Oscillation with coercive force

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Solution: *The particular solution can be found by the ansatz* $x_p(t) = be^{-t/\tau}$ *with* $b = \beta\tau^2/(m - \gamma\tau + \kappa\tau^2)$. *Depending on whether* $\gamma^2 - 4\kappa m >, =, < 0$ *we have for a.* $\gamma^2 - 4\kappa m \equiv \Gamma^2 > 0$

$$x(t) = e^{-\frac{\gamma t}{2m}} \left(Ae^{-\frac{\Gamma t}{2m}} + Be^{\frac{\Gamma t}{2m}} \right) + \frac{\beta\tau^2 m\ e^{-\frac{t}{\tau}}}{(m - \frac{\gamma\tau}{2})^2 - \frac{\Gamma^2\tau^2}{4}}\ ;$$

for b. $\gamma^2 - 4\kappa m = 0$

$$x(t) = e^{-\frac{\gamma t}{2m}} (A + Bt) + \frac{\beta\tau^2 m\ e^{-\frac{t}{\tau}}}{(m - \frac{\gamma\tau}{2})^2}\ ;$$

for c. $\kappa m - \gamma^2 \equiv \Omega^2 > 0$

$$x(t) = e^{-\frac{\gamma t}{2m}} \left(A\cos\frac{\Omega t}{2m} + B\sin\frac{\Omega t}{2m} \right) + \frac{\beta\tau^2 m\ e^{-\frac{t}{\tau}}}{(m - \frac{\gamma\tau}{2})^2 + \frac{\Omega^2\tau^2}{4}}\ .$$

Through the boundary conditions $x(0) = 0$, $\dot{x}(0) = 0$ *we can determine* A, B. *Furthermore, we immediately see that* $x(\infty) = 0$, $\dot{x}(\infty) = 0$ *for all of the above mentioned solutions. Since the work of the two non-conservative forces equals the difference between the kinetic energy and the work for conservative forces, that is,* $A = 0$, *the entire work of* $F(t)$ *has been dissipated by* F_R.