Physical pendulum on a spiral spring

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Solution: *The retroactive force exerts a torque* $|\vec{M}| = |\vec{r} \times \vec{F}_R| = rD\phi$. *The torque due to the inertia of the mass is* $\vec{M}_T = \vec{I}\vec{\omega}$. *With that the equation of motion gets,*

$$I\ddot{\phi} + rF_R\phi = 0 \ .$$

The inertial moment of the beam is,

$$\begin{aligned} I_\omega &= \frac{1}{V}\int_{viga} \varrho(\mathbf{r})(y^2+z^2)d^3r = \frac{\varrho_0}{V}\int_{-c/2}^{c/2}\int_{-b/2}^{b/2}\int_{-a/2}^{a/2}(y^2+z^2)dxdydz \\ &= \frac{\varrho_0 abc}{12V}(a^2+c^2) = \frac{\varrho_0}{6}(a^2+c^2) \ , \end{aligned}$$

The rest is trivial.