Oscillation with coercive force

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Solution: a. The differential equation is,

 $m\ddot{x} + \gamma \dot{x} + Dx = mg - V\rho_{was}g + F_0 \cos \omega t \; .$

b. The resting position of the mass in air is $x_l = \frac{mg}{D} \simeq 0.98$ m and in water $x_w = z + \frac{m-V\rho_{was}}{D}g \simeq 0.57$ m. This ansatz leads to the known differential equation,

$$m\ddot{z} + \gamma\dot{z} + Dz = F_0\cos\omega t$$
.

c. The resonance frequencies with and without friction are related by,

$$\omega_w = \sqrt{\omega_0^2 - \frac{\gamma^2}{2m^2}} \; .$$

Hence, $\gamma = m\sqrt{2\omega_0^2 - 2\omega_w^2} \simeq 154 \text{ kg/s}.$