Energy of normal modes

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Solution: *We have,*

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\begin{aligned}
E_{tot} &= E_{kin,1} + E_{pot,1} + E_{kin,2} + E_{pot,2} + E_{cpl} \\
&= \tfrac{m}{2} v_1^2 + \tfrac{mg}{2L} x_1^2 + \tfrac{m}{2} v_2^2 + \tfrac{mg}{2L} x_2^2 + \tfrac{k}{2}(x_1 - x_2)^2 \\
&= \tfrac{m}{2}(L\dot{\theta}_1)^2 + \tfrac{mg}{2L}(L\theta_1)^2 + \tfrac{m}{2}(L\dot{\theta}_2)^2 + \tfrac{mg}{2L}(L\theta_2)^2 + \tfrac{k}{2}L^2(\theta_1 - \theta_2)^2 \\
&= \tfrac{m}{2}(L\dot{\Psi})^2 + \tfrac{m}{2} gL\Psi^2 + \tfrac{m}{2}(L\dot{\aleph})^2 + \tfrac{m}{2} gL\aleph^2 + kL^2\aleph^2 \\
&= \tfrac{m}{2}(L\dot{\Psi})^2 + \tfrac{m}{2}\omega_\Psi^2(L\Psi)^2 + \tfrac{m}{2}(L\dot{\aleph})^2 + \tfrac{m}{2}\omega_\aleph^2(L\aleph)^2 = E_\Psi + E_\aleph \ .
\end{aligned}
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