Coupled springs

Philippe W. Courteille, 05/02/2021

Solution: *We choose the origin of the coordinate system at the center of the ring and the equilibrium position at* $\mathbf{r}$*. Then the forces are balanced holds* $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$*. Following the scheme the suspension points are at,*

$$\mathbf{a}_1 = R\begin{pmatrix} -\frac{1}{2}\sqrt{3} \\ \frac{1}{2} \end{pmatrix} \quad , \quad \mathbf{a}_2 = R\begin{pmatrix} \frac{1}{2}\sqrt{3} \\ \frac{1}{2} \end{pmatrix} \quad , \quad \mathbf{a}_3 = R\begin{pmatrix} 0 \\ -1 \end{pmatrix} ,$$

where $a_1 = a_2 = a_3$*. Hence,* $-k_1(\mathbf{a}_1 - \mathbf{r}) - k_2(\mathbf{a}_2 - \mathbf{r}) - k_3(\mathbf{a}_3 - \mathbf{r}) = 0$*. Finally we get for the components,*

$$x_0 = \frac{k_1 - k_2}{k_1 + k_2 + k_3}\frac{R}{2}\sqrt{3} \quad , \quad y_0 = \frac{k_1 + k_2 - 2k_3}{k_1 + k_2 + k_3}\frac{R}{2} .$$

The equations of motion are, letting the springs be equal, $k_1 = k_2 = k_3$*,*

$$m\ddot{x} = -k\left(x + \tfrac{R}{2}\sqrt{3}\right) - k\left(x - \tfrac{R}{2}\sqrt{3}\right) - kx = -3kx$$
$$m\ddot{y} = -k\left(y - \tfrac{R}{2}\right) - k\left(y - \tfrac{R}{2}\right) - k\left(y + R\right) = -3ky .$$