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Measuring the separation of the sodium D-doublet with a Michelson interferometer

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Abstract

Revisiting a method proposed by Fizeau in 1862, in this paper we measure the separation of the Na-doublet (the wavelength difference $\Delta\lambda$ between the two emission D-lines of the sodium spectrum) with a didactical Michelson interferometer. We describe the setup, how the measurements have been done and develop a mathematical model in order to explain the principal features of the collected data. Discussing the limits of this model, we suggest further experimental and theoretical extensions of the experience, also focusing on the didactical aspects to show how this experiment could bring advanced modern physics topics into high schools.

Supplementary material for this article is available online

Keywords: sodium doublet, Michelson interferometer, beats, interferometry

(Some figures may appear in colour only in the online journal)

1. Introduction

The Michelson interferometer (MI) is one of the best known but still intriguing instruments. From a historical point of view, among the experiments dealing with Newtonian absolute space, Earth movement and the detection of the ether wind, those with the MI are generally regarded as foundational [1]. From a didactical point of view, the MI allows educators to present many experiments—interference conditions, measure of wavelengths and refractive indexes—easy to perform and really supporting the understanding of the basic concepts of the wave model [2]. However, supported by instruments now available in school laboratories, the

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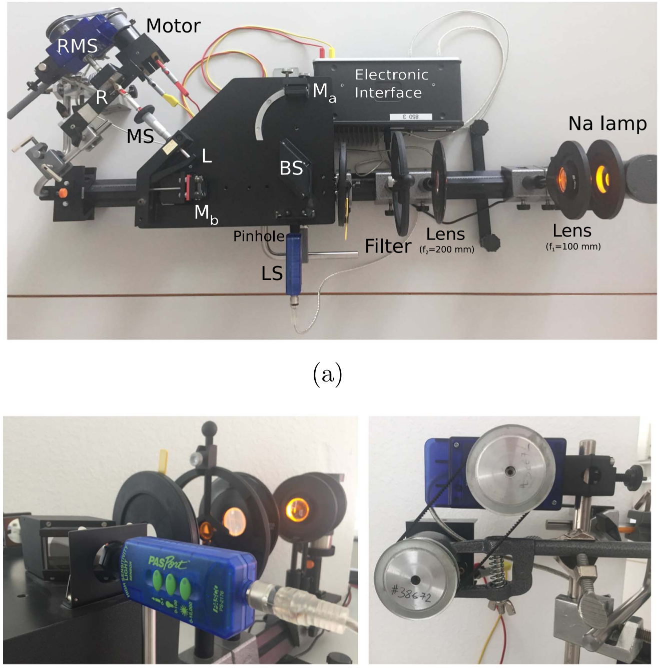


(a)

(b) (c)

Figure 1. (a) Top view of the experimental setup, (b) the light sensor and the optics and (c) the mechanical coupling between the motor and the RMS.

MI allows one to also attempt experiments of greater conceptual complexity or experimental difficulty as the one we present in this paper: the measure of the separation of the Na-doublet, i.e. the wavelength difference $\Delta\lambda$ between the two emission D-lines of the sodium spectrum. To our knowledge, the separation $\Delta\lambda$ was determined for the first time in 1862 by Fizeau [3], using the Newton's rings method. His high experimental skill and persistence allowed him to observe a light contrast modulation over about 50 beat periods each averaging 983 fringes, determining the value $\Delta\lambda = 0.60$ nm, a result of astonishing precision for the time. After remarkable remakes of Fizeau's experiment [4, 5], today the most popular way to measure the separation $\Delta\lambda$ in introductory university physics labs is to use a MI [6–9], but as we will show, the high performance of modern data acquisition systems makes possible gathering data at a suitable high sampling rate and for a sufficiently long time so that their analysis becomes accessible for high school students too.

2. Setup and experimental results

Figure 1 shows the experimental setup: a Na-spectral lamp, two lenses ($f_1 = 100$ mm, $f_2 = 200$ mm), a 10 nm band-pass filter centered at 589 nm, a MI with a beam splitter (BS), a fixed mirror (M_a) and a mobile mirror (M_b), a rotary motion sensor (RMS) to measure the M_b displacement, a

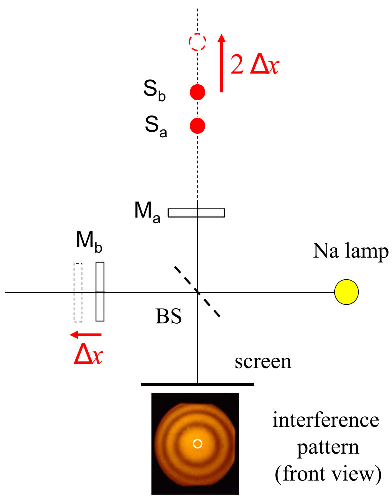


Figure 2. Schematic representation of the light paths: a displacement Δx of the mirror M_b induces an increment $2\Delta x$ of the optical path. The diameter of the image on the screen is about 3.0 cm; the white circle marks the region seen by the light sensor (provided with a pinhole of 1.5 mm of diameter) during the measurements.

light sensor (LS) with a pinhole, an electronic interface to collect data[4]. M_b is moved by a device made by a electrical motor, two pulleys coupled by a toothed belt (one is fixed on the RMS axis, working also as a turns counter), a 1:100 reducer (R) and a 0.5mm/turn micrometrical screw (MS) coupled to a 1:5 lever (L), so that by our operative conditions 100 turns of the RMS correspond to 1 turn of the MS, and to a 100 μm displacement of M_b.

The fringes pattern actually seen on the screen (in the following replaced by the LS, figure 2) depends on the position and inclinations of the mirrors. As well-known, to observe circular fringes, i.e. a fringes pattern made by concentric circles, the geometry of the MI must be regulated in order to align along a direction normal to the screen the symmetry axis of the system of rotational hyperboloids originated by the two point-like interfering virtual sources S_a and S_b, associated to the mirrors M_a and M_b.

When the mobile mirror M_b is driven away, the fringes on the screen begin to spread out from the center of the image, so that the light intensity captured by the LS oscillates. But in the case of the Na-lamp, this oscillation is accompanied also by a periodical variation of fringes visibility, i.e. of the contrast between adjacent maxima and minima. Figure 3 presents a typical sequence of circular fringes observed while M_b moves: the clear fringe contrast visible in figure 3(a) fades gradually until figure 3(e), where it is almost imperceptible, to raise then to the initial visibility in figure 3(h). In fact, as we started the measurement providing the LS with a pinhole of 1.5 mm of diameter (to capture efficiently light intensity in a small region, ideally at a point), we can clearly recognize the intensity changes caused by the passage of single fringes, as evident in the data of figure 4(a), registered during 50 s. But on a much longer time of about 2.5 hours (about 2300 RMS turns, figure 4(b)), we also registered a

[4] We use the Osram 15 W Na/10 Spectroscopic lamp, powered with a Leybold 451 30 Universal choke for spectrum lamps. The Michelson interferometer is an Ealing 25-9069, the rotary motion sensor is a PASCO PS-2120A and the high sensitivity light sensor is the PASCO PS-2176, equipped with a handmade circular pinhole of 1.5 mm diameter, in order to collect the maximum light without losing resolution. Data are collected with the PASCO UI-5000 850 universal interface and elaborated with the Capstone software, http://pasco.com.

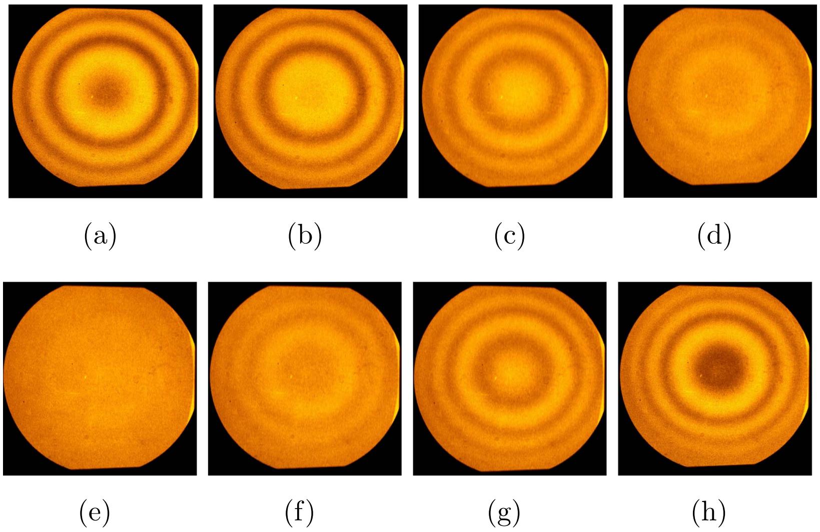


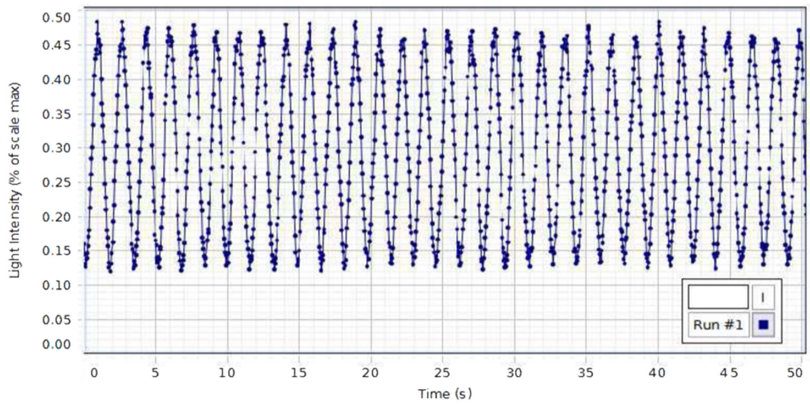
Figure 3. Typical sequence of circular fringes observed on the screen: the displacement of the mirror M_b between (a) and (h) is about 0.3 mm, i.e 300 turns of the RMS. The diameter of each pattern is about 3.0 cm.

periodical modulation of the fringes visibility, whose minima are easily identified as beat nodes.

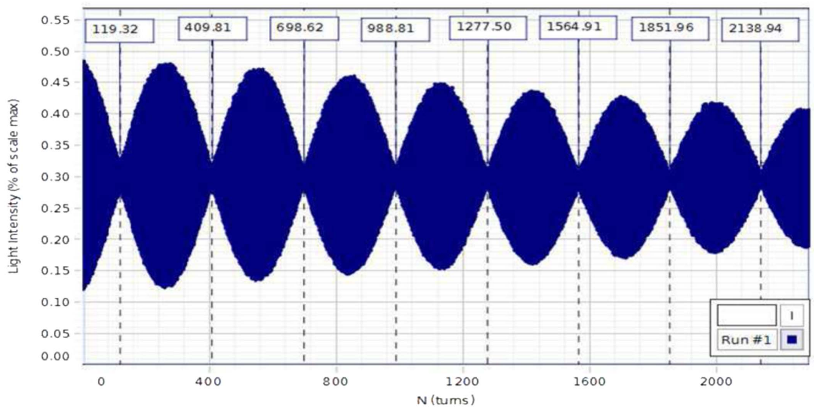
To reproduce Fizeau's result it is sufficient to count the number of fringes between two consecutive beat nodes, i.e. in half a modulation period, as we did after reporting the numerical values of the RMS turns where the beat nodes occur (figure 4(b)), determined by spreading out the time scale of the registered data and exploiting their intrinsic symmetry. Implementing this procedure we get a mean value of (986 ± 4) fringes in a beat. The uncertainty takes into account the incertitude in the beat nodes positions, a few irregularities in data registration and a drift of 0.5 fringes/h due to the change in room temperature during the measurements (revealed making the equipment work after decoupling the RMS, removing the belt).

The good fringes' definition and the regularity observed during the counting, suggest a faster procedure. Considering not all the data, but only some consequent intervals of 25 RMS turns (1/4 MS turn each), sampled randomly along the whole data set in figure 4(b), the number of fringes per MS turn can be obtained quite accurately. Increasing the number of these intervals this value does not change indeed significantly, while its standard deviation becomes rapidly smaller than the experimental incertitude, estimated in one fringe. With groups of 12 consecutive intervals we get in particular (85.4 ± 1.0) fringes per MS turn. Independently, the modulation of the fringes' visibility allows one to determine the MS turns versus the corresponding beat nodes numbers, as shown in figure 5. From the slope we get (2.885 ± 0.003) turns of the MS per beat, and therefore (986 ± 12) fringes per beat, in accordance with both the direct counting result and Fizeau's historical value[5].

[5] The fringe counting procedures can seem rough, but lead to good results. The data from figure 4(b) have been analyzed also with a FFT procedure, but since the ratio between the frequency difference $\Delta\nu$ and the average frequency ν of the signal is of the order of 1/986, our school's software was not able to resolve the two peaks. Also a direct data fit based on the model discussed in section 3 (considering also the partial coherence effect) gives unsatisfactory results because to obtain well separated fringes, above all where the amplitude of the oscillating light intensity is about zero, a sample rate of at least 25 Hz is needed, and our school's software was not able to manage the high amount of data.



(a)



(b)

Figure 4. Data registered by the light sensor with a sampling rate of 25 Hz: (a), example of a short time zoom of 50 s; (b), a longer data collection for the light intensity versus the number N of the RMS turns (sampling rate, 25 Hz): the markers have been set at the beat nodes positions to determine the beats length. Data are available for those interested.

3. Modeling, data analysis and results

It is not easy to describe mathematically the whole intensity distribution observed on the screen of the MI because the actual geometry of the observed interference pattern is strongly affected by the position and the inclination of the mirrors. However, since our light sensor collects the light on a small area (figure 2), we can consider the intensity distribution locally, ideally at a point P, where the total signal can be regarded as the incoherent superposition of two monochromatic interference patterns [10], each one produced by one of the two components of the Na-doublet. Indeed, assuming that the MI is balanced and that the position $x = x(t)$ of the mobile mirror M_b equals zero when the two MI arms are of equal length, for every value of x allowing interference (i.e. x smaller than the coherence lengths of the interfering signals), each of the two Na D-lines of brightness I_{0jP}, wave number $k_j = 2\pi/\lambda_j$ and coherence length l_j, contributes to the intensity at the

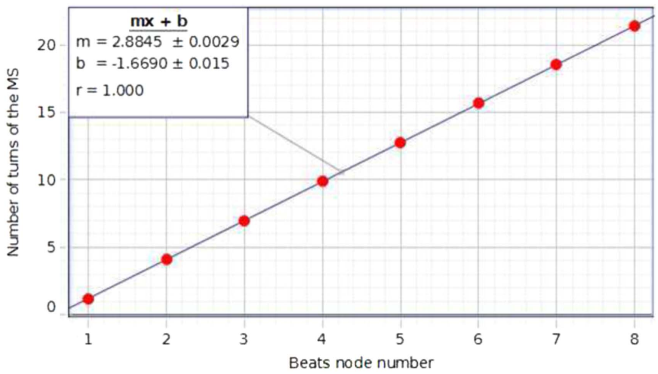


Figure 5. Number of turns of the micrometrical screw (MS) versus the beat nodes number: the slope allows one to determine the number of the fringes per beat.

point P with [11]:

$$I_{jP}(x) = I_{0jP}\cos^2(k_j x) - \frac{1}{2} I_{0jP}\frac{x}{l_j}\cos(2k_j x) \qquad j = 1, 2. \tag{1}$$

The first term describes the interference pattern produced in the ideal case by two perfectly coherent sources, while the second one arises from the normalized correlation function involved in cases of partial coherence [11], and is responsible for the general amplitude decrease in figure 4(b).

Since the coherence lengths of each component of our Na-source, determined experimentally[6], are both at least one order of magnitude greater than the maximal optical path difference used in our experiment, to explain the main features of the measured data we neglect the second term in equation (1) for both lines, assuming therefore that the total intensity at a point P is conveniently modeled by:

$$I_{\mathrm{tot},P} = I_{1P}(x) + I_{2P}(x) = I_{01P}\cos^2(k_1 x) + I_{02P}\cos^2(k_2 x), \tag{2}$$

so that only the time dependence of the position $x = x(t)$ of the mobile mirror $\mathrm{M_b}$ determines the time dependence of the light intensity. With this assumption, using standard trigonometry and omitting the subscript P, we expand equation (2) as:

$$\begin{aligned} I_{\mathrm{tot}}(x) &= \frac{I_{01}}{2}[1 + \cos(2k_1 x)] + \frac{I_{02}}{2}[1 + \cos(2k_2 x)] \\ &= \frac{I_{01} + I_{02}}{2} + \frac{I_{02}}{2}[\cos(2k_1 x) + \cos(2k_2 x)] + \frac{I_{01} - I_{02}}{2}\cos(2k_1 x) \\ &= \frac{I_{01} + I_{02}}{2} + \frac{I_{01} - I_{02}}{2}\cos(2k_1 x) + I_{02}\cos(k_1 x + k_2 x)\cos(k_1 x - k_2 x). \end{aligned} \tag{3}$$

Starting from the left in the last line, we have therefore a constant term, fixing the intensity value around which oscillations are observed; an oscillating term, determining the beat nodes amplitude (if the two components have the same brightness this term is indeed zero); a modulating term representing the beats. To show how this model explains the main features of our measurements, in figure 6(a), we represent separately each term, calculated assuming fictitious parameters values, while in 6(b), we show their sum, the only quantity we can directly measure.

[6] The coherence lengths l_1 and l_2 are determined from the positions and the FWHM of the D-lines in the spectrum of our Na-lamp, registered with the Czerny–Turner spectrograph (10 m focal—316 lines/mm) of the IRSOL (Istituto Ricerche Solari, Locarno, http://irsol.ch/?lan=en). We get lower limits $l_1 \geqslant 2.1$ cm for the D1 line and $l_2 \geqslant 2.7$ cm for the D2 line.

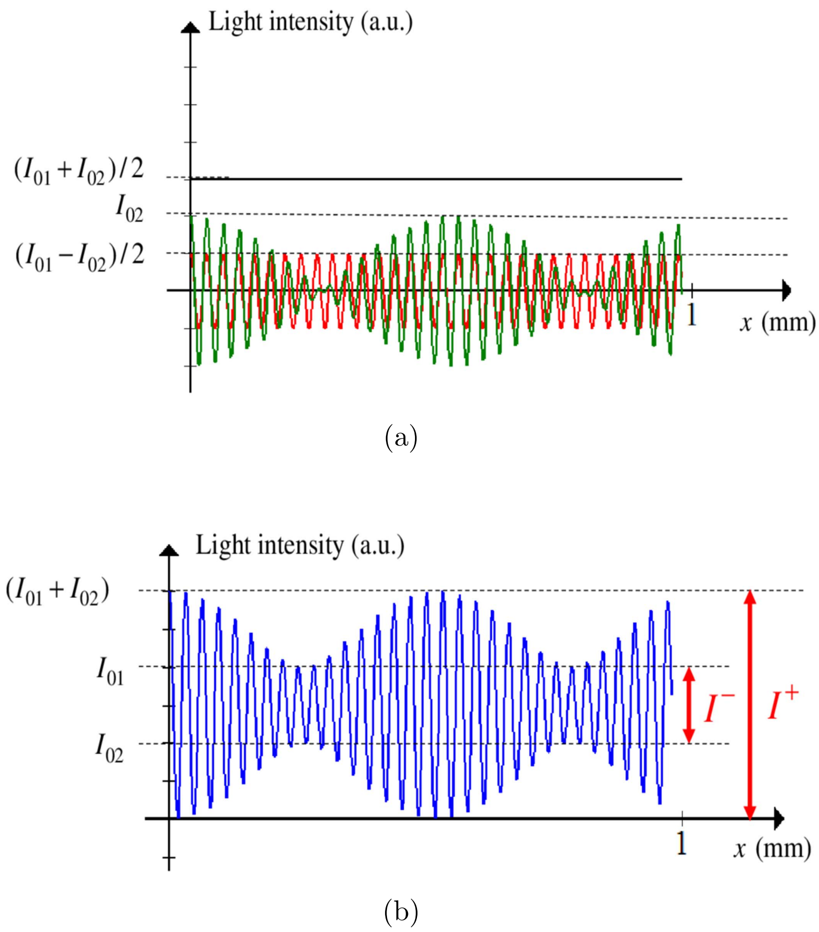


Figure 6. Qualitative illustration of the intensity model of equation (3), for $k_1 = 100\,\mathrm{rad/mm}$, $k_2 = 94\,\mathrm{rad/mm}$ and $I_{02}/I_{01} = 1/2$: (a) each term separately: constant (black), oscillating (red), modulating (green); 4(a), their sum. The quantities $I^+ = (I_{01} + I_{02})$ and $I^- = (I_{01} - I_{02})$ are respectively the maximal and minimal amplitude of the beats, which values are directly correlated to the fringes visibility (equation (7)). In this simplified model the intensity minimum reaches zero, unlike measurements, even if performed in a completely darkened room (figure 4(b)).

The separation $\Delta\lambda$ can be calculated from the modulating term. Indeed, since the two wavelengths in the Na D-doublet are very close in value, defining their mean value $\overline{\lambda} = (\lambda_1 + \lambda_2)/2$, we can approximate the modulating term as:

$$\cos(k_1 x - k_2 x) = \cos\left(2\pi \frac{\lambda_1 - \lambda_2}{\lambda_1 \lambda_2} x\right) \approx \cos\left(2\pi \frac{\Delta\lambda}{\overline{\lambda}^2} x\right). \tag{4}$$

Equation (4) means that measuring the (spatial) modulation period allows one to determine the doublet separation, i.e. the wavelength difference $\Delta\lambda$ of the two components. The mean value $\overline{\lambda}$ can be determined from the number of the fringes in a beat N_f, and the corresponding displacement Δx of the mirror M_b, calculated by the number of MS turns (100 μm each).

From the measured values and the interferometric relation between the number of fringes N_f and the mirror displacement Δx:

$$2\Delta x = N_f\, \lambda, \tag{5}$$

we then obtain $\overline{\lambda} = (585 \pm 6)$ nm, in accordance with the values reported in the literature [12] for the wavelengths measured in vacuum, respectively $\lambda_1 = 589.0$ nm and $\lambda_2 = 589.6$ nm. Finally, we get the doublet separation $\Delta\lambda$ considering the periodicity of the modulating term in equation (4):

$$\Delta\lambda = \frac{\overline{\lambda}^2}{\Delta x} = \frac{\overline{\lambda}}{N_f} = \frac{585 \pm 6}{986 \pm 8}\text{nm} = (0.59 \pm 0.01)\text{nm}, \tag{6}$$

in accordance with both the Fizeau's result and today's accepted value $\Delta\lambda = (0.597 \pm 0.001)$ nm measured in vacuum [13].

4. Didactical perspective and further developments

In this paper we show how to determine the separation $\Delta\lambda$ of the sodium D-doublet with a MI and lab equipment accessible to high schools. This experiment is of didactical interest because it allows students to use several aspects of wave theory in an intriguing application of interferometry. Since a long measurement time is required, only the experiment setup can be really presented in classroom. On the other hand, the measures, the data analysis and the results can be discussed in detail with the students.

Even if the main goal we looked for, i.e. the determination of the separation $\Delta\lambda$ of the sodium D-doublet, is achieved, it is possible to make students appreciate further investigations, recognizing how tools developed in the present experiment still work in more elaborate contexts[7].

For example we consider the actual beats modulation, connected to the difference in brightness of the two D-lines, as stressed before. Our model (equation (3)) suggests that the ratio of their intensities can be determined directly from the values of the maximal amplitude $I^+ = (I_{01} + I_{02})$ and the minimal amplitude $I^- = (I_{01} - I_{02})$ in the beats (figure 6(b)), getting:

$$\frac{I_{02}}{I_{01}} = \frac{I^+ - I^-}{I^+ + I^-}. \tag{7}$$

The measured values of consecutive beats minima I^- and maxima I^+ in figure 4(b), regardless of their position along the registered data[8], give a mean value for the ratio $I_{02}/I_{01} = 0.84 \pm 0.06$, a value significantly higher than the 0.50 reported in the literature [13]. Nevertheless, we confirmed our mean value experimentally, by a numerical integration on the precision D-lines profile obtained for our Na-lamp with the IRSOL spectrograph,

[7] Alternatively, in our education system such an activity could represent the experimental focus of a *lavoro di maturità*, a didactical activity which allows the students, with the help of teachers, to investigate and develop some issues of particular interest during the two last years of the high school, from a research perspective.

[8] As already pointed out, in the measured data the signal does not reach zero even after subtracting the background. We identified three possible causes for such a behavior: while the linear amplitude decrease (figure 4(b)) suggests that the partial coherence effect (equation (1)) is not entirely negligible, it is also necessary to consider the finite aperture of the sensor pinhole as well as a not ideal balancing of the MI. Each of these factors requires a detailed analysis that goes beyond the scope of this paper; but since each of them rescales the beats' amplitude symmetrically with respect to the value of the constant term in equation (3), one could argue that equation (7) should be still valid, provided we use the rescaled quantities.

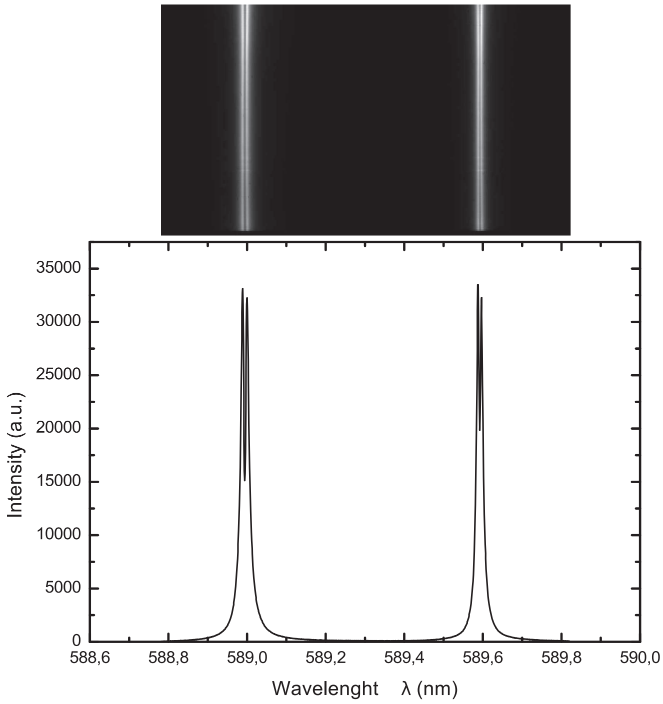


Figure 7. The observed spectrum and the precision D-line intensity profile obtained for our Na-lamp with the IRSOL spectrograph (see footnote 6).

which shows an evident self absorption effect in the two lines, though to a different extent [14][9]. This discrepancy between experimental and theoretical values becomes therefore an opportunity to show students how, at a certain precision level, consistent results analysis can rise a number of engaging issues which had not been expected initially.

Moreover, this experiment suggests other challenging extensions: the same method can be applied to also resolve the $H_\alpha - D_\alpha$ lines doublet of a hydrogen–deuterium mixture[10] (it should be $\Delta\lambda = 0.18$ nm), permitting one to visualize and quantify the effect of the finite nuclear mass of different isotopes in light spectra.

Acknowledgments

One of us (MD) is indebted to his retired colleague E Pfister, who introduced him to this fascinating experiment more than 35 years ago when he was a young theoretical physicist faced for the first time, as a teacher, with the challenge of making physics interesting to high school students. We are also grateful to our colleagues O Foà Häusermann for her assistance in taking the images of figure 3 and S. Prinz for his assistance in data analysis, as well as to GM Graf (ETH Zürich) for useful discussions and to M Bianda (IRSOL Locarno) for his assistance in the pinhole realization, for lending us the interferometric filter as well for his precious collaboration by recording and evaluating the spectrum of our Na-lamp.

[9] Self-absorption depends strongly on the experimental conditions of the Na gas (temperature, pressure, stability of the thermodynamical conditions), so a comparison with the values in the literature can be only indicative.
[10] This requires however a H-D-light source of sufficient intensity and stability.

Appendix

The goal of our paper is to encourage high school teachers to use the MI for experiments that without excessive efforts go beyond the classical measurements of wavelength or refraction index: if the necessary devices are already available at the school, it is also possible to easily organize an experiment like the determination of the separation of the Na-doublet. We therefore mention only as a special further development the possibility of using the same data to also determine the relative intensity of the two D-lines, since they are both very much influenced by a self absorption effect (strongly dependent on the lamp characteristics) in which each one is apparently doubled and four peaks appear. In our opinion, without a detailed discussion (requiring a separate paper) this behavior could be misunderstood, and students could be rather confused. However, since one of our reviewers strongly suggested sharing our experimental findings, figure 7 shows the observed spectrum and the precision D-line intensity profiles, both obtained with the IRSOL spectrograph (a facility certainly not available for most schools).

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