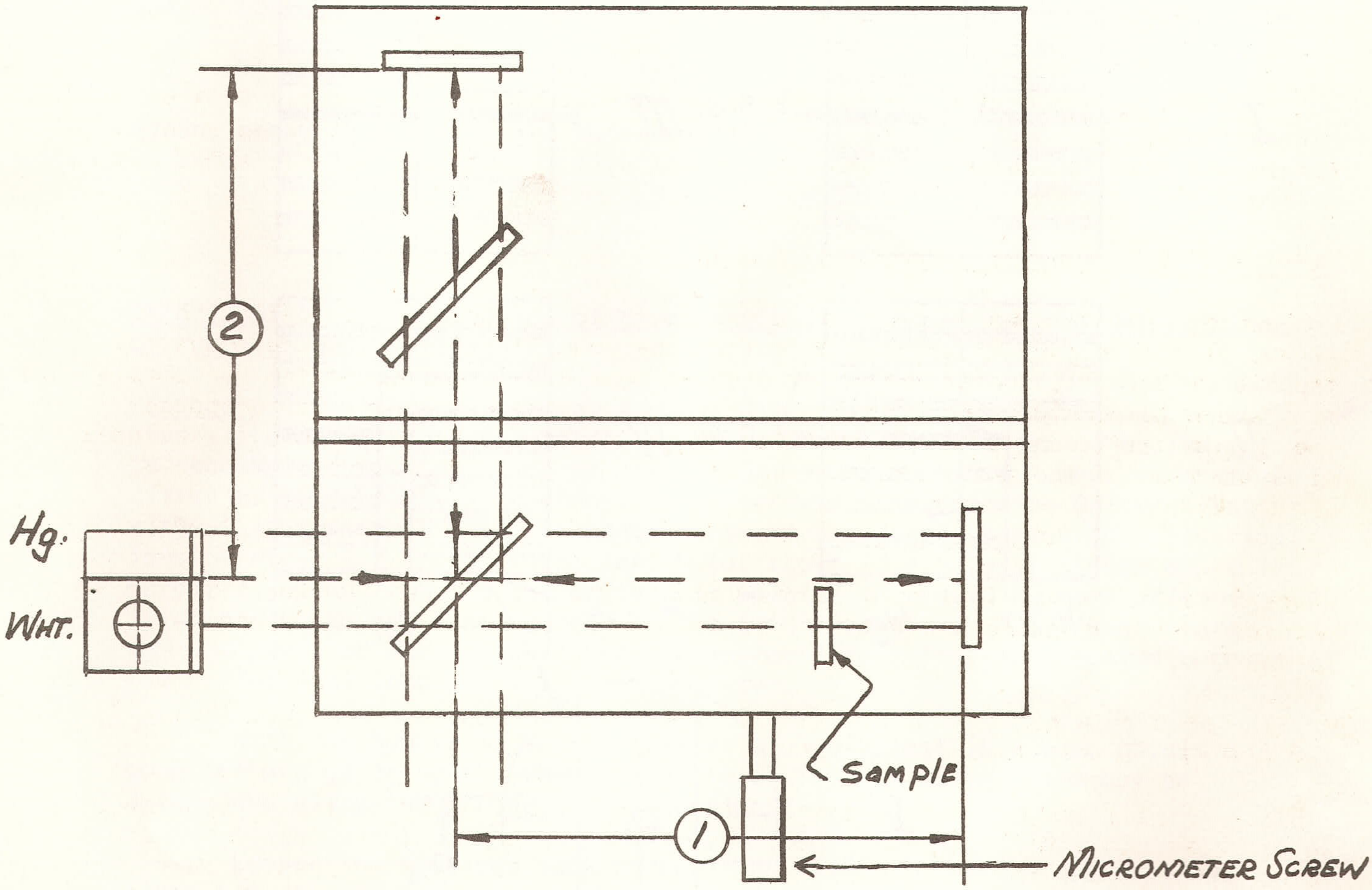
A cell can be readily constructed from a piece of brass tubing about 4 cm in diameter and 6 to 7 cm long. The ends of the tubing should be turned true in a lathe and a side tube attached for pumping. Thin plate glass squares, carefully cleaned with alcohol, are then fitted to the ends with Tackiwax (Central Scientific Co., Catalog No. 11444). The cell is placed in the uncompensated arm of the interferometer and attached to a ballast bottle of about five-liters capacity, fitted with a stop cock for admitting air and for connection to a vacuum pump. On pumping down the system the fringe pattern alters in a staccato fashion, very rapidly at first and then more slowly as vacuum is attained. Leaks can be detected if the pattern is observed to alter when pumping is complete. By admitting air slowly through the stop-cock it is possible to count the fringe displacement from vacuum to atmospheric pressure. For gases other than air the determination can be made by admitting the gas via the stop-cock. The ballast bottle permits fine control of the rate of fringe displacement without the use of a needle valve. For air, a displacement of about 60 fringes is obtained for a 6-cm cell; reasonable accuracy for the refractive indices of gases can be attained.

EXPERIMENT 6: Determining Thickness of Thin Transparent Films of Organic or Inorganic Materials.

A method of determining the thickness of such films is outlined here. This method will serve to determine: (1) Refractive index, if thickness is known; and (2) Thickness, if refractive index is known.

Procedure:

1. Set up the Interferometer as shown below. First the instrument is adjusted to show white light fringes in left half of the field and showing the black band (see Fig. I next page).

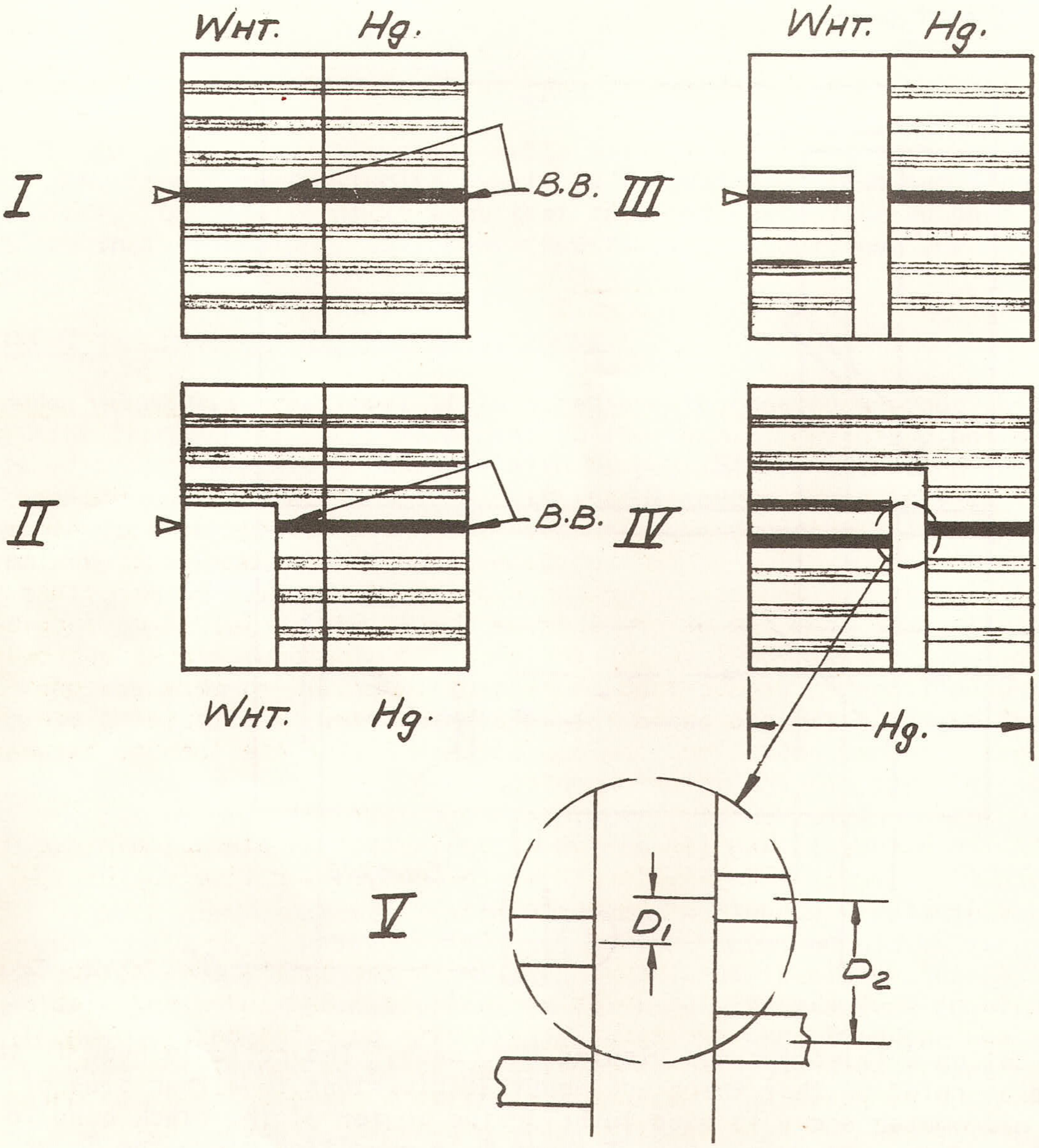


2. Set up a telescope for viewing the bands. The reticule used in the ocular should be ruled so that there are about 20 divisions to either side of center. The micrometer screw is used to bring the center of the black band to the center line of the reticule. Fig. I shows this condition minus the image of the reticule. Substituted, in the drawing, for the center line of the reticule is the symbol, ▷.

3. If now the film to be measured is placed somewhere in optical path #1 of the instrument and positioned so as to appear in part of the white fringe area as shown in Fig. II, it will be noticed that the white light fringes have disappeared in the portion of light path that now passes through the film. This is due to the fact that the film has caused the optical path #1 to appear longer due to the refractive index of the film. To return the black band so it shows in the file, the optical path #2 must be lengthened.

4. The micrometer screw is slowly turned toward higher readings so that the carriage moves farther from the beam splitter.

While this is done a careful count must be made of the Hg light fringes, one by one, as they pass any given fixed position until the black band is visible through the film and in its original horizontal position as in Fig. III



5. To determine the thickness, the formula given in Experiment 5 is used as follows:

$$t = \frac{\lambda_{air} \Delta n}{2(\mu - 1)}$$

where t = Thickness
Δn = Number of fringes passed over
μ = Index of refraction of the film material
λ_{air} = Wave length of Hg light = 5460Å

For example, suppose $\Delta n = 10$, $\mu = 1.5$, then

$$t = \frac{10 \times 5460}{2 \times .5} = 54,600 \text{ Å} = 5.46 \text{ micron} = .00546 \text{ mm.}$$

6. In measurement of very thin films where the displacement is less than one fringe (see Fig. V) measure as follows:

$$D_1 = 1/4 D_2$$

and D_2 corresponds to 1/2 wave length path difference ($\Delta n = 1/2$). Therefore

$$\Delta n = 1/4 \times 1/2 = 1/8$$

Again, assuming $\mu = 1.5$, the thickness is given as before by

$$t = \frac{1/8 \times 5460}{2 \times .5} = 582 \text{ Å}$$

EXPERIMENT 7: Determination of Wave Length Differences for the Balmer Lines of Hydrogen and Deuterium.

Procedure: For this experiment a Heavy Water Balmer Tube light source is used with the M-4 Interferometer. A Number 16 Wratten filter or equivalent is necessary for observation of the red Balmer lines, and a Number 45 Wratten filter or equivalent is necessary for observation of the blue Balmer lines without interference from the other lines of the Balmer series. A cylindrical lens of about 2.5 inches focal length (or about 15 diopters) placed approximately 2 inches from the Balmer tube is helpful in providing more uniform illumination to the field viewed in the interferometer. A diffuser plate (ground glass or waxed paper) and the appropriate filter are located between the cylindrical lens and the interferometer.

With the Number 16 Wratten filter or equivalent in place, obtain a good pattern of fringes. The wavelength of the red Balmer line may be determined using the procedure of Experiment 1.

Loosen the carriage lock screw. Now, with the bull's eye in view, place a thumb on each side of the interferometer base, and index and middle fingers on each side of the carriage. Very gently push the carriage until the bull's eye disappears. Place a centimeter scale on top of the beam splitter and the compensator. Measure the distance between the index marks in the top center of the beam splitter and compensator frames. Estimate distances to 0.1 millimeters. Again gently push the carriage. The bull's eye will reappear and then again disappear. Measure the distance between index marks. Repeat this procedure for five to ten successive disappearances of the bull's eye. In reducing the data only the initial and final measurements are used. However, a reasonable uniformity of the differences between intermediate measurements ensures that a disappearance of the bull's eye has not been missed in moving the interferometer carriage.

Between successive disappearances of the bull's eye, we have moved the carriage one more wavelength for the shorter wavelength line than for the longer wave length line.

$$2(d_2-d_1) = \lambda_1 \Delta n = \lambda_2(\Delta n + 1)$$

Where (d_2-d_1) is the distance the carriage is moved between successive disappearances of the bull's eye. However,

$$\lambda_1 = \lambda_2 + \Delta\lambda$$

Thus, $$\lambda_2 = \Delta n \Delta\lambda$$

$$2(d_2-d_1) = \frac{\lambda_2 \lambda_1}{\Delta\lambda}$$

Since λ_1 is approximately equal to λ_2, we have:

$$\Delta\lambda = \frac{\lambda^2}{2(d_2-d_1)}$$

An example of data taken and its reduction is given below.

$$\lambda = \frac{2(d_2-d_1)}{\Delta n} = \frac{4 \times 10^{-3} (D_2-D_1)}{\Delta n} = \frac{4 \times 10^{-3}(17.23 - 12.28)}{300} = 6.6 \times 10^{-5} \text{ cm.}$$

Disappearance of Bull's Eye	Distance between Index Marks (cm)	Distance Between Disappearances
0	7.85	
1	7.96	0.11
2	8.08	0.12
3	8.20	0.12
4	8.32	0.12
5	8.45	0.13
6	8.57	0.12
7	8.69	0.12
8	8.80	0.11
9	8.92	0.12

$$d_2-d_1 = \frac{1.07}{9} = 0.119$$

$$\Delta\lambda = \frac{\lambda^2}{2(d_2-d_1)} = \frac{(6.6 \times 10^{-5})^2}{0.238} = 1.83 \times 10^{-8} \text{ cm.}$$

More accurate values for the red lines are:

λ = 6563 Angstrom Units for $H\alpha$ and $\Delta\lambda$ = 1.79 Angstrom Units.

With the Number 45 Wratten filter or equivalent in place, the experiment may be performed for the blue Balmer line. An example of data taken and its reduction is given below.

$$\lambda = \frac{2(d_2-d_1)}{\Delta n} = \frac{4 \times 10^{-3}\,(D_2-D_1)}{\Delta n} = \frac{4 \times 10^{-3}(15.58-11.90)}{300} = 4.9 \times 10^{-5} \text{ cm.}$$

Disappearance of bull's eye	Distance between Index Marks (cm)	Distance Between Disappearances
0	8.03	
1	8.13	0.10
2	8.22	0.09
3	8.31	0.09
4	8.40	0.09
5	8.49	0.09

$$d_2-d_1 = 0.46/5 = 0.092$$

$$\Delta\lambda = \frac{\lambda^2}{2(d_2-d_1)} = \frac{(4.9 \times 10^{-5})^2}{0.184} = 1.31 \times 10^{-8} \text{ cm.}$$

More accurate values for the blue lines are:

λ = 4861 Angstrom Units for Hβ and $\Delta\lambda$ = 1.33 Angstrom Units.

Discussion: According to the Bohr theory, the wavelength of a spectrum line can be expressed by the formula:

$$\frac{1}{\lambda} = \frac{2\pi^2 me^4 Z^2}{ch^3}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Where
- λ is wavelength
- m is the mass of the electron
- e is the charge of the electron
- Z is the charge of the atomic nucleus
- c is the velocity of light
- h is Planck's constant
- n_1 is the quantum number of the initial state
- n_2 is the quantum number of the final state.

The Rydberg constant $R = 2\pi^2 me^4/ch^3$ so that

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right).$$

However, the electron does not rotate about a stationary nucleus but instead, both the electron and the nucleus rotate about the center of mass of the system. Thus the mass of the electron m should be replaced by the reduced mass $\frac{\mu}{1 + \frac{m}{M}}$ where M is the mass of the nucleus.

For the nucleus of hydrogen M = 1837 m and for the nucleus of deuterium M = 3674 m. Thus the Rydberg constant is slightly different for deuterium than for hydrogen. For hydrogen R = 109677.759. For deuterium R = 109707.387. For a nucleus of infinite mass R = 109737.424.

In the Bohr theory formula above Z = 1 for hydrogen and deuterium. For the Balmer series $n_1 = 2$. The red line H_α corresponds to $n_2 = 3$ and the blue line H_β corresponds to $n_2 = 4$.

References:

H. E. White, Introduction to Atomic Spectra, McGraw-Hill, 1934, pp. 27-38.

G. Herzberg, Atomic Spectra and Atomic Structure, Dover 1944, pp. 19-26, 182-183.

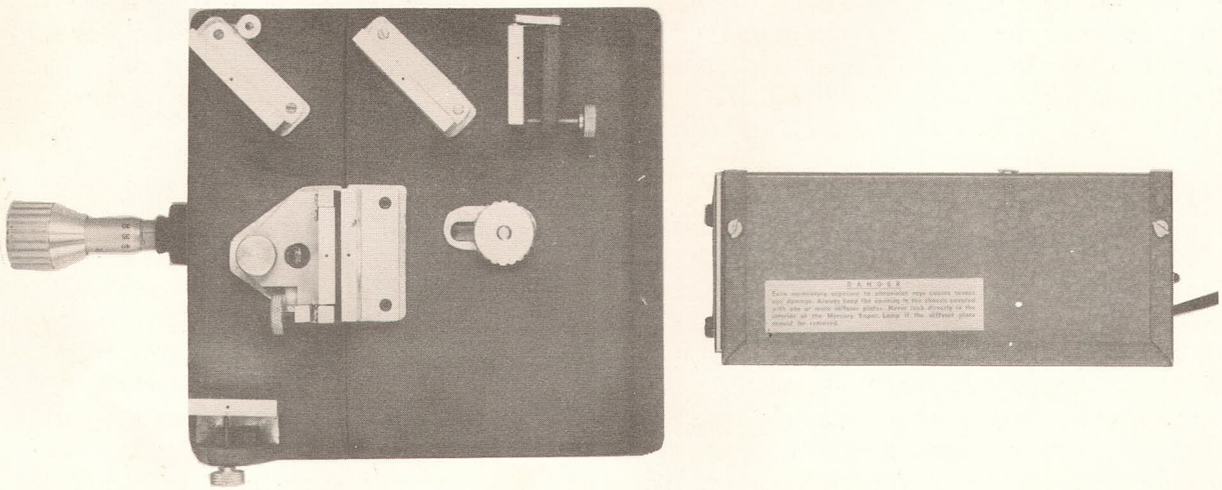
PART II

FABRY-PEROT OPTICS

ASSEMBLY: If the Fabry-Perot Optics are ordered with the M-4 Interferometer, unpacking and assembly instructions are the same as for the Michelson Optics (see page 1). If the Fabry-Perot Optics or the Michelson Optics are ordered later, they may be easily installed by removing the masking tape. Then the optical frames can be attached with screws to the drilled and tapped holes.

Under no conditions should the Fabry-Perot mirrors ever be touched with anything but a soft brush--such as the camel's hair brush, which is included with each order--or lens paper. Any cleaning fluid of any kind will seriously decrease the reflectivity of the mirrors and may possibly necessitate recoating.

LIGHT SOURCE: Both mercury and sodium light sources will be needed for Fabry-Perot experiments. The Atomic Laboratories' Mercury Light Source is quite satisfactory. For Fabry-Perot experiments it is positioned as shown below.



TELESCOPE: A 6 to 10 power telescope which can focus at infinity is desirable for performance of the following experiments. It is helpful, but not essential, that the telescope be provided with a reticle.

ADJUSTMENT: Loosen the carriage lock screw and adjust the carriage until the Fabry-Perot mirrors are about 1 millimeter apart. Large distances between mirrors makes adjustment more difficult. Now tighten the carriage lock screw.

The next step is to bring the mirror mounted on the base into exact parallelism with the mirror mounted on the carriage. This adjustment can be accomplished as follows.

Turn on the light source and observe multiple reflections in the mirrors. The initial steps in adjustment are accomplished more easily without the telescope in place. A black card having a pinhole in the center placed between the diffuser plate and the Fabry-Perot is sometimes helpful in observing the multiple reflections.

Bring the multiple reflections into coincidence by appropriate turning of the two adjusting screws. The orientation of the base-mounted plate is adjusted about the horizontal axis by the center adjusting screw, and about the vertical axis by the adjusting screw on the right side. If turning of the side adjusting screw is not adequate to bring the multiple images into coincidence, the Allen head screw next to the side adjusting screw may need adjustment.

When coincidence of the multiple images is obtained, remove the black pinhole card. A set of circular fringes should be visible. Precise adjustment for parallelism of the plates is now accomplished. If by moving the eye up and down the circular fringes appear to shrink and expand, a slight adjustment of the center adjusting screw is necessary. If by moving the eye from side to side the circular fringes appear to shrink and expand, a slight adjustment of the side adjusting screw is necessary. These adjustments should be continued until the shrinking and expanding of the circular fringe pattern is minimized and symmetrical about the center.

DISCUSSION: In the Michelson interferometer, the incoming light is split into two beams, each of which travels a different path before they are brought together to interfere. In the Fabry-Perot interferometer, the incoming light is split into many beams--the multiple reflections observed as the plates were being adjusted. The fringes become sharper as the number of reflections increases. The number of reflections increases as the coating on the mirrors (plates) becomes thicker and less transparent. In this instrument, the number of interfering beams is between 10 and 20, making the width of the fringes about 1/10 to 1/20 the distance between successive fringes. (In the Michelson interferometer, the fringes are just one-half as wide as the distance between fringes.) Wavelength measurements are therefore much more accurate when made using a Fabry-Perot interferometer. In most frequent practice, the two mirrors (usually called plates) are separated by fixed spacers in an arrangement called an etalon.

EXPERIMENT I: Measurement of Sodium Doublet Separation.

Procedure: The sodium doublet consists of two spectral lines in the yellow having wavelengths of 5890 and 5896 Angstrom Units. The 5890 Å line is twice as intense as the 5896 Å line.

Use a sodium light source (e.g. Cenco Number 87300) to establish a fringe pattern. When a good fringe pattern has been obtained, turn the micrometer head until the rings due to the weaker line (the less intense rings) are halfway between the brighter rings. Take a reading of the micrometer head. Now turn the micrometer head toward larger reading values until the weaker rings coincide with the stronger ones and then separate until the weaker rings are again halfway between the rings due to the stronger line. Take a new reading of the micrometer head. The formula for the fringe system of a Fabry-Perot Interferometer is:

$$m\lambda = 2\mu t \cos\theta$$

Where:
- m is the order of interference.
- λ is the wavelength of the light.
- μ is the index of refraction of the medium between the mirrors.
- t is the separation between the mirrors.
- θ is the angle measured from the normal to the mirrors.

With air for the medium between the mirrors, we have $\mu \cong 1$. At the center of the fringe pattern $\cos\theta = 1$. Our equation becomes:

$$m\lambda = 2t$$

For our first reading we have:

$$2t_1 = m_1\lambda_1 = (m_1 + n + 1/2)\lambda_2$$

Where λ_1 is greater than λ_2. The last term on the right-hand side means that the order of the shorter wavelength ring system must differ from that of the longer wavelength ring system by an odd half integer. This is so because the ring patterns have been adjusted to fall midway between each other.

For our second reading we have:

$$2t_2 = m_2\lambda_1 = (m_2 + n + 3/2)\lambda_2$$

(Note that if we had started with the plates in contact with each other, the quantity n would not have appeared in the two equations immediately preceding.)

By subtraction we obtain:

$$2(t_2 - t_1) = (m_2 - m_1)\lambda_1 = (m_2 - m_1 + 1)\lambda_2$$

$$(m_2 - m_1)(\lambda_1 - \lambda_2) = \lambda_2$$

$$(m_2 - m_1) = \frac{\lambda_2}{\lambda_1 - \lambda_2}$$

$$2(t_2 - t_1) = \frac{\lambda_1\lambda_2}{\lambda_1 - \lambda_2}$$

Since λ_1 and λ_2 are approximately equal, we then obtain:

$$\lambda_1 - \lambda_2 = \frac{\lambda^2}{2(t_2 - t_1)} .$$

The separation $(t_2 - t_1)$ is evaluated as in the case of the Michelson interferometer as:

$$t_2 - t_1 = 0.10\,(D_2 - D_1)\,K$$

Where $(D_2 - D_1)$ is the change of the micrometer reading as read in millimeters, and K is the ratio of carriage movement to micrometer reading.

$$K = 0.020$$

so:

$$(t_2 - t_1) = 2.0 \times 10^{-3}\,(D_2 - D_1).$$

Finally, the doublet separation is given by:

$$\Delta\lambda = \frac{\lambda^2}{0.4(D_2 - D_1)} \text{ cm.} = \frac{3.47}{(D_2 - D_1)} \text{ Å}$$

$$\Delta\lambda = \frac{\lambda^2}{4 \times 10^{-3}(D_2 - D_1)} \text{ cm.} = \frac{86.8}{(D_2 - D_1)} \text{ Å}$$

The procedure just described assumes that the viewing telescope is not fitted with a reticle. If the telescope is fitted with a reticle, then it is no longer essential that the weaker fringe pattern be precisely centered in the stronger fringe pattern. This will be discussed in greater detail below.

By use of the method described above, where the fringe pattern due to the weaker line has been displaced one order with respect to the fringe pattern due to the stronger line by turning the micrometer head, the following data were obtained.

D_2 (in.)	D_1 (in.)	$\lambda_2 - \lambda_1$ (Å)
25.50	11.18	6.06
19.90	5.40	5.98
22.52	8.06	6.00

Table I

Combining these results we obtain an average value for $\lambda_2 - \lambda_1$ of 6.01 Å.

When a telescope having a graduated reticle is used, a more precise determination of the difference in wavelength between the members of the sodium doublet may be obtained. A reticle containing 5 to 10 millimeters graduated in tenths of a millimeter is preferable. A reticle containing 0.2 to 0.4 inches graduated in 0.005 inches would be adequate.

One side of the fringe pattern is observed by the telescope. This is accomplished by moving the telescope so that there is a slight angle between the axis of the telescope and the normal to the Fabry-Perot plates.

With both weak and strong fringe patterns resolved from each other, and with the fringe patterns as sharp as fine adjustment of the adjusting screws will permit, measure radially from the center of the pattern the position at least some ten fringes, starting several fringes from the center of the pattern. After measuring the fringes, locate the first fringe measured to be sure the fringe pattern has not altered while the measurements were being made. Thermal equilibrium and absence of vibration are necessary for obtaining stable fringe patterns. Read the micrometer. Turn the micrometer head some fifteen to twenty-five revolutions toward higher readings while watching the fringes. The weak and strong fringe patterns will first merge and then again be resolved.

Again measure the position of at least ten fringes outward starting several fringes from the center of the pattern. Check the location of the first fringe measured to determine stability of the pattern. Again read the micrometer.

Analysis of the data is accomplished by use of the "Off-Centre Reduction Method" described on pages 133 and 134 of High Resolution Spectroscopy by S. Tolansky, Pitman Publishing Corporation, 1947.

At the center of a fringe system ($\cos\theta = 1$) we have $m_o\lambda = 2t$. For a circular fringe not in the center of the pattern we have $m\lambda = 2t\cos\theta$. Thus $m = m_o\cos\theta$ where θ is the angular radius of a ring of order of interference p. The order of interference of the p th ring in the system is $m = m_o - p$ (with a Fabry-Perot interferometer the highest order of interference is at the center of the fringe pattern).

To a close approximation $m = m_o\cos\theta$ can be written $m = m_o - m_o\frac{\theta^2}{2}$ for the small angles under consideration. The angular radius (in radians) for the p th ring is therefore $\theta = \sqrt{\frac{2p}{m_o}}$. Thus, in proceeding outward from the center of a fringe pattern the successive circular fringes close up in accordance with a parabolic formula since the squares of the radii are in arithmetic progression.

If

$$S_2W_2 = \Delta_2\ ,\ S_3W_3 = \Delta_3\ ,\ \text{etc.}$$

$$W_1W_2 = \Delta_{1W},\ W_2W_3 = \Delta_{2W},\ \text{etc.} \qquad \text{(See Figure 2)}$$

$$S_2S_3 = \Delta_{2S},\ S_3S_4 = \Delta_{3S},\ \text{etc.}$$

then to a reasonably close approximation the fraction of an order dm between the strong and weak line is given by

$$dm = \frac{2\Delta_2}{(\Delta_{1W} + \Delta_{2S})} = \frac{2\Delta_3}{(\Delta_{2W} + \Delta_{3S})} = \ldots \text{ for the initial micrometer reading}$$

Thus a series of independent values of dm is obtained and sufficient values lead to a good average. A diagram of the appearance of the fringe pattern and how the Δ's are obtained appears below.

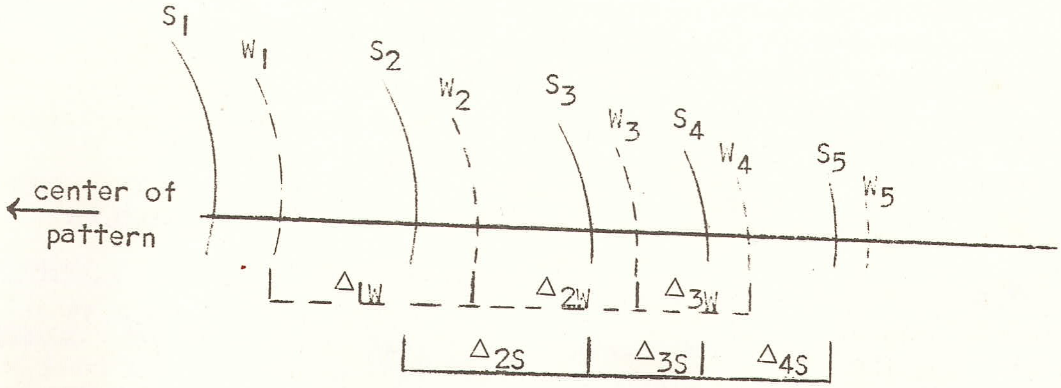


Fig. 2

In figure 2 members of the fringe pattern due to the stronger line of the sodium doublet have been designated by the letter S, and those due to the weaker line by the letter W. From the measurements taken of the fringe pattern at the initial micrometer reading, the Δs (Δ_1, Δ_2, etc.) will represent the distance from a strong fringe to the next weak fringe outward from the center of the pattern. From the measurements taken at the final micrometer reading the Δs will represent the distance from a weak fringe to the next strong fringe outward from the center of the pattern.

The formula for the difference in wavelength is:

$$\Delta\lambda = \lambda^2 \frac{dm_f - (-dm_i)}{2dt} = \lambda^2 \frac{dm_f + dm_i}{2 \times 2 \times 10^{-3}\,(D_f - D_i)}$$

Where $\Delta\lambda$ and λ are in centimeters and D_f and D_i are the final and initial micrometer readings in millimeters. λ is approximately 5.89×10^{-5} cm. The initial value of the fractional order dm_i is indicated as negative in the formula above because it has been measured in the opposite sense from the final value, dm_f.

An example of data taken and its reduction is given below.

D_i = 7.77 millimeters

$w_1 = 1.50$
$s_2 = 2.15$
$w_2 = 2.55$
$s_3 = 3.15$
$w_3 = 3.50$
$s_4 = 4.05$
$w_4 = 4.35$
$s_5 = 4.85$

$\Delta_2 = 0.40$
$\Delta_3 = 0.35$
$\Delta_4 = 0.30$

$\Delta_{1w} = 1.05$
$\Delta_{2w} = 0.95$
$\Delta_{3w} = 0.85$

$\Delta_{2s} = 1.00$
$\Delta_{3s} = 0.90$
$\Delta_{4s} = 0.80$

$$dm_1 = \frac{0.80}{2.05} = \frac{0.70}{1.85} = \frac{0.60}{1.65} = 0.377$$

$D_f = 19.60$ millimeters

$s_1 = 1.15$
$w_1 = 1.60$
$s_2 = 2.00$
$w_2 = 2.45$
$s_3 = 2.80$
$w_3 = 3.20$
$s_4 = 3.50$
$w_4 = 3.90$

$\Delta_1 = 0.40$
$\Delta_2 = 0.35$
$\Delta_3 = 0.30$

$\Delta_{1s} = 0.85$
$\Delta_{2s} = 0.80$
$\Delta_{3s} = 0.70$

$\Delta_{1w} = 0.85$
$\Delta_{2w} = 0.75$
$\Delta_{3w} = 0.70$

$$dm_f = \frac{0.80}{1.70} = \frac{0.70}{1.55} = \frac{0.60}{1.40} = 0.450$$

$$\Delta\lambda = (5.89)^2 \times 10^{-10} \frac{0.450 + 0.377}{2 \times 2 \times 10^{-3} (19.60 - 7.77)}$$

$$= 86.8 \times 10^{-8} \frac{0.827}{11.83}$$

$$= 6.06 \times 10^{-8} \text{ cm.}$$

= 6.06 Angstrom Units with a probable error of 0.16 Angstrom Units.

Discussion: The sodium doublet spectral lines are a result of the electronic transitions $3^2S_{1/2} - 3^2P_{1/2}$ (λ 5896) and $3^2S_{1/2} - 3^2P_{3/2}$ (λ 5890). The wavelength difference is an indication of the difference in energy between the electronic states having the electron spin angular momentum aligned antiparallel (λ 5896) and parallel (λ 5890) with the electron orbital angular momentum around the nucleus.

$$\lambda\nu = c$$

Where ν is frequency in cycles per second and c is the velocity of light. Therefore:

$$\Delta\nu = -\frac{c}{\lambda^2}\Delta\lambda$$

The energy difference between the two electronic levels is:

$$\Delta E = h\Delta\nu = -\frac{hc}{\lambda^2}\Delta\lambda$$

Where $h = 6.62 \times 10^{-27}$ erg - second is Planck's constant.

The accepted value of $\Delta\lambda$ for the sodium doublet is 5.96 Å which may be compared with the experiment quoted above. Using the value $\Delta\lambda = 5.96 \times 10^{-8}$ cm., the above formula for ΔE gives the result:

$$\Delta E = \frac{6.62 \times 10^{-27} \times 3 \times 10^{10} \times 5.96 \times 10^{-8}}{(5.89)^2 \times 10^{-10} \times 1.6 \times 10^{-12}}$$

$$= 2.13 \times 10^{-3} \text{ ev.}$$

EXPERIMENT 2: Observation of Hyperfine Structure in the Mercury Green Line.

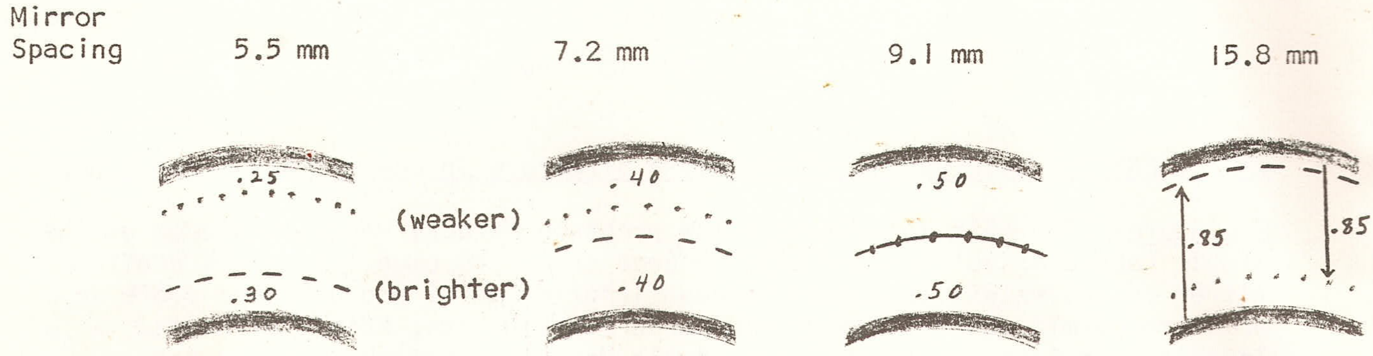
Procedure: For this experiment a low pressure Mercury vapor lamp such as the Atomic Laboratories' Mercury Light Source should be used. A No. 74 Wratten filter or equivalent is placed between interferometer and diffuser plate in order to permit observation of the mercury green line, λ5461, without interference from the other lines of the Mercury spectrum. Due to the faintness of the lines to be observed, one of the two diffuser plates on the Atomic Laboratories Mercury Light Source may be removed to enhance the intensity of the fringes. However, care should be taken to prevent direct exposure of the eye to the exposed lamp in the light source.

When good fringes have been obtained, place the telescope in a position such that the adjusting screws of the Fabry-Perot are accessible while the fringes are under observation through the telescope. Since fringes are much more conveniently obtained with a spacing of about 1 mm between the mirrors, and observation of the mercury hyperfine structure requires considerably increased spacing between the mirrors, this increase in spacing must be carefully accomplished in order not to lose the fringe pattern.

First loosen the carriage lock screw. Then, with a thumb on each side of the interferometer base, and index and middle fingers on each side of the carriage, very gently push the carriage away in order to increase the distance between the mirrors. (The carriage is moved by hand to achieve a sufficiently large separation.) Meanwhile carefully observe the fringe pattern for loss of sharpness of the fringes. When the fringes have become blurred, cease pushing and use the adjusting screws to restore sharpness to the fringes. Using this procedure, the spacing between the mirrors can be increased from one-half to one millimeter at a time.

As the spacing between the mirrors is gradually increased, a faint fringe will appear to move outward from each bright fringe and a yet fainter fringe will appear to move inward from each bright fringe. This effect will first become apparent when the spacing between the mirrors is about 3 millimeters. When the spacing between the mirrors is some 5 millimeters, each faint fringe will appear to have moved about one fourth of the distance between bright fringes. When the spacing between the mirrors is some 10 millimeters, the faint fringes will appear to overlap about halfway between bright fringes. When the spacing between mirrors is some 20 millimeters, the faint fringes will appear to have become superimposed on the bright fringe once removed from the bright fringe from which they started.

At mirror spacings of about 5, 10, and 15 millimeters, make estimates of the location of the faint fringes with respect to the bright fringes. At these positions accurately measure the separation between the mirrors. This can be accomplished by carefully laying a millimeter scale across the top of the mirror frames and sighting downward. Parallax errors can be minimized by moving the head back and forth and taking the scale reading at the point where the mirror surface disappears from the eye's view. With care, accuracies of 0.1 mm can be achieved. Some typical observations appear below.



Portions of two adjacent bright fringes are shown with the relative position of the fainter fringes between them.

<u>Discussion</u>: Mercury in nature consists of about 70% of isotopes having even mass numbers and 30% having odd mass numbers. The hyperfine structure observed here is due to the nuclear spin possessed by the odd isotopes Hg^{199} and Hg^{201}. The energy level diagrams are shown below.

λ 5461 ($6^3P_2 - 7^3S_1$)

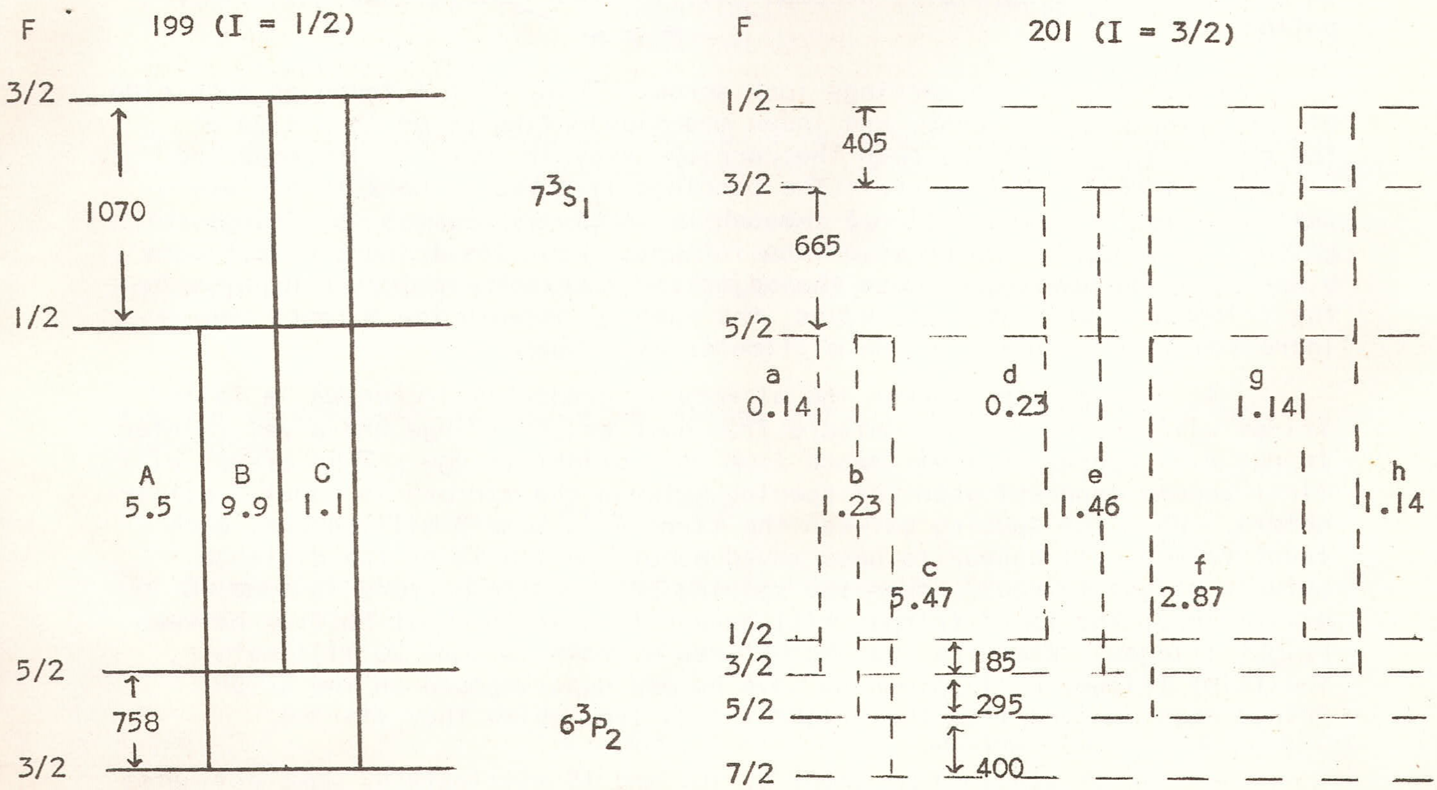


Fig. 3

F represents the vector sum of the nuclear spin and total electronic angular momentum vectors. I is the nuclear spin. The numbers between levels indicate level separations in thousandths of wave numbers (wavenumber = $\frac{1}{\text{wavelength in cm.}}$.) The numbers below the letters indicate the percentage intensity of the hyperfine spectrum line due to that transition. With the above level structure, a diagram of intensity appears as follows:

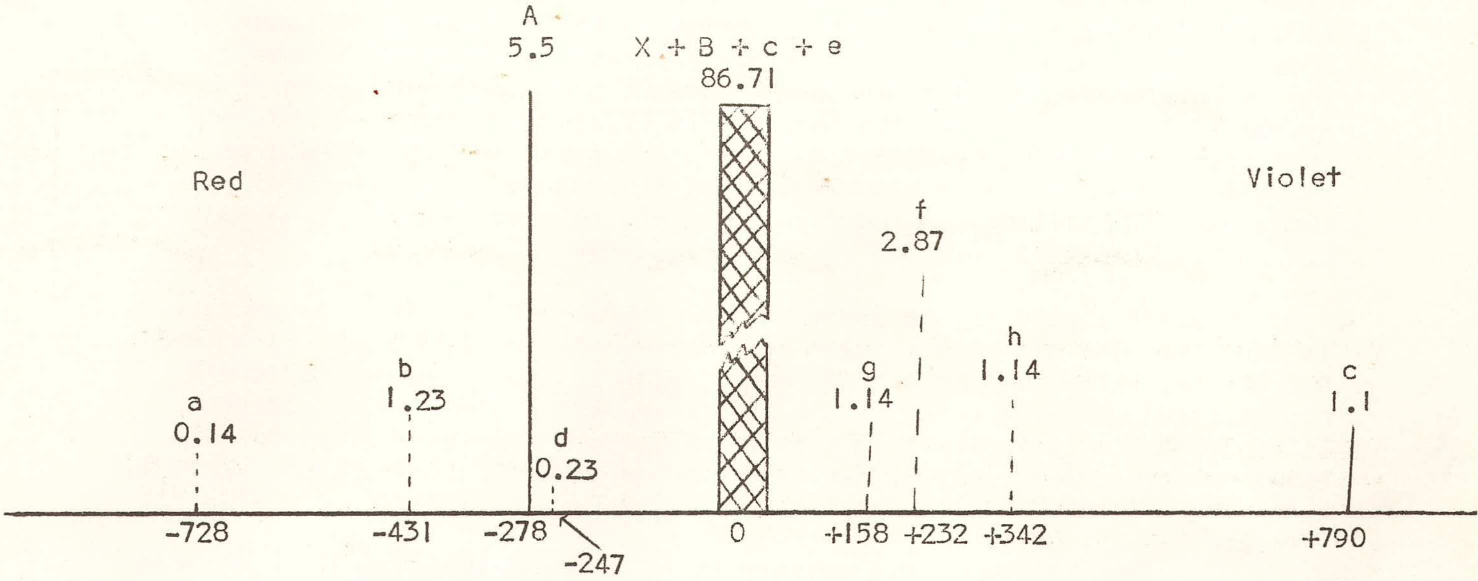


Fig. 4

X is the line due to the even mass isotopes. The letters and numbers are similar to those of the previous diagram. The numbers on the abscissa represent deviations from the center of the intense line in thousandths of a wave number. This research may be found in articles by H. Schüler and J. E. Keyston, Zeitschrift für Physik 72, 423 (1931), and H. Schüler and E. G. Jones, Zeitschrift für Physik 74, 631 (1932).

In wave numbers we find that the free range between orders is the reciprocal of twice the mirror spacing in centimeters. That is, since $m\lambda = m/\sigma = 2t\cos\theta$, where σ represents wave number (units of $cm.^{-1}$) then for $dm = 1$ we have $d\sigma = 1/2t$ (note $\cos\theta \cong 1$).

Using the relationship $d\sigma = \frac{1}{2t}$ for the difference in wave numbers between two bright fringes, the relationship of the faint fringes can be found. Using as an example the typical observation with a mirror spacing of 5.5 mm illustrated previously, we see that the faint fringe illustrated by dashes has moved outward from the center of the fringe pattern (that is, toward the violet) some three tenths the distance between bright fringes. Since $d\sigma = 0.91\ cm^{-1}$ between bright fringes, the faint fringe is some 0.27 cm^{-1} from the bright fringe toward the violet. This is approximately the position of the components "f" and "h" due to the isotope Hg^{201}. Similarly we see that the faint fringe illustrated by dots has moved inward toward the center of the fringe pattern (that is, toward the red) about one-fourth the distance between bright fringes. The faint fringe is thus some 0.23 cm^{-1} from the bright fringe toward the red. This is approximately the position of the component "A" due to the isotope Hg^{199}.

EXPERIMENT 3: Determination of Wave Length Differences for the Balmer Lines of Hydrogen and Deuterium.

Procedure: For this experiment a Heavy Water Balmer Tube Light Source is used. A Number 16 Wratten filter or equivalent is needed for observation of the red Balmer lines and a Number 45 Wratten filter or equivalent is necessary for observation of the blue Balmer lines without interference from the other lines of the Balmer series. A cylindrical lens of about 2.5 inches focal length placed approximately 2 inches from the Balmer tube is necessary in order to provide reasonably uniform illumination to the field viewed through the interferometer. A diffuser plate (ground glass or waxed paper) and the appropriate filter are located between the cylindrical lens and the interferometer. A viewing telescope of six to ten power having a graduated reticle is used in making measurements of the fringe pattern. A reticle containing 10 millimeters graduated in tenths of a millimeter or containing 0.4 inches graduated in 0.005 inches is suitable.

Obtain a good fringe pattern with the Fabry-Perot Mirrors practically in contact. The distance between the mirrors should be between 0.1 mm and 0.2 mm. Turn the micrometer head some ten to fifteen revolutions toward higher readings while watching the fringe pattern. The initial somewhat broad fringes will separate into pairs of somewhat narrower fringes. The outer fringe of each pair is due to deuterium and the inner fringe is due to hydrogen.

One side of the fringe pattern is to be measured using the telescope with a graduated reticle. This is accomplished by moving the telescope so that there is a slight angle between the axis of the telescope and the normal to the Fabry-Perot plates.

With both the hydrogen and the deuterium fringe patterns well resolved from each other, and with the fringe patterns as sharp as fine adjustment of the adjusting screws will permit, measure radially from the center of the pattern the position of at least eight fringes, starting with a deuterium fringe several fringes from the center of the pattern. After measuring the fringes locate the first fringe measured to be sure the fringe pattern has not altered while the measurements were being made. Thermal equilibrium and absence of vibration are necessary for obtaining stable fringe patterns. Read the micrometer.

Turn the micrometer head some eight to ten revolutions toward higher readings while watching the fringe patterns. The outer (deuterium) fringe of each pair will separate further from the inner (hydrogen) fringe. Again measure radially from the center of the pattern the position of at least eight fringes, starting with a deuterium fringe several fringes from the center of the pattern. Check the location of the first fringe measured to determine stability of the pattern. Again read the micrometer.

Analysis of the data is accomplished as described in Experiment 1. However, the formula for the difference in wavelength is:

$$\Delta\lambda = \lambda^2 \frac{dm_f - dm_i}{2 \times 2 \times 10^{-3} (D_f - D_i)}$$

since the initial value of the fractional order, dm_i, has been measured in the same sense as the final value, dm_f.

An example of data taken and its reduction is given below. The W's (for weak) of Experiment I have been replaced by D's (for deuterium), and the S's (for strong) have been replaced by H's (for hydrogen). The Δ's (Δ_1, Δ_2, etc.) represent the distance from a hydrogen fringe to the next deuterium fringe outward from the center of the pattern for measurements taken at both the initial and final micrometer readings.

D_i = 18.17 millimeters

$D_1 = 4.90$

$H_2 = 6.00$ $\qquad\qquad \Delta_{1D} = 1.70$

$\qquad \Delta_2 = 0.60$

$D_2 = 6.60$ $\qquad\qquad\qquad \Delta_{2H} = 1.50$

$H_3 = 7.50$ $\qquad\qquad \Delta_{2D} = 1.40$

$\qquad \Delta_3 = 0.50$

$D_3 = 8.00$ $\qquad\qquad\qquad \Delta_{3H} = 1.30$

$H_4 = 8.80$ $\qquad\qquad \Delta_{3D} = 1.20$

$\qquad \Delta_4 = 0.40$

$D_4 = 9.20$ $\qquad\qquad\qquad \Delta_{4H} = 1.10$

$H_5 = 9.90$

$$dm_i = \frac{1.20}{3.20} = \frac{1.00}{2.70} = \frac{0.80}{2.30} = 0.364$$

D_f = 23.50 millimeters

$D_1 = 4.30$

$H_2 = 5.40$ $\qquad\qquad \Delta_{1D} = 1.90$

$\qquad \Delta_2 = 0.80$

$D_2 = 6.20$ $\qquad\qquad\qquad \Delta_{2H} = 1.60$

$H_3 = 7.00$ $\qquad\qquad \Delta_{2D} = 1.50$

$\qquad \Delta_3 = 0.70$

$D_3 = 7.70$ $\qquad\qquad\qquad \Delta_{3H} = 1.40$

$H_4 = 8.40$ $\qquad\qquad \Delta_{3D} = 1.20$

$\qquad \Delta_4 = 0.50$

$D_4 = 8.90$ $\qquad\qquad\qquad \Delta_{4H} = 1.20$

$H_5 = 9.60$

$$dm_f = \frac{1.60}{3.50} = \frac{1.40}{2.90} = \frac{1.00}{2.40} = 0.452$$

$$\Delta\lambda = (6.56)^2 \times 10^{-10} \frac{0.452 - 0.364}{2 \times 2 \times 10^{-3}\ (23.50 - 18.17)}$$

$$= 4.30 \times 10^{-8}\ \frac{0.088}{0.213}$$

$$= 1.78 \times 10^{-8} \text{ cm.}$$

= 1.78 Angstrom Units with a probable error of 0.20 Angstrom Units.

Vrey, Brickweddè, and Murphy (Phys Rev <u>40</u>, 1, 1932) measured $\Delta\lambda$ to be 1.79 Angstrom Units in discovering the presence of the deuterium isotope. The calculated value for $\Delta\lambda$ is 1.787 Angstrom Units.

With the Number 45 Wratten filter or equivalent in place, the experiment may be performed for the blue Balmer lines. An example of data taken and its reduction in this case is given below.

D_i = 10.62 millimeters

$D_1 = 1.00$			
$H_2 = 3.30$		$\Delta_{1D} = 3.10$	
	$\Delta_2 = 0.80$		
$D_2 = 4.10$			$\Delta_{2H} = 2.10$
$H_3 = 5.40$		$\Delta_{2D} = 1.90$	
	$\Delta_3 = 0.60$		
$D_3 = 6.00$			$\Delta_{3H} = 1.60$
$H_4 = 7.00$		$\Delta_{3D} = 1.50$	
	$\Delta_4 = 0.50$		
$D_4 = 7.50$			$\Delta_{4H} = 1.30$
$H_5 = 8.30$		$\Delta_{4D} = 1.20$	
	$\Delta_5 = 0.40$		
$D_5 = 8.70$			$\Delta_{5H} = 1.20$
$H_6 = 9.50$			

$$dm_i = \frac{1.60}{5.20} = \frac{1.20}{3.50} = \frac{1.00}{2.80} = \frac{0.80}{2.40} = 0.335$$

D_f = 16.88 millimeters

$D_1 = 3.40$			
$H_2 = 4.40$		$\Delta_{1D} = 1.80$	
	$\Delta_2 = 0.80$		
$D_2 = 5.20$			$\Delta_{2H} = 1.50$
$H_3 = 5.90$		$\Delta_{2D} = 1.30$	
	$\Delta_3 = 0.60$		
$D_3 = 6.50$			$\Delta_{3H} = 1.20$
$H_4 = 7.10$		$\Delta_{3D} = 1.10$	
	$\Delta_4 = 0.50$		
$D_4 = 7.60$			$\Delta_{4H} = 1.10$
$H_5 = 8.20$		$\Delta_{4D} = 1.10$	
	$\Delta_5 = 0.50$		
$D_5 = 8.70$			$\Delta_{5H} = 1.00$
$H_6 = 9.20$			

$$dm_f = \frac{1.60}{3.30} = \frac{1.20}{2.50} = \frac{1.00}{2.20} = \frac{1.00}{2.10} = 0.474$$

$$\Delta\lambda = (4.86)^2 \times 10^{-10} \frac{0.474 - 0.335}{2 \times 2 \times 10^{-3}\ (16.88-10.62)}$$

$$= 2.36 \times 10^{-8} \frac{0.139}{0.250}$$

= 1.31 Angstrom Units with a probable error of 0.14 Angstrom Units.

Vrey and his co-workers measured $\Delta\lambda$ to be 1.33 Angstrom Units for the blue Balmer lines. The calculated value for $\Delta\lambda$ is 1.323 Angstrom Units.

Discussion: In applying quantum theory to his theory of the hydrogen and hydrogen-like atoms, Bohr made the assumption that the frequency of a spectrum line is proportional to the difference between two energy states. This, combined with his assumptions regarding the electron's orbits in a coulomb field about the nucleus and the quantization of electronic orbital angular momentum, leads to the expression:

$$\frac{1}{\lambda} = \frac{2\pi^2 me^4 Z^2}{ch^3} \quad \frac{1}{n_1^2} = \frac{1}{n_2^2}$$

Where:
- λ is the wavelength of the spectrum line in cm.
- m is the mass of the electron = 9.035×10^{-28} gm.
- e is the electronic charge = 4.770×10^{-10} abs. e.s.u.
- h is Planck's constant = 6.62×10^{-27} erg sec.
- c is the velocity of light = 3.0×10^{10} cm/sec.
- Z is the nuclear charge (one for hydrogen, two for ionized helium, three for doubly ionized lithium, etc.)

n_1 is the quantum number of the initial state.
n_2 is the quantum number of the final state.
$R = \frac{2\pi^2 me^4}{ch^3}$ is the Rydberg constant for an atom with nucleus of infinite mass = R_∞.

In general, the electron and the nucleus both rotate about a common center of gravity. The so-called reduced mass should be used in the equation for the Rydberg constant. The expression for the reduced mass is:

$$\mu = \frac{m}{1 + \frac{m}{AM_p}}$$

Where M_p is the atomic mass unit and A is the atomic weight.

Then the Rydberg constant becomes:

$$R_A = \frac{R_\infty}{1 + \frac{m}{AM_p}} = \frac{R_\infty}{1 + \frac{1}{1837\ A}}$$

since $\frac{M_p}{m} = 1837$.

Substituting the appropriate constants:

$$R_\infty = 109737.303$$

Z = 1 for hydrogen. Solving an equation for the difference in wavelength between isotopic hydrogen lines we have :

$$\Delta\lambda = \frac{1}{R_\infty \frac{M_p}{m}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)} \left(\frac{A-1}{A}\right)$$

Or, solving for A:

$$A = \frac{1}{1 - \Delta\lambda R_\infty \frac{M_p}{m}\left(\frac{1}{n_1^2} \quad \frac{1}{n_2^2}\right)}$$

For the Balmer lines $n_1 = 2$, for the red Balmer line $n_2 = 3$, and for the blue Balmer line $n_2 = 4$. Thus, a measurement of the wavelength difference provides an estimate of the atomic weight of deuterium.

References:

H.E. White, Introduction to Atomic Spectra, McGraw-Hill, 1934, pp. 27-38.

G. Herzberg, Atomic Spectra and Atomic Structure, Dover, 1944, pp. 19-26, 182-183.

PART III

Replacement Optics: As stated previously, the M-4 Interferometer may be ordered with Michelson Optics, with Fabry-Perot Optics, or with a combination of both. If one set of optics is ordered (such as the Michelson Optics) and the customer later wishes to order the Fabry-Perot set, this may easily be accomplished. All sets are shipped mounted in frames and ready for insertion in the holes that are already drilled and tapped. Simply remove the masking tape on the M-4 chassis and carriage, and the optics are ready for immediate installation. The necessary screws are included in each shipment. Because of the difficulty of mounting the optics and frames, they are never sold unmounted.

Michelson Optics mounted (Complete Set). $150.00

Fabry-Perot Optics mounted (Complete Set). $175.00

Individual Optics, mounted:

A. Front surface mirror . . . . $ 40.00

B. Beam Splitter. . . . . $ 40.00

C. Compensator. . . . . $ 30.00

D. Fabry-Perot Mirror $100.00

NOTE: All Michelson optics are guaranteed to be flat to within 1/4 wave length of green light over their entire surfaces (front surface mirrors over mirrored surfaces). Fabry-Perot mirrors are guaranteed to be within 1/6 wave length of green light over their matching surfaces.