Rotational oscillation of a disk

Philippe W. Courteille, 05/02/2021

Solution: *The angular acceleration of the disc M the inertial moment of which is $I = \frac{M}{2}R^2$, is given by,*

$$\frac{M}{2}R^2\vec{\alpha} = I\vec{\alpha} = \vec{\tau}_m + \vec{\tau}_k = \mathbf{R} \times m\mathbf{g} - \mathbf{R} \times k\mathbf{x} = (Rmg - Rkx)\hat{\mathbf{e}}_\theta \ .$$

As the body does not slide, the angle of the disc is linked to the mass displacement, $x = R\theta$. We get,

$$\alpha = \ddot{\theta} = \frac{2mg - 2kx}{MR} = \frac{2mg}{MR} + \frac{2k}{M}\theta \ .$$

Substituting $\theta \equiv \tilde{\theta} - \frac{mg}{kR}$:

$$\ddot{\tilde{\theta}} = \frac{2mg}{MR} - \frac{2k}{M}\left(\tilde{\theta} - \frac{mg}{kR}\right) = -\frac{2k}{M}\tilde{\theta} \ .$$

That is, we have a harmonic oscillation around the angle $\theta_0 = \frac{mg}{kR}$ with frequency $\omega_0 = \sqrt{\frac{2k}{M}}$.