Spring-mass system

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Solution: *a. Since the stretch of the spring corresponds to* $y_0 = -\frac{mg}{k}$*, we can calculate the spring constant,*

$$k = \frac{mg}{-y_0} = 526\ N/m\ ,$$

and the oscillation frequency is,

$$\omega_0 = \sqrt{\frac{k}{m}}\ .$$

So we can describe the motion by the function,

$$y(t) = y_m \cos \omega_0 t + y_0\ .$$

The total energy at the upper turning point is,

$$E = E_{mol} + E_{grv} = \frac{k}{2}(y_m + y_0)^2 + mg(y_m + y_0) = -0.079\ J\ .$$

b. The gravitational potential energy at the lower turning point is

$$E_{grv} = mg(-y_m + y_0) = -0.74\ J\ .$$

c. The total potential energy at the lower turning point is equal to the total energy (a), because the kinetic energy disappears.
d. The instantaneous velocity being,

$$v(t) = -y_m \omega_0 \sin \omega_0 t\ ,$$

the maximum kinetic energy is,

$$E_{kin} = \frac{m}{2} y_m^2 \omega_0^2 = 0.13\ J\ .$$

The figure shows the energy conservation using the expressions of the formula (1.28). The red line shows the temporal transformation of kinetic energy, the green line of the spring, the blue of gravitation, the cyan of the potential, and black of the total energy.