Lorentz model of light-atom interaction

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Solution: *The differential equation is,*

$$m\ddot{x} + m\Gamma\dot{x} + m\omega_0^2 x = e\mathcal{E}_0 e^{i\omega t} \ .$$

The ansatz $x \equiv x_0 e^{i\omega t}$ *leads to the oscillation amplitude,*

$$x_0(\omega) = \left| \frac{e\mathcal{E}_0}{m} \frac{1}{\omega_0^2 - \omega^2 - i\Gamma\omega} \right| \ .$$

For small detunings $\delta = \omega - \omega_0 \ll \omega$ *vale*

$$x_0(\omega) \simeq \frac{e\mathcal{E}_0}{m} \left| \frac{1}{2\omega_0(\omega_0 - \omega) + i\Gamma\omega_0} \right| = \frac{e\mathcal{E}_0}{m\omega_0} \left| \frac{1}{\delta - i\frac{\Gamma}{2}} \right| \ .$$

We also have,

$$\phi(\omega) = \frac{Im\ \alpha}{Re\ \alpha} = \frac{-\gamma\omega^2}{\omega^2 - \omega_0^2} \xrightarrow{\omega \to \omega_0} \frac{\gamma}{2\Delta} \ .$$