# Resolution of the damped oscillator equation

*Philippe W. Courteille, 05/02/2021*

**Solution:** *To determine the frequency of the damped oscillation described by this equation, we choose the function,*

$$x(t) = Ae^{-\gamma t}\cos\omega t$$

*with the derivatives:*

$$\begin{aligned} v(t) &= -\gamma Ae^{-\gamma t}\cos\omega t - \omega Ae^{-\gamma t}\sin\omega t \\ a(t) &= (\gamma^2-\omega^2)Ae^{-\gamma t}\cos\omega t + 2\gamma\omega Ae^{-\gamma t}\sin\omega t\ . \end{aligned}$$

*Entering the differential equation and collecting the cos and sin terms separately:*

$$\begin{aligned} (\gamma^2-\omega^2)-\gamma\frac{b}{m}+\frac{k}{m} = 0 \qquad &and \qquad 2\gamma\omega-\omega\frac{b}{m} = 0 \\ \gamma = \frac{b}{2m} \qquad &and \qquad \omega = \sqrt{\gamma^2-\gamma\frac{b}{m}+\frac{k}{m}} = \sqrt{\omega_0^2-\frac{b^2}{4m^2}} \end{aligned}$$

*com* $\omega_0 \equiv \sqrt{k/m}$.