

Pendulum coupled to a spring

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Solution: The inertial moment is $I = ML^2$. The equation of motion is,

$$I\vec{\alpha} = \vec{\tau} = \vec{L} \times M\vec{g} + \vec{a} \times \vec{F}_{ml} .$$

Hence,

$$\begin{aligned} ML^2\ddot{\theta} &= -LMg \sin \theta - akx = -LMg \sin \theta - aka\theta \\ 0 &= \ddot{\theta} + \frac{g}{L} \sin \theta + \frac{ka^2}{ML^2} \theta \simeq \ddot{\theta} + \left(\frac{g}{L} + \frac{ka^2}{ML^2} \right) \theta \\ \omega_0 &= \sqrt{\frac{g}{L} + \frac{ka^2}{ML^2}} . \end{aligned}$$