

Three coupled pendulums

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Solution: *In the example of three oscillators we have,*

$$\begin{pmatrix} \omega_0^2 + \beta^2 & \beta^2 & 0 \\ \beta^2 & \omega_0^2 + 2\beta^2 & \beta^2 \\ 0 & \beta^2 & \omega_0^2 + \beta^2 \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} = 0 .$$

The matrix eigenvectors and eigenvalues are:

$$\begin{aligned} (1, 1, 1) & \quad \text{for} \quad \omega^2 = \omega_0^2 \\ (1, 0, -1) & \quad \text{for} \quad \omega^2 = \omega_0^2 + \beta^2 \\ (1, -2, 1) & \quad \text{for} \quad \omega^2 = \omega_0^2 + 3\beta^2 . \end{aligned}$$