Oscillation of a rolling cylinder

Philippe W. Courteille, 05/02/2021

Solution: *The inertial moment is* $I = \frac{M}{2}R^2$. *The springs exert the forces,*

$$F_{mol} = -kx + k(L - x) \ .$$

Hence we have the equations of motion,

$$M\ddot{x} = F_{mol} - F_{at}$$
$$I\ddot{\theta} = I\frac{\ddot{x}}{R} = dF_{mol} - RF_{at} \ .$$

Using $x = R\omega$ *and eliminating the friction, we get,*

$$I\ddot{\theta} = I\frac{\ddot{x}}{R} = \frac{M}{2}R^2\frac{\ddot{x}}{R} = dF - RF_{at} = d[-kx + k(L - x)] - R[-kx + k(L - x) - M\ddot{x}] \ .$$

Solving by $\ddot{x}$,

$$\ddot{x} + \frac{2k}{M}\left(1 + \frac{d}{R}\right)(2x - L) = 0 \ .$$

The frequency is,

$$\omega_0 = \sqrt{\frac{4k}{M}\left(1 - \frac{d}{R}\right)} \ .$$