Super- and subradiance

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Solution: *a. The equations of motion of these three carts are,*

$$
\begin{aligned}
m\ddot{x}_1 &= -k(x_1 - x_2) - \gamma\dot{x}_1 \\
M\ddot{x}_2 &= -k(x_2 - x_1) - k(x_2 - x_3) - \Gamma\dot{x}_2 \\
m\ddot{x}_3 &= -k(x_3 - x_2) - \gamma\dot{x}_3 \ .
\end{aligned}
$$

b. Para $M \to 0$*,*

$$
\begin{aligned}
m\ddot{x}_1 &= -kx_1 + \tfrac{1}{2}kx_1 + \tfrac{1}{2}kx_3 - \tfrac{1}{2}\Gamma\dot{x}_2 - \gamma\dot{x}_1 \\
kx_2 &= \tfrac{1}{2}kx_1 + \tfrac{1}{2}kx_3 - \tfrac{1}{2}\Gamma\dot{x}_2 \\
m\ddot{x}_3 &= -kx_3 + \tfrac{1}{2}kx_1 + \tfrac{1}{2}kx_3 - \tfrac{1}{2}\Gamma\dot{x}_2 - \gamma\dot{x}_3 \ .
\end{aligned}
$$

Substituting,

$$
\begin{aligned}
m\ddot{x}_1 &= -\tfrac{1}{2}k(x_1 - x_3) - \tfrac{1}{2}\Gamma\dot{x}_2 - \gamma\dot{x}_1 \\
m\ddot{x}_3 &= -\tfrac{1}{2}k(x_3 - x_1) - \tfrac{1}{2}\Gamma\dot{x}_2 - \gamma\dot{x}_3 \ .
\end{aligned}
$$

Considering the normal modes,

$$
m\ddot{\Psi} = -\Gamma\dot{x}_2 - \gamma\dot{\Psi} \qquad , \qquad m\ddot{\aleph} = -k\aleph - \gamma\dot{\aleph} \ .
$$

Obviously, in the absence of dissipation γ *in the movement of the individual oscillators, we have two modes. One of the modes, called* Ψ*, is subject to dissipation* Γ *linked to the coupling between the oscillators. This mode is called superradiant, as it delivers its energy quickly to the environment. The other mode, called* $\aleph$*, is free of dissipation and is therefore called subradiant.*